وزارة التعليم العالي والبحث العلمي
الجامعة التقنية الجنوبية
المعهد التقني العما رة
قسم التقنيات الكهربائية

الحقيبة التدريسية لمادة

الدوائر الكهربائية والقياسات /2

المرحلة الاولى

الفصل الدراسي الثاني

اع داد
شويكار محمود جاس م
رئيس مهندسين اقدم

اسم المادة	لغة التدريس	السنة الدراسية	الفصل الدراسي	الفرع	الساعات الاسبوعية			الوحدات
					ن	ع	م	
الدوائر الكهربائية والقياسات ٢	الانكليزية	الأولى	الثاني	جميع الفروع	٢	٢	٤	٤

المفردات

الاسبوع	تفاصيل المفردات
الاول	تطبيق النظريات كنظرية نورتن ونظرية ثفنن والتطابق على دوائر التيار المتناوب مع حل امثلة
الثاني	القدرة في دوائر التيار المتناوب ويشمل حساب القدرة في – دوائر تحتوي على مقاومة فقط – دوائر تحتوي على محاثة فقط – دوائر تحتوي على متسعة فقط – دائرة تحتوي على مقاومة ومحاثة ومتسعة على التوالي والتوازي – تعريف القدرة الفعالة وكيفية حسابها – القدرة غير الفعالة وكيفية حسابها
الثالث	القدرة الظاهرية الكلية (تعريفها) – كيفية رسم مثلث القدرة – معامل القدرة – تعريفه وتأثيره على دوائر التيار المتناوب – كيفي تحسين معامل القدرة – مع امثلة تطبيقية
الرابع	نظرية نقل اعظم قدرة ممكنة في دوائر التيار المتناوب – اشتقاق العلاقات الخاصة بها – مع امثلة
الخامس	الطرق العملية في قياس المقاومات ذات القيم العالية والمتوسطة والصغيرة – باستخدام الاوميتر في حالة التوالي والتوازي – طريقة الاميتر والفولتميتر – طريقة التعويض – باستخدام قنطرة ويتستون – طرقة مقسم الجهد – طريقة التبديل – مع حل امثلة على كل طريقة
السادس	دوائر التيار المتناوب ذات ثلاثة اطوار – تعريفه وكيفية توليد تيار متناوب طور واحد – طورين – ثلاثة اطوار – مع رسم كل دائرة توصيلات الشكر النجمي والمثلثي في دوائر التيار المتناوب ذات ثلاثة اطوار والعلاقات الخاصة لحساب تيار وفولتية الخط والطور والقدرة الكلية وقدرة الخط – قدرة الطور – مميزات كل ربط عند استخدامه في الاحمال المتزنة وغير المتزنة مع حل امثلة
السابع	حل امثلة تطبيقية حول التيار المتناوب ذو ثلاثة اطوار وبالتوصيلات المثلثي والنجمي مع الاحمال المتزنة وغير المتزنة
الثامن	طرق قياس القدرة للأحمال ذات ثلاثة اطوار – جهاز الواطميتر كيفية ربطه بالدائرة لقياس القدرة الفعالة – وحساب القدرة غير الفعالة والقدرة الظاهرية مع حل مثال

	قياس القدرة باستخدام واطميتر وجهد – كيفية ايجاد القدرة الكلية بهذه الطريقة وفي حالة التوصيل النجمي والمثلثي – باستخدام واطميترين – استخدام ثلاثة واط ميترات
التاسع	المغناطيسية – الدائرة المغناطيسية – مقدمة عن المغناطيسية القطب الشمالي والجنوبي – انواع المواد المغناطيسية – الصفات الاساسية للمواد المغناطيسية وتعريفها وتشمل المجال المغناطيسي – الفيض المغناطيسي – القوة الدافعة المغناطيسية – كثافة الفيض المغناطيسي والعوامل التي تؤثر على الفيض المغناطيسي – النفاذية وتأثيرها– الدوائر المغناطيسية وتطبيق قوانين كيرشوف عليها
العاشر	حل امثلة تطبيقية على المغناطيسية
الحادي عشر	الحث الذاتي للملف (الحث الكهرومغناطيسي) – تعريفه – العلاقات الخاصة لايجاد الحث الذاتي للملف – الحث المتبادل بين ملفين – والعلاقات لايجاد الحث المتبادل وحسب نوعية ربط الملفين ويشمل : ربط توالي تعاضدي وتعاكسي
الثاني عشر	منحنيات نمو واضمحلال التيار من الدائرة الحثية – شرح هذه الدائرة وتأثيرها في التيار المستمر – العلاقة العامة لنمو واضمحلال التيار في الملف – رسم التيار وحساب ثابت الزمن – حل امثلة شحن وتفريغ المكثفات ويشمل استخدام المتسعة في دوائر التيار المستمر العلاقة العامة لشحن وتفريغ المكثف ورسم التيار – تأثير ثابت الزمن مع حسابه – حل امثلة
الثالث عشر	اجهزة القياس وتشمل – انواع اجهزة القياس – طبيعة عملها – اجهزة القياس ذات الملف المتحرك – تركيبه واستخدامه في قياس الفولتية والتيار مع ذكر مميزاته وعيوبه ورسم الجهاز
الرابع عشر	جهاز القياس ذو القلب الحديدي – تركيبه وكيفية استخدامه في القياس – مميزاته وعيوبه ورسم مخطط الجهاز
الخامس عشر	اجهزة القياس الواط ميتر – تركيبه – رسم مخطط الجهاز – ترتيبه في الدائرة الكهربائية لقياس القدرة – معادلات العزوم – مميزاته – عيوبه – جهاز الاوسلسكوب – رسم الجهاز – تركيبه – كيفية تشغيله واستخدامه

Generation of A.C Voltage

If coil rotate in magnetic field or magnetic field rotate inside the coil there is an alternating e. m. f. generate in the coil. The generated emf is proportional to the number of turns of coil, magnetic field strength, and the angle between the coil and magnetic field.

$$e = BLv \sin\theta$$

From this:

$$L = Length\ of\ the\ conductor.$$

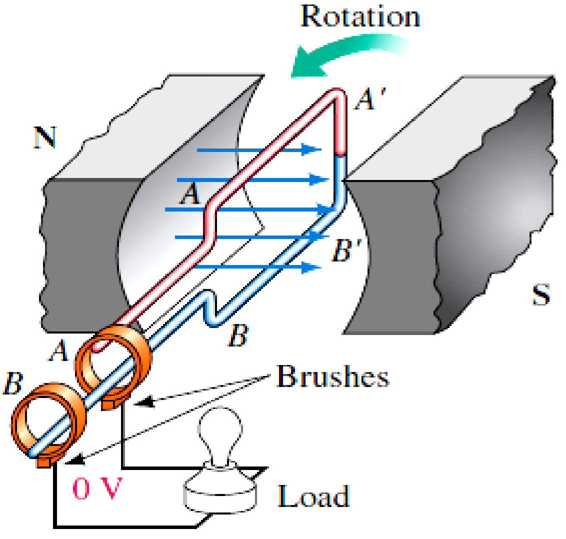
$$v = Velocity\ of\ conductor.$$

$$B = Flux\ density.$$

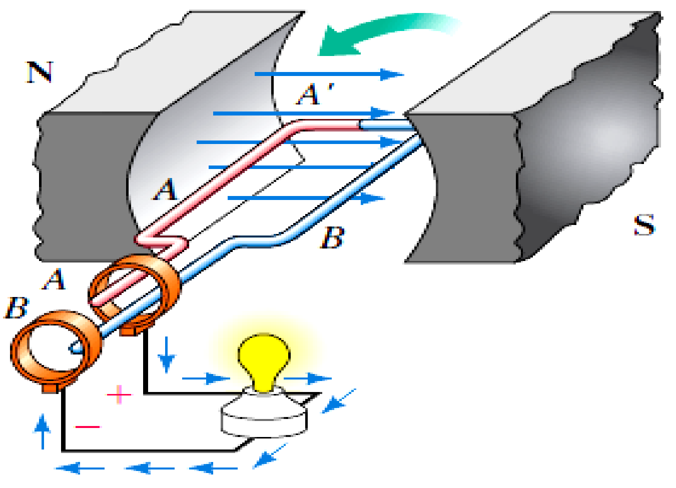
$$\theta = angle\ between\ field\ to\ conductor.$$

$$e = generated\ AC\ emf$$

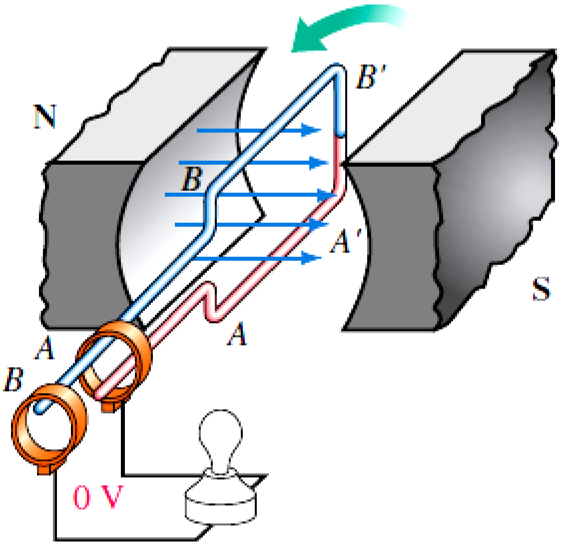
The generated AC emf value is depending upon the sine value of the angle between the magnetic field and conductor.



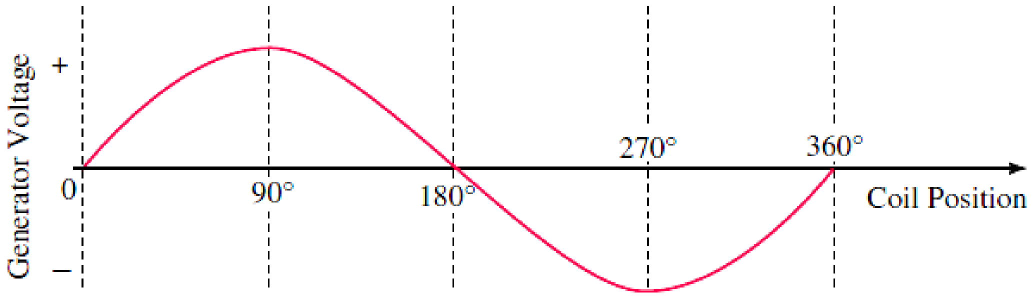
(a) 0° Position: Coil sides move parallel to flux lines. Since no flux is being cut, induced voltage is zero.



(b) 90° Position: Coil end *A* is positive with respect to *B*. Current direction is out of slip ring *A*.

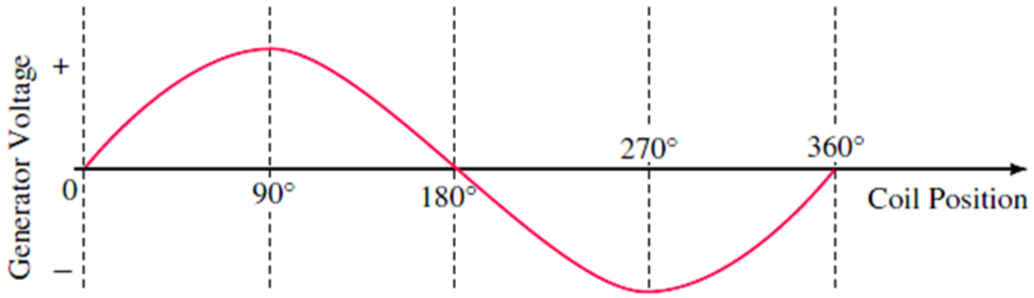


(c) 180° Position: Coil again cutting no flux. Induced voltage is zero.



Sinusoidal Alternating waveforms

Waveform : the path traced by aquantity such as the voltage in figure (1)



Cycle:

An alternating current complete set of one positive half cycle and one negative half cycle is called one cycle.

Time period:

The time taken by an alternating quantity to complete one cycle is called time period. It is denoted by the letter "T".

Frequency:

The number of cycle per second is called the frequency of the alternating quantity. The unit is hertz (Hz).

$$f = \frac{1}{T}$$

Instantaneous value:

The alternating quantity changes at every time.

$$v = V_{max} \sin \omega t \quad or \quad i = I_{max} \sin \omega t$$

Maximum value:

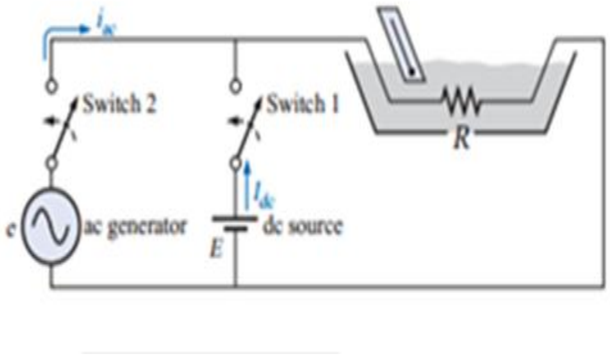
The maximum value positive or negative of an alternating quantity is known as its maximum value. Denoted by "I_{max} or V_{max}".

Effective value and RMS value:

The effective value of an alternating current is given by that DC current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time also is called root mean square value RMS. The voltmeter and ammeter are read the effective value only.

$$RMS\ value = \frac{I_{max}}{\sqrt{2}} \quad or \quad \frac{V_{max}}{\sqrt{2}} = 0.707 V_{max}$$

EFFECTIVE (rms) VALUES



$$I_{eq(dc)} = I_{eff} = 0.707 I_m$$

$$I_m = \sqrt{2} I_{eff} = 1.414 I_{eff}$$

$$E_{eff} = 0.707 E_m$$

Average value:

The average value is calculated by the averages of the maximum value of alternating quantity at different instances.

$$\boldsymbol{Average\ value = \frac{2I_{max}}{\pi} \quad or \quad \frac{2V_{max}}{\pi}}$$

***Example 1**:find the period of aperiodic waveform with afrequency of 60 Hz*

Solution

$$T = \frac{١}{f} = \frac{١}{٦٠} = ٠.٠١٦٦٧□ \ or\ ١٦.٦٧□□$$

***Example2**:determine the afrequency of the waveform of figure*

Solution

From the figure T=(25ms ─ 5ms)=20ms

$$f = \frac{١}{T} = \frac{١}{٢٠ \times ١٠^{-٣}} = ٥٠\ HZ$$

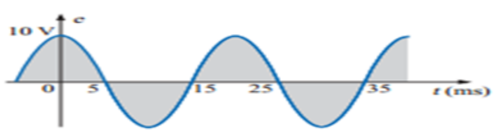


FIG.
Example .

The sine wave

$$\boldsymbol{Radian} = \left(\frac{\pi}{180°}\right) \times (\boldsymbol{degree})$$

$$\boldsymbol{Degree} = \left(\frac{180°}{\pi}\right) \times (\boldsymbol{Radian})$$

The velocity with witch the raduis vector rotates about the center called the angular velocity , can be determined from the following equation :

$$\boldsymbol{Angular\ velocity\ (\omega)} = \frac{\boldsymbol{distance\ (degree\ or\ radian)}}{\boldsymbol{time\ (second)}}$$

***Example* 3: determine the angular velocity of the sine wave having afrequency of 60 hertz (H_Z)**

Soltion : $\boldsymbol{\omega = 2\pi f = 2 \times 3.14 \times 60 = 377 \frac{rad}{s}}$

General format for sinusoidal voltage and current

The sinusoidal waveform is $\boldsymbol{A_m \sin \alpha}$

For electrical quantity such as current and voltage

$$\boldsymbol{i = I_m \sin \alpha = I_m \sin(\omega t)}$$

$$\boldsymbol{v = V_m \sin \alpha = V_m \sin(\omega t)}$$

Example 4*: *An alternating current varying sinusoidally with afrequency of 50 (H_Z) has RMS value of 20A , write down the equation for the instantaneous value and find this value a) 0·0025second , b) 0·0125second , c) what the instantaneous current be 14.14A .

Soltion :

$$\boldsymbol{I_m = \sqrt{2} \times RMS\ value = 20\sqrt{2} = 28 \cdot 2A}$$

$$\boldsymbol{\omega = 2\pi f = 2\pi \times 50 = 100\pi \frac{rad}{s}}$$

$$\boldsymbol{i(t) = I_m \sin(\omega t) = 28 \cdot 2 \sin(100\pi t)\ A}$$

a)

$$i(0\cdot 0025) = I_m\, sin(\omega t)$$
$$= 28\cdot 2\, sin(100\pi t) = 28\cdot 2\, sin(100\times 180\times 0\cdot 0025) = 19\cdot 94\ \ A$$

b)

$$i(0\cdot 0125) = I_m \sin(\omega t)$$
$$= 28\cdot 2\sin(100\pi t) = 28\cdot 2\sin(100\times 180\times 0\cdot 0125) = -19\cdot 94\ \ A$$

c)

$$i(t) = 14\cdot 14 = 28\cdot 2\, sin(100\pi t) = 28\cdot 2\, sin(100\times 180\times t)$$

$$\frac{14\cdot 14}{28\cdot 2} = 0\cdot 501 = sin(100\times 180\times t)$$

$$\sin^{-1}(0\cdot 501) = 100\times 180\times t \qquad \Rightarrow t = \frac{sin^{-1}(0\cdot 501)}{100\times 180} = \frac{30}{18000} = 0\cdot 00166s$$

***Example 5**: An alternating current of frequency 50 hertz (H_Z) has amaxhmum value of 100A calculate, a)its write down the equation for the instantaneous value and find this value a) $\frac{1}{600}$ s , b) what the instantaneous current be 86.6A .*

Soltion : $I_m = 100A$

$$\omega = 2\pi f = 2\pi\times 50 = 100\pi \frac{rad}{s}$$

$$i(t) = I_m \sin(\omega t) = 100\sin(100\pi t)\ \ A$$

$$i\left(\frac{1}{600}\right) = 100\, sin\left(100\times 180\times \frac{1}{600}\right) = 50\ \ A$$

b) $i(t) = 86\cdot 6 = 100\, sin(100\pi t) = sin(100\times 180\times t)$

$$\frac{86\cdot 6}{100} = 0\cdot 866 = sin(100\times 180\times t)$$

$$\sin^{-1}(0\cdot 866) = 100\times 180\times t \qquad \Rightarrow t = \frac{sin^{-1}(0\cdot 866)}{100\times 180} = \frac{60}{18000} = \frac{1}{300}s$$

PHASE RELATIONS

$$A_m \sin(\omega t \pm \theta)$$

where θ is the angle in degrees or radians that the waveform has been shifted.

If the waveform passes through the horizontal axis with a *positive-going* (increasing with time) slope *before* 0°, as shown in Fig. the expression is

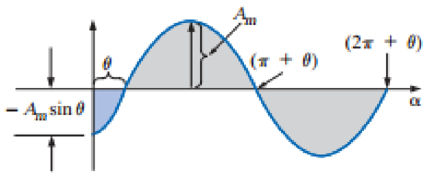
$$A_m \sin(\omega t + \theta)$$

$$\sin(\omega t + 90°) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

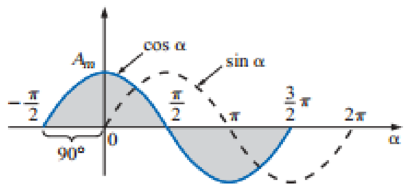
$$\sin \omega t = \cos(\omega t - 90°) = \cos\left(\omega t - \frac{\pi}{2}\right)$$

$$\begin{aligned} \cos \alpha &= \sin(\alpha + 90°) \\ \sin \alpha &= \cos(\alpha - 90°) \\ -\sin \alpha &= \sin(\alpha \pm 180°) \\ -\cos \alpha &= \sin(\alpha + 270°) = \sin(\alpha - 90°) \\ &\text{etc.} \end{aligned}$$

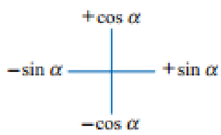
$$\begin{aligned} \sin(-\alpha) &= -\sin \alpha \\ \cos(-\alpha) &= \cos \alpha \end{aligned}$$



Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope after 0°.



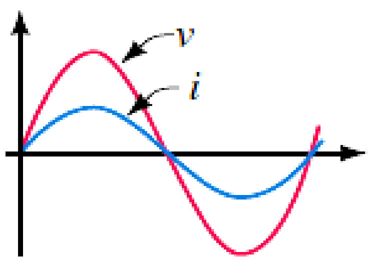
Phase relationship between a sine wave and a cosine wave.



Graphic tool for finding the relationship between specific sine and cosine functions.

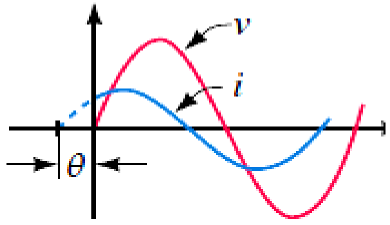
In phase:

If waveform of two AC quantities (voltage or current) get the maximum and zero at same time then they are said to be in phase.



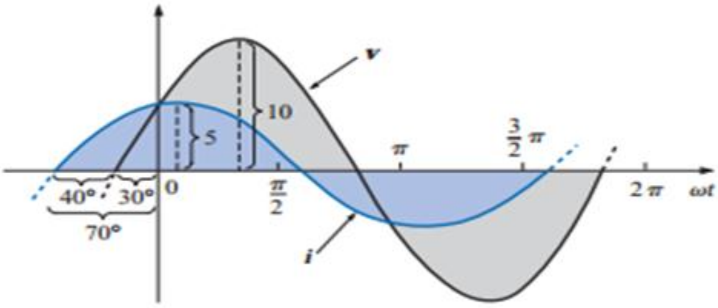
Out of phase:

If in AC circuit two quantities namely voltage or current waves get the maximum and zero at different value then they are said to be out of phase.



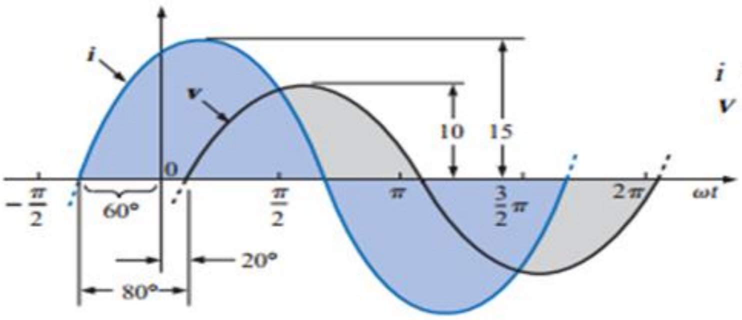
Current leads

i **leads** ***v*** **by 40°, or** ***v*** **lags** ***i*** **by 40°.**

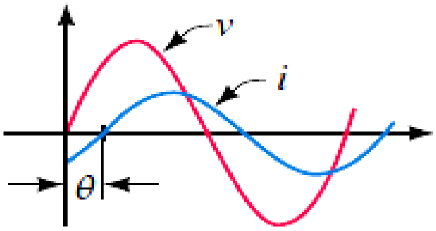


$v = 10 \sin(\omega t + 30°)$
$i = 5 \sin(\omega t + 70°)$

i **leads** ***v*** **by 80°, or** ***v*** **lags** ***i*** **by 80°.**



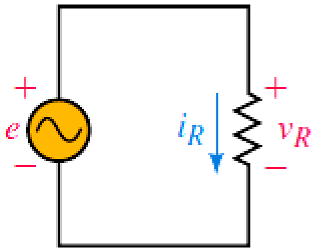
$i = 15 \sin(\omega t + 60°)$
$v = 10 \sin(\omega t - 20°)$

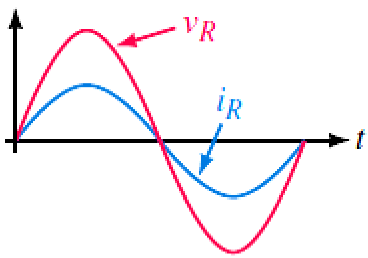


Current lags

AC circuit with pure resistance:

A circuit without inductance and capacitance is called pure resistance circuit as shown in figure.



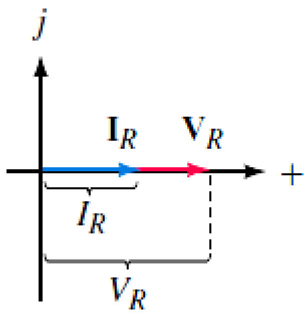


By applying Ohm's law:

$$i_R = \frac{v_R}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R}\sin \omega t = I_m \sin \omega t$$

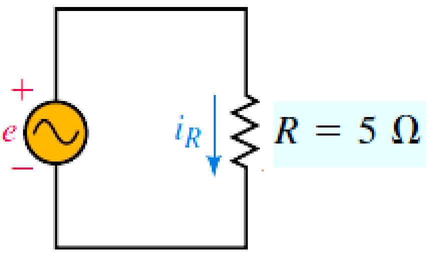
where $I_m = V_m/R$

Transposing, $V_m = I_m R$



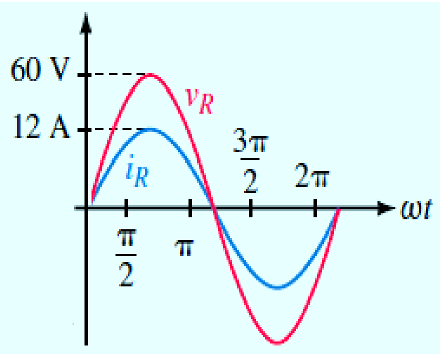
Note that current and voltage as in phase:

Example-6: For the circuit shown in figure find the value of i_R *, if* *v(t)=60sinωt.*

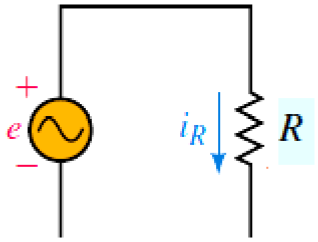


Solution:

$$i_R = \frac{v}{R} = \frac{60\sin(\omega t)}{5} = 12\sin(\omega t)\ A$$



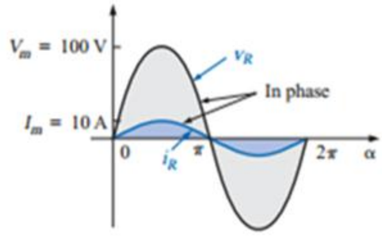
***Example-7**: For the circuit shown in figure if R= 10Ω, find the sinusoidal expression for the current if :-*



d) $v(t) = 100\sin(377t)$

e) $v(t) = 25\sin(377t + 60°)$

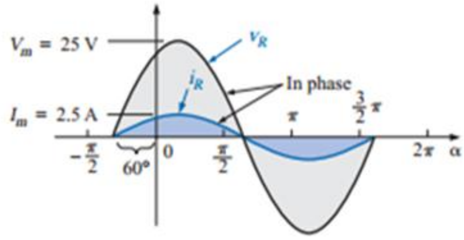
Solution:a) $I_m = \frac{V_m}{R} = \frac{25}{10} = 10\text{A}$



$i(t) = 10\sin(377t + 60°)$

b) $I_m = \frac{V_m}{R} = \frac{100}{10} = 2 \cdot 5\text{A}$

i (t)=2·5 sin (377t+60°)

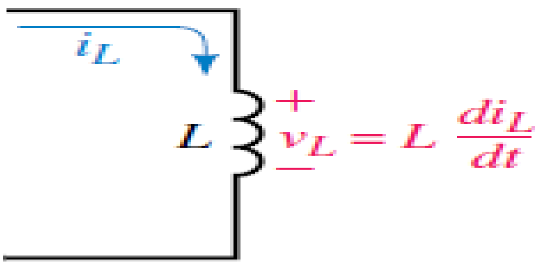


AC circuit with Purely Inductive Load:

Consider a purely inductive circuit with an inductor connected to an AC generator, as shown in Figure.

$$v_L = L\frac{di_L}{dt} = L\frac{d}{dt}(I_m \sin\omega t) = \omega L I_m \cos\omega t = V_m \cos\omega t$$

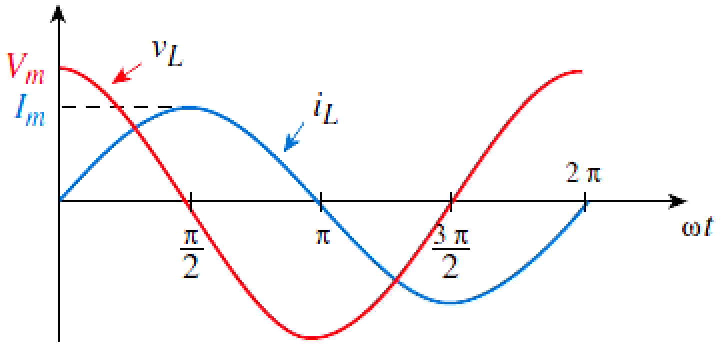
Utilizing the trigonometric identity [$\cos\omega t = \sin(\omega t + 90^{\circ})$you can writethis as:

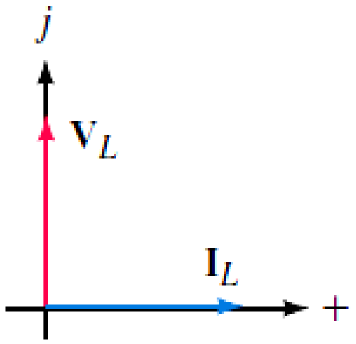


$$v_L = V_m \sin(\omega t + 90°)$$

$$V_m = \omega L I_m$$

Voltage and current waveforms are shown in Figure (a), and phasors inFigure(b). As you can see, *for a purely inductive circuit, current lagsvoltage by 90°*





From Equation above, we see that the ratio V_m to I_m is:

$$\frac{V_m}{I_m} = \omega L$$

This ratio is defined as inductive reactance and is given the symbol *XL*.Since the ratio of volts to amps is ohms, reactance has units of ohms.

$$X_L = \frac{V_m}{I_m} \quad (\Omega)$$

Combining Equations above yields:

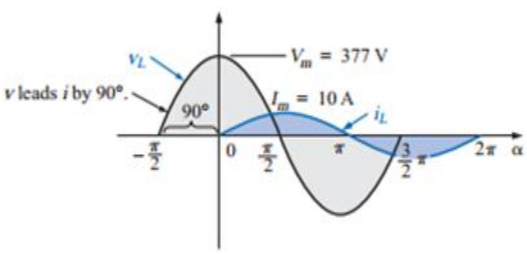
$$X_L = \omega L \quad (\Omega)$$

Example-8*: *find the sinusoidal expression for the voltage across the coil (0·1 H) if the current through the coil is :-

A)

$$V_m = I_m X_L = 10 \times 37 \cdot 7 = 377\ V$$

$$X_L = \omega L = 377 \times 0 \cdot 1 = 37 \cdot 7 \Omega$$



$$V_m = I_m X_L = 10 \times 37 \cdot 7 = 377\,V$$

The voltage is leads the current by 90°

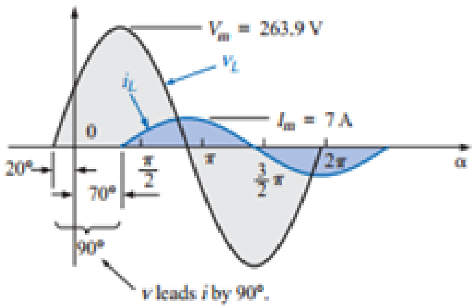
$$v(t) = 377 \sin(377t^\circ + 90)$$

B)

$$V_m = I_m X_L = 7 \times 37 \cdot 7 = 263 \cdot 9\,V$$

$$X_L = \omega L = 377 \times 0 \cdot 1 = 37 \cdot 7\Omega$$

$$V_m = I_m X_L = 7 \times 37 \cdot 7 = 263 \cdot 9\,V$$



The voltage is leads the current by 90°

$$v(t) = 263 \cdot 9 \sin(377t^\circ - 70 + 90) = 263 \cdot 9 \sin(377 + 20)$$

Where ω is radians per second $\omega = 2\pi f$

Example 9: ***The voltage across a 0.2H inductance is v_L = 100 sin(400t - 70°) V. Determine i_L and sketch it***

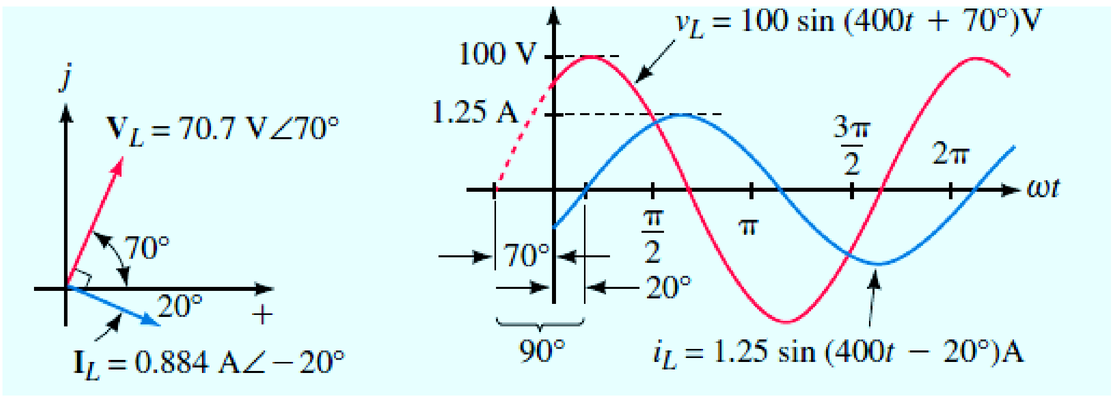
Solution:

ω = 400 rad/s therefore $x_L = \omega L = 400 \times 0.2 = 80\Omega$

$$I_m = \frac{V_m}{x_L} = \frac{100}{80} = 1.25A$$

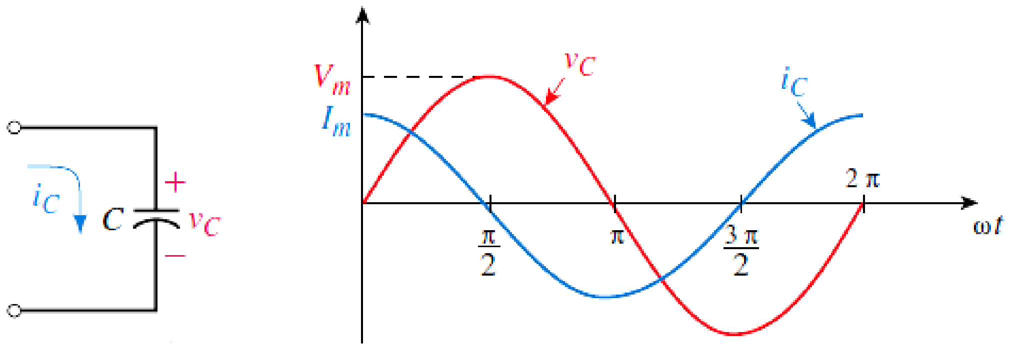
The current lags the voltage by 90° therefore:

$i_L = 1.25 \sin(400t - 20^\circ)$ **A as indicated in figure below**



AC circuit with Purely Capacitive Load:

In the purely capacitive case, both resistance R and inductance L are zero. The circuit diagram is shown in Figure



For capacitance, current is proportional to the rate of change of voltage:

$$i_C = C\frac{dv_C}{dt} = C\frac{d}{dt}(V_m \sin \omega t) = \omega C V_m \cos \omega t = I_m \cos \omega t$$

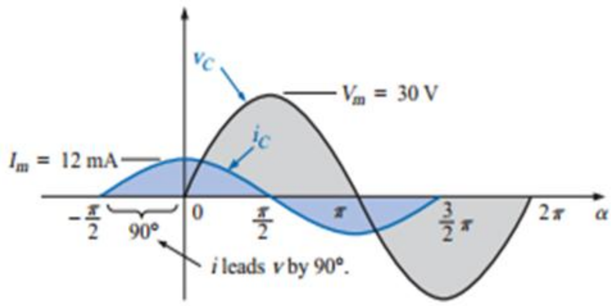
Using the appropriate trigonometric identity, this can be written as:

$$i_C = I_m \sin(\omega t + 90°)$$

Where $I_m = \omega C V_m = \frac{V_m}{X_c}$

X_c is called the capacitance reactance

***Example10:** The voltage across a 1-μf capacitor is $V_C = 30 \sin(400t)$ V. Determine i_c and sketch it with V_C*



Solution: $\omega = 400$ **rad/s therefore** $X_C = \frac{1}{\omega C} = \frac{1}{400\times1\times10^{-6}} = 2500\Omega$

$$I_m = \frac{V_m}{x_C} = \frac{30}{2500} = 0\cdot0120A = 120mA$$

The current leads the voltage by 90° therefore:

$i_c = 0\cdot012 \sin(400t + 90)$ **A as indicated in figure below**

Example 11: For the following pairs voltages and currents ,determine whether the element involved is acapacitor , an inductance or resistor and determine the value of C , L or R

E) $v = 100\sin(\omega t + 40°)$

$i = 20\sin(\omega t + 40°)$

F) $v = 1000\sin(377t + 10°)$

$i = 20\sin(377t - 80°)$

G) $v = 500\sin(157t + 30°)$

$i = 1\sin(157t + 120°)$

H) $v = 50\cos(\omega t + 20)$

$i = 5\sin(\omega t + 110°)$

Solutio:

A) $v = 100\sin(\omega t + 40°)$

$i = 20\sin(\omega t + 40°)$ **since V and I in phase, the element is R**

$R = \frac{V_m}{I_m} = \frac{100}{20} = 5\Omega$

B) $v = 1000\sin(377t + 10°)$

$i = 20\sin(377t - 80°)$ **since V leads I by 90^0, the element is L**

$X_L = \frac{V_m}{I_m} = \frac{1000}{5} = 200\Omega$

$$X_L = \omega L \Rightarrow L = \frac{X_L}{\omega} = \frac{200}{377} = 0 \cdot 531H$$

C) $v = 500\sin(157t + 30°)$

$i = 1\sin(157t + 120°)$ **since I leads V by 90^0, the element is C**

$$X_C = \frac{V_m}{I_m} = \frac{500}{1} = 500\Omega$$

$$X_C = \frac{1}{\omega L} = \frac{1}{377 \times C} \Rightarrow C = \frac{1}{377 \times 500} = 12 \cdot 74\mu F$$

D) $v = 50\cos(\omega t + 20) = 50\ \sin(\omega t + 20 + 90) = 50\sin(\omega t + 110°)$

$i = 5\sin(\omega t + 110\ °)$ **since V and I in phase, the element is R**

$R = \frac{V_m}{I_m} = \frac{50}{5} = 5\Omega$

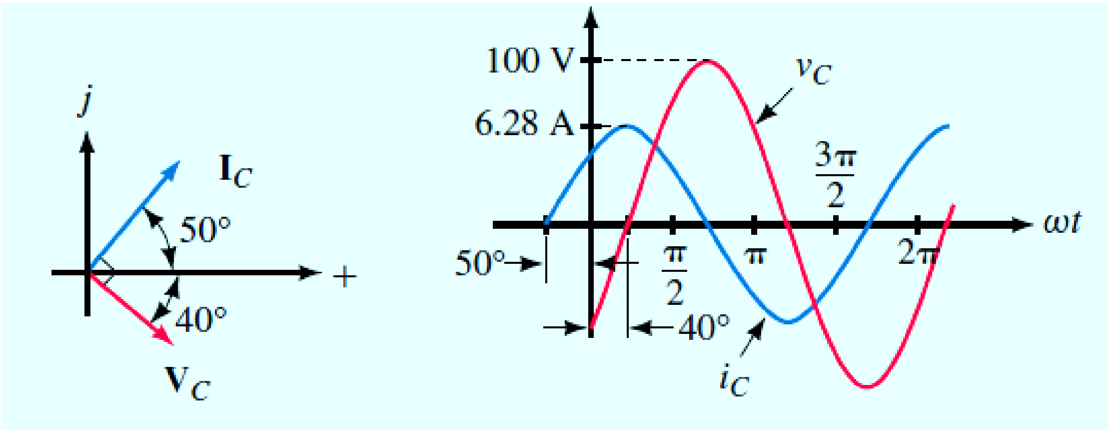
***Example-12**: The voltage across a 10-mF capacitance is v_C= 100sin(ωt -40°) V and f = 1000 Hz. Determine i_C and sketch its waveform.*

Solution:

$$X_C = \frac{1}{\omega L} = \frac{1}{2\pi f C} = \frac{1}{2 \times 3\cdot 14 \times 1000 \times 10 \times 10^{-3}} = 0\cdot 015\Omega$$

$I_m = \frac{V_m}{X_C} = \frac{100}{0\cdot 015} = 6\cdot 667A$

Since current leads voltage by 90°, i_C= 6.28 sin(6283*t* + 50°) A as indicatedin Figure below.



Impedance:

The opposition that a circuit element presents to current in the phasor domain is defined as its impedance.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{(ohms)}$$

For resistor:

$$\mathbf{Z}_R = R$$

For inductance:

$$\mathbf{Z}_L = j\omega L = jX_L$$

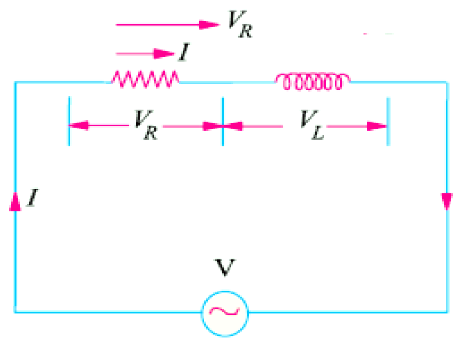
For capacitance:

$$\mathbf{Z}_C = -j\frac{1}{\omega C} = -jX_C \quad \text{(ohms)}$$

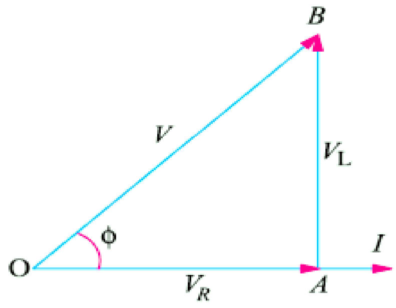
Series AC circuit:

AC through Resistance and Inductance:

A pure resistance R and inductance L are shown in figure(a) is connected in series.



Figure(a)



Figure(b)

Let V=r.m.s. value of the applied voltage; I=r.m.s. value of the resultant current.

$$V_R = IR \quad \textit{voltage drop across } R(\textit{in phase with } I)$$

$$V_L = I \times X_L \quad \textit{voltage drop across inductance } (\textit{lag of } I \textit{ by } 90)$$

The applied voltage V is:

$$V = \sqrt{V_R^2 + V_L^2} \qquad or \;\; V = I \times Z$$

$$\textit{where } Z \textit{ is the impedance of the circuit}$$

$$Z = \sqrt{R^2 + X_L^2}$$

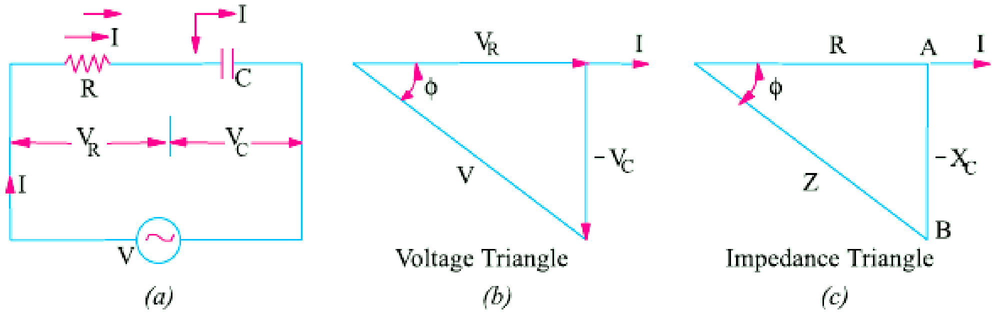
From figure(b) it is clear that the applied voltage V leads the current I by an angle φ such that:

$$\tan\varphi = \frac{V_L}{V_R} = \frac{X_L}{R}$$

AC through Resistance and Capacitance:

The circuit is shown in figure(a). Here V_R = I.R = drop across R in phase with I. V_C = I.X_C = drop across capacitor lagging I by 90°.

Ac capacitive reactance X_C is taken negative; V_C is shown along negative direction of Y-axis in the voltage triangle figure(b).



$$V = \sqrt{V_R^2 + (-V_C)^2} \qquad or\ \ V = I \times Z$$

$$where\ Z\ is\ the\ impedance\ of\ the\ circuit$$

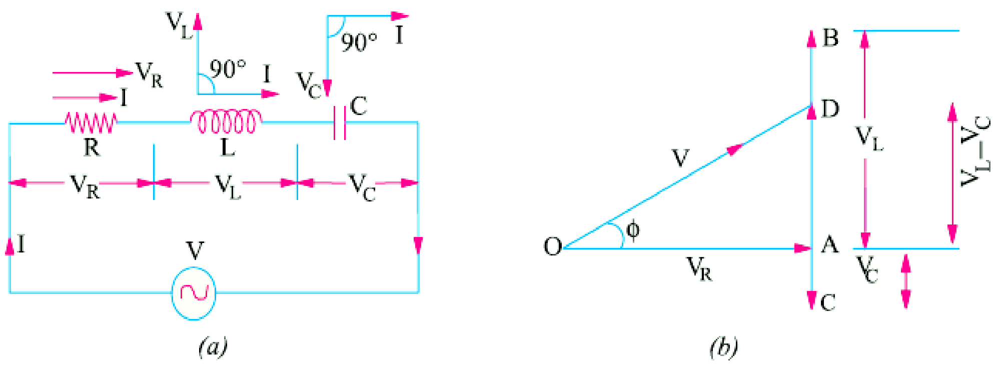
$$Z = \sqrt{R^2 + (-X_C)^2}$$

From figure(b) it is clear that the current I leads theapplied voltage V by an angle φ such that:

$$\tan\varphi = \frac{-V_C}{V_R} = \frac{-X_L}{R}$$

Resistance; Inductance and capacitance in series:

The three are shown in figure(b) joined in series across an AC supply of r.m.s. voltage V



Let $V_R = I.R\ = voltage\ drop\ across\ R \qquad In\,phase\,with\ \ I$

$$V_L = I.X_L\ = voltage\ drop\ across\ L \qquad Leading\ \ I\ by\ 90^{\circ}$$

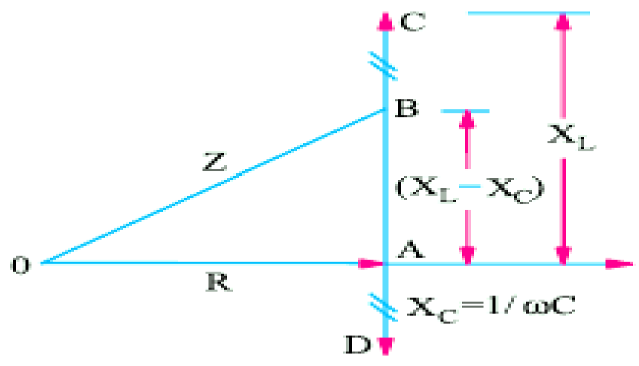
$$\mathrm{V_C = I.X_C = voltage\ drop\ across\ C \qquad Lagging\ by\ 90^{\circ}}$$

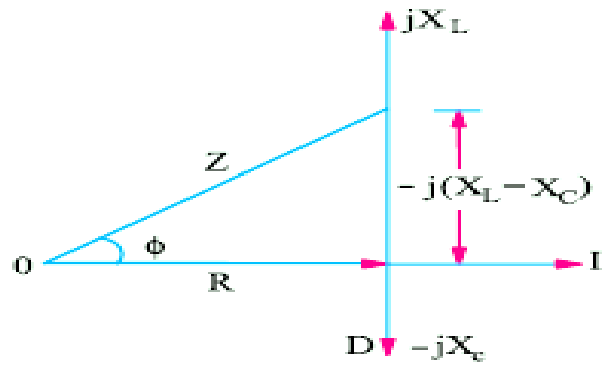
In the voltage triangle of figure(b):

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} \quad ; \quad V = Z \times I$$

Where Z is the impedance of the circuit $Z = \sqrt{R^2 + (X_L - X_C)^2}$

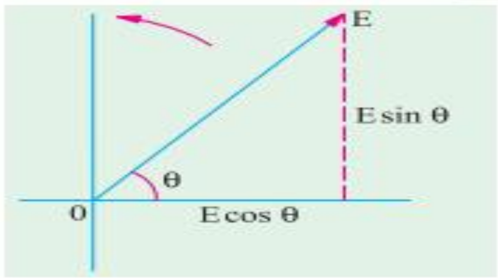
Phase angle φ is given by: $\tan\varphi = \frac{(X_L - X_C)}{R}$

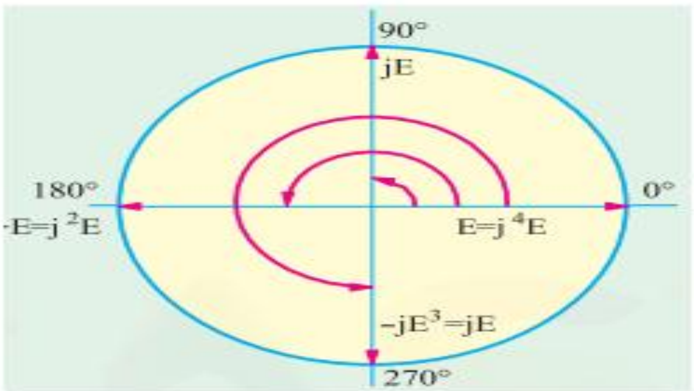




Trigonometrical Form of Vector

$\mathbf{E} = E(\cos\theta + j\sin\theta)$





$j = 90°$ ccw rotation $= \sqrt{(-1)}$

$j^2 = 180°$ ccw rotation $= [\sqrt{(-1)}]^2 = -1;$

$j^3 = 270°$ ccw rotation $= [\sqrt{(-1)}]^3 = -\sqrt{(-1)} = -j$

$j^4 = 360°$ ccw rotation $= [\sqrt{(-1)}]^4 = +1;$

$j^5 = 450°$ ccw rotation $= [\sqrt{(-1)}]^5 = -\sqrt{(-1)} = j$

It should also be noted that $\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$

Polar Form of Vector Representation

Polar form (conventional) $\mathbf{E} = E\angle \pm\theta$.

. Addition and Subtraction of Vector Quantities

Rectangular form is best suited for addition and subtraction of vector quantities.

; $\mathbf{E_1} = a_1 + jb_1$ and $\mathbf{E_2} = a_2 + jb_2$

Addition. $\mathbf{E} = \mathbf{E_1} + \mathbf{E_2} = a_1 + jb_1 + a_2 + jb_2 = (a_1 + a_2) + j(b_1 + b_2)$

The magnitude of resultant vector **E** is $\sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2}$

The position of **E** with respect to *X*-axis is $\theta = \tan^{-1}\left(\frac{b_1 + b_2}{a_1 + a_2}\right)$

Subtraction. $\mathbf{E} = \mathbf{E_1} - \mathbf{E_2} = (a_1 + jb_1) - (a_2 + jb_2) = (a_1 - a_2) + j(b_1 - b_2)$

Magnitude of $\mathbf{E} = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$

Its position with respect to *x*-axis is given by the angle $\theta = \tan^{-1}\left(\frac{b_1 - b_2}{a_1 - a_2}\right)$

Multiplication and Division of Vector Quantities

(i) Multiplication – *Rectangular form*

Let the two vectors be given by $\mathbf{A} = a_1 + jb_1$ and $\mathbf{B} = a_2 + jb_2$

$\therefore \quad \mathbf{A} \times \mathbf{B} = \mathbf{C} = (a_1 + jb_1)(a_2 + jb_2) = a_1a_2 + j^2b_1b_2 + j(a_1b_2 + b_1a_2)$

$= (a_1a_2 - b_1b_2) + j(a_1b_2 + b_1a_2)$

The magnitude of $\mathbf{C} = \sqrt{[(a_1a_2 - b_1b_2)^2 + (a_1b_2 + b_1a_2)^2]}$

In angle with respect to *X*-axis is given by $\theta = \tan^{-1}\left(\frac{a_1b_2 + b_1a_2}{a_1a_2 - b_1b_2}\right)$

(ii) Division – *Rectangular Form*: $\frac{\mathbf{A}}{\mathbf{B}} = \frac{a_1 + jb_1}{a_2 + jb_2} = \frac{(a_1 + jb_1)(a_2 - jb_2)}{(a_2 + jb_2)(a_2 - jb_2)}$

$$\frac{\mathbf{A}}{\mathbf{B}} = \frac{(a_1a_2 + b_1b_2) + j(b_1a_2 - a_1b_2)}{a_2^2 + b_2^2} = \frac{a_1a_2 + a_1b_2}{a_2^2 + b_2^2} + j\frac{b_1a_2 + a_1b_2}{a_2^2 + b_2^2}$$

(iii) Multiplication – *Polar Form*

Let $\mathbf{A} = a_1 + jb_1 = A\angle\alpha$

$\mathbf{B} = a_2 + jb_2 = B\angle\beta$

$\therefore \quad \mathbf{AB} = A\angle\alpha \times B\angle\beta$

$$\frac{\mathbf{A}}{\mathbf{B}} = \frac{A\angle\alpha}{B\angle\beta} = \frac{A}{B}\angle(\alpha - \beta)$$

Example-13 *the R –L circuit , R=3.5Ω and L=0.1 H find a) the current through the circuit ,b) power factor , if a 50Hz V= 220<30° is applied across the circuit .*

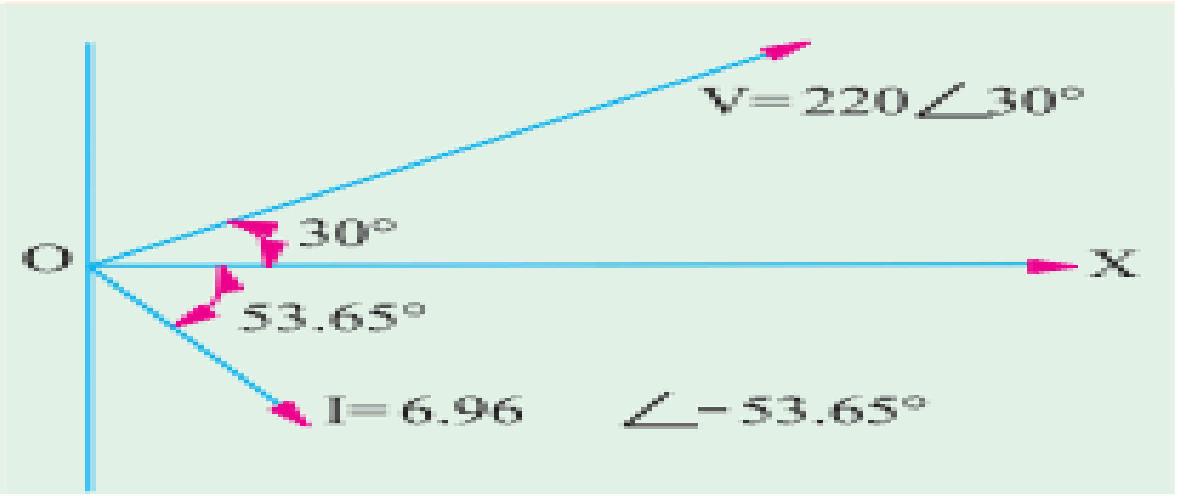
a) $\omega = 2\pi f = 3 \times 3\cdot 14 \times 50 = 314\, {}^{rad}/_{s}$

$$X_L = \omega L = 314 \times 0\cdot 1 = 31\cdot 42\Omega$$

$$Z = R + (X_L) = 3\cdot 5 + j(31\cdot 42) = 31\cdot 6\angle 83\cdot 65\Omega$$

$$I = \frac{V}{Z} = \frac{220\angle 30^\circ}{31\cdot 6\angle 83\cdot 65^\circ} = 6\cdot 96\angle -53\cdot 65^\circ A$$

f) $\cos\varphi = \cos\left(30^\circ + (-83\cdot 65^\circ)\right) = \cos(-53\cdot 65) = 0\cdot 11\ lagging$



Example-14 *A 60Hz sinusoidal voltage V = 141sinωt is applied to a series R-L circuit. The value of the resistance and the inductance are 3Ω and 0.0106H respectively.*

Write the expression for the instantaneous current in the circuit.

Compute the r.m.s. value and the phase of the voltage across the resistance and the inductance.

Solution:

$V_m = 141$ V; $V = 141/\sqrt{2} = 100\,V \quad \therefore\ V = 100 + j0$

$X_L = 2\pi \times 60 \times 0.0106 = 4\,\Omega.\ Z = 3 + j4 = 5\angle 53.1^\circ$

(i) $I = V/Z = 100\angle 0^\circ / 5\angle 53.1^\circ = 20\angle -53.1^\circ$

Since angle is minus, the current lags behind the voltage by 53.1

(ii) $I_m = \sqrt{2} \times 20 = 28.28; \quad \therefore\ i = 28.28 \sin(\omega t - 53.1^\circ)$

(iii) $V_R = IR = 20\angle -53.1^\circ \times 3 = 60\angle -53.1^\circ$ volt.

$V_L = jIX_L = 1\angle 90^\circ \times 20\angle -53.1^\circ \times 4 = 80\angle 36.9^\circ$

Example 15 *:in the circuit shown in figure V=(0+j10) and I=(0.8+j0.6)determine the value of R and X and also if X is inductance or capacitance*

Solution:

$$Z = \frac{V}{I} = \frac{0 + j10^\circ}{0\cdot 8 + j0\cdot 6} = \frac{10\angle 90^\circ}{1\angle 36\cdot 9^\circ} = 1\angle 53\cdot 1 = 6 + j8$$

Hence R=6Ω and

X_L=8Ω

***Example-16**: A pure resistance of 50Ω is in series with a pure capacitance of 100µf. The series combination is connected across 100v ; 50Hz supply. Find:*

The impedance.

Current.

Phase angle.

Voltage across resistor and capacitor.

Solution:

(a) $Z = \sqrt{50^2 + 32^2} = 59.4\ \Omega$ (b) $I = V/Z = 100/59.4 = 1.684$ A

(c) p.f. $= R/Z = 50 = 59.4 = 0.842$ (lead) (d) $\phi = \cos^{-1}(0.842) = 32°36'$

(e) $V_R = IR = 50 \times 1.684 = 84.2$ V $V_C = IX_C = 32 \times 1.684 = 53.9$ V

***Example17:** A resistance of 20Ω an inductance of 0.2H and a capcitance of 100µf are connected in series across 220V; 50Hz mains. Detemine:*

Impedance.

Current.

Voltage across R; L; and C.

Solution:

Solution. $X_C = 0.2 \times 314 = 63\ \Omega$, $C = 10\ \mu F = 100 \times 10^{-6} = 10^{-4}$ farad

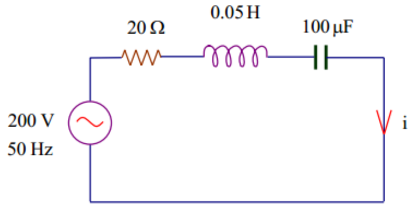
$X_C = \dfrac{1}{\omega C} = \dfrac{1}{314 \times 10^{-4}} = 32\ \Omega$, $X = 63 - 32 = 31\ \Omega$ (*inductive*)

(a) $Z = \sqrt{(20^2 + 31)^2} = 37\ \Omega$ (b) $I = 220/37 = 6$ A (approx)

(c) $V_R = I \times R = 6 \times 20 = 120$ V; $V_L = 6 \times 63 = 278$ V, $V_C = 6 \times 32 = 192$ V

***Example18:** A resistance of 20Ω an inductance of 0.05H and a capcitance of 100µf are connected in series across 200V; 50Hz mains. Detemine:*

Impedance ,Current ,Voltage across R; L; and C ,and draw phasor daigram .



$$\omega = 2\pi f = 3 \times 3\cdot 14 \times 50 = 314 \, {}^{rad}/_{s}$$

$$X_L = \omega L = 314 \times 0\cdot 05 = 15\cdot 7\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 100 \times 10^{-6}} = 31\cdot 84\Omega$$

$$Z = R + (X_L - X_C) = 20 + j(15\cdot 7 - 31\cdot 84) = (20 - j16\cdot 14)\Omega$$

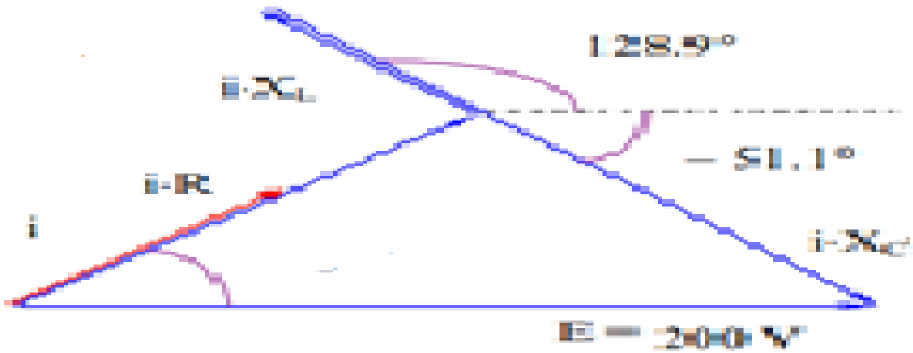
$$Z = 20 - j16\cdot 14 = 25\cdot 7\angle -38\cdot 9^\circ$$

$$I = \frac{V}{Z} = \frac{200\angle 0^\circ}{25\cdot 7\angle -38\cdot 9^\circ} = 7\cdot 78\angle 38\cdot 9^\circ A$$

$$V_R = IR = (7\cdot 78\angle 38\cdot 9^\circ)(20\angle 0^\circ) = 155\cdot 6\angle 38\cdot 9^\circ V$$

$$V_L = IX_L = (7\cdot 78\angle 38\cdot 9^\circ)(15\cdot 7\angle 90^\circ) = 122\cdot 146\angle 128\cdot 9^\circ V$$

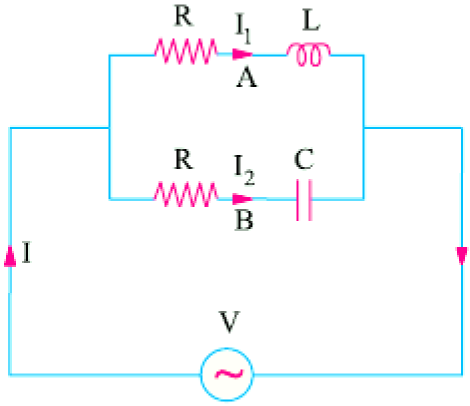
$$V_C = IX_C = (7\cdot 78\angle 38\cdot 9^\circ)(31\cdot 84\angle -90^\circ) = 247\cdot 4\angle -51\cdot 1^\circ V$$



AC parallel circuit:

Consider the circuits shown in figure below. Here two branch A and B have been joined in parallel across an r.m.s supply of V volts. The voltage across two parallel branches A and B is the same. But currents through them are different.

For branch A



$$Z_1 = \sqrt{(R_1^2 + X_L^2)} \;:\; I_1 = \frac{V}{Z_1}$$

Current I_1 lags behind the applied voltage by φ_1

For branch B

$$Z_2 = \sqrt{(R_2^2 + X_C^2)} \;:\; I_2 = \frac{V}{Z_2}$$

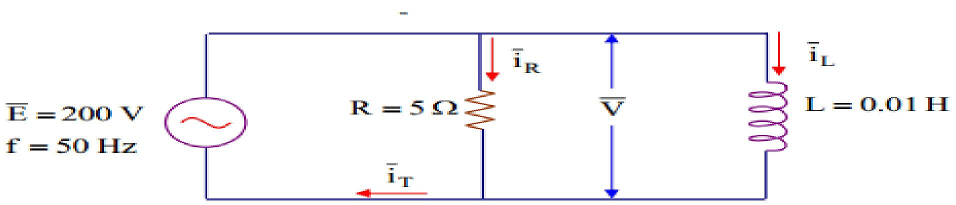
The total current I is:

$$I = I_1 + I_2 = \frac{V}{Z_1} + \frac{V}{Z_2} = V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right) = V(Y_1 + Y_2) = VY$$

$$where\;\; Y = total\ admittance = Y_1 + Y_2$$

It should be noted that admittance are added for parallel branches; whereas for branches in series it is the impedances which are added.

Example19: *for the circuit shown find (I_L and I_R) .*

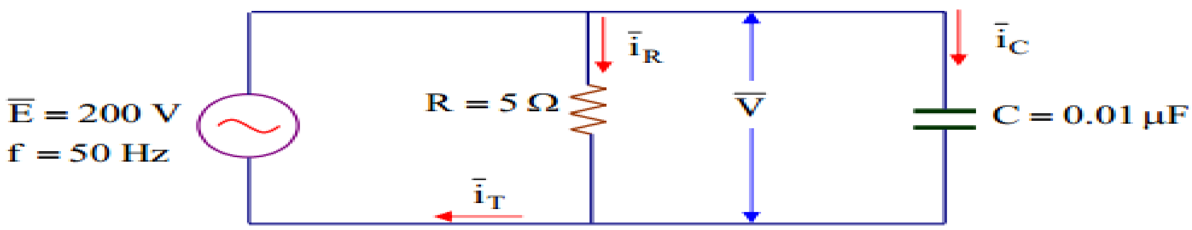


$$\bar{i}_R = \frac{E \angle 0^\circ}{R \angle 0^\circ} = \frac{E}{R} \angle 0^\circ = \frac{200\ V \angle 0^\circ}{5\ \Omega \angle 0^\circ} = 40\ A \angle 0^\circ$$

$$\bar{i}_L = \frac{E}{\omega L} \angle -90^\circ = \frac{E}{2\ \pi\ f\ L} \angle -90^\circ$$

$$\bar{i}_L = \frac{200\ V}{2\ \pi \cdot 50 \cdot 1 \times 10^{-2}\ \Omega} \angle -90^\circ = 63.7\ A \angle -90^\circ$$

Example20: *for the circuit shown find (I_c , I_R and I_T) .*



Solution:

$$I_R = \frac{E}{R} = \frac{200\angle 0°}{5\angle 0°} = 40\angle 0°\text{A}$$

$$I_c = \frac{E}{X_c} = \omega EC\angle 90° = 200 \times 2 \times 50 \times 3.14 \times 0.01 \times 10^{-6} = 62.8 \times 10^{-3}\angle 90°\text{A}$$

$$I_T = I_R + I_C = 40\angle 0° + 62.8 \times 10^{-3}\angle 90° =$$

***Example21*:** *Two circuit the impedance of which are given by* $Z_1 = 10 + j15\Omega$ *and* $Z_2 = 6 - j8\Omega$ *are connected in parallel. If the total current applied is 15A. what are the current taken by each branch.*

Solution. Let $\mathbf{I} = 15\angle 0°$; $\mathbf{Z}_1 = 10 + j15 = 18\angle 57°$

$\mathbf{Z}_2 = 6 - j8 = 10\angle -53.1°$

Total impedance, $\mathbf{Z} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(10 + j15)(6 - j8)}{16 + j7}$

$= 9.67 - j3.6 = 10.3\angle -20.4°$

Applied voltage is given by

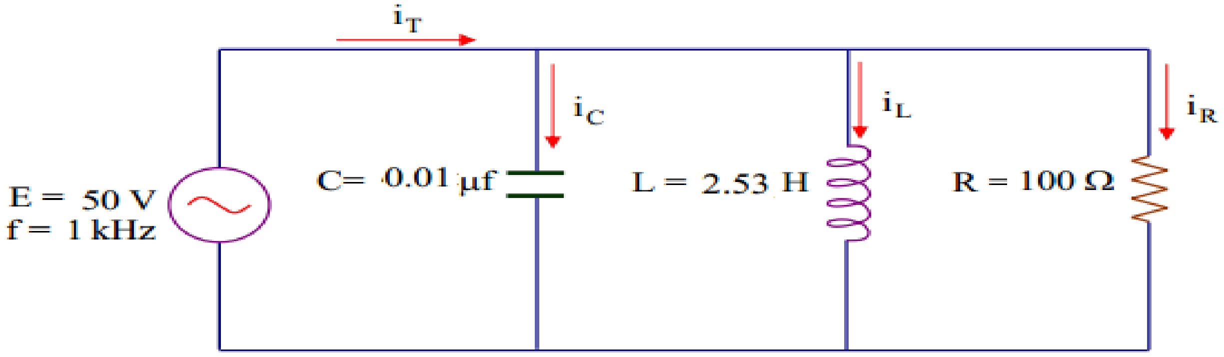
$\mathbf{V} = \mathbf{IZ} = 15\angle 0° \times 10.3\angle 20.4° = 154.4\angle 20.4°$

$\mathbf{I}_1 = \mathbf{V}/\mathbf{Z}_1 = 154.5\angle 20.4°/18\angle 57° = 8.58\angle 77.4°$

$\mathbf{I}_2 = \mathbf{V}/\mathbf{Z}_2 = 154.5\angle 20.4°/10\angle 53.1°$

$= 15.45\angle 32.7°$

<u>Example22</u>: for the circuit shown find (I_c, I_R, I_L and I_T) .



$$i_R = \frac{50\ V\ \angle 0^\circ}{100\ \Omega\ \angle 0^\circ} = 0.5\ A\ \angle 0^\circ$$

$$i_L = \frac{50\ V\ \angle 0^\circ}{(2\times\pi\times 1000\times 2.53)\,\Omega\ \angle 90^\circ} = 3.145\ mA\ \angle -90^\circ$$

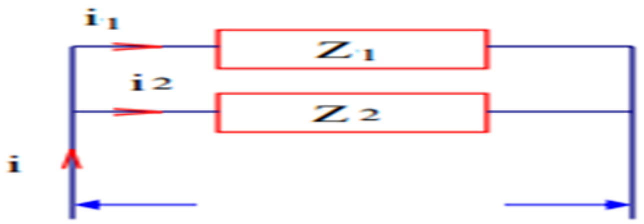
$$i_C = \frac{50\ V\ \angle 0^\circ}{\left(1/2\times\pi\times 1000\times 0.01\times 10^{-6}\right)\Omega\ \angle -90^\circ} = 3.145\ mA\ \angle 90^\circ$$

$$i_T = i_R + i_L + i_C = 0.5\ A\ \angle 0^\circ + 3.145\ mA\ \angle -90^\circ + 3.145\ mA\ \angle 90^\circ$$

$$i_T = 0.5\ A\ \angle 0^\circ$$

<u>Example23</u>: for the circuit the shown if $Z_1 = 6 + j8\Omega$ and $Z_2 = 8 - j6\Omega$ are connected in parallel. If the applied voltage is (100V , 60Hz) ,what are the current taken by each branch.

Solution :



$$Z_1 = 6 + j8 = 10\ \Omega\ \angle 51.3^\circ$$

$$Z_2 = 8 - j6 = 10\ \Omega\ \angle -36.9^\circ$$

$$E = 100\ V \angle 0^\circ$$

$$i_1 = \frac{100\angle 0^\circ}{10\angle 51.3^\circ} = 10\ A\ \angle -51.3^\circ$$

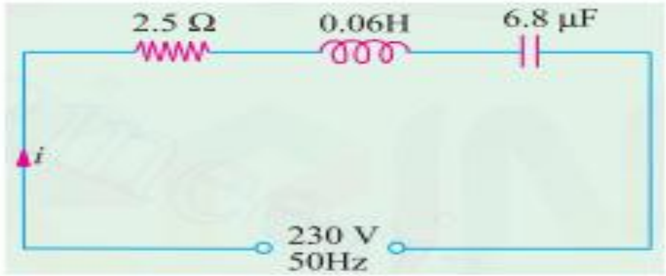
$$i_2 = \frac{100\angle 0^\circ}{10\angle -36.9^\circ} = 10\ A\ \angle 36.9^\circ$$

$$Z_t = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(6 + j8)(8 - j6)}{(6 + j8)(8 - j6)}$$

$$Z_t = \frac{96 + j28}{14 + j2} = \frac{100\angle 16.26^\circ}{14.14\angle 8.13^\circ} = 7.1\ \Omega\ \angle 8.13^\circ$$

$$i = \frac{E}{\overline{Z}_t} = \frac{100\angle 0}{7.1\angle 8.13} = 14.1\ A\ \angle -8.13^\circ$$

Example24: *for the circuit the shown find i*



Power in the AC circuit

Apparent power (S): It is given by product of r.m.s. values of applied voltage and circuit current.

$$S = VI = I^2 Z \qquad (VA)$$

Active power (P): It is the power which is actually dissipated in the circuit resistance.

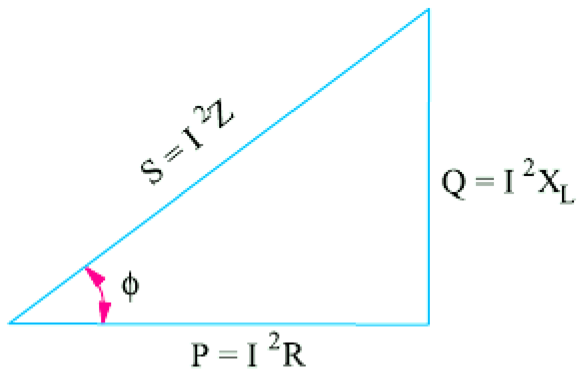
$$P = VI \cos\varphi = I^2 R \qquad (watt)$$

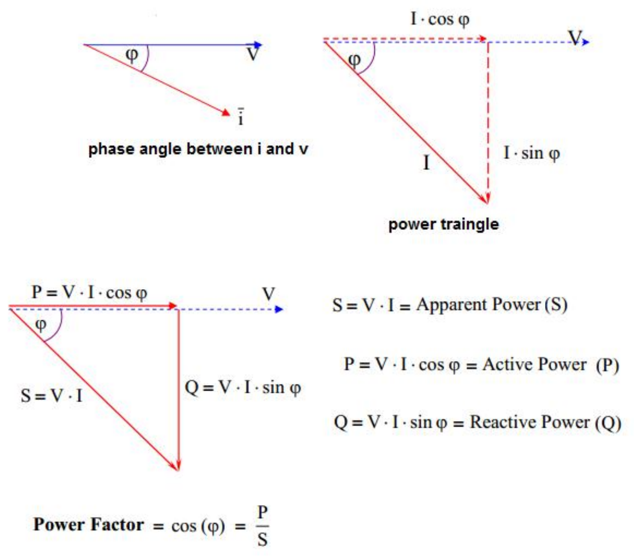
Reactance power (Q): it is the power developed in the reactance of the circuit.

$$Q = I.X.\cos\varphi = I^2.X$$

These three powers are shown in the power triangle of figure below.

$$S = \sqrt{P^2 + Q^2}$$





Power factor

$$\cos \varphi = \frac{R}{Z} = \frac{P}{S}$$

Example 25 *A 60Hz sinusodial voltage $v = 141 \sin \omega t$ is applied to a series R-L circuit. The values of the resistance and the inductance are 3Ω and 0.0106H respectively.*

Find the average power dissipated by the circuit.

Calculate the power factor of the circuit.

Solution. $V_m = 141$ V; $V = 141/\sqrt{2} = 100\ V \quad \therefore\ V = 100 + j0$

$X_L = 2\pi \times 60 \times 0.0106 = 4\ \Omega.\ Z = 3 + j4 = 5 \angle 53.1°$

$I = V/Z = 100 \angle 0°/5\angle 53.1° = 20 \angle -53.1°$

Since angle is minus, the current lags behind the voltage by 53.1

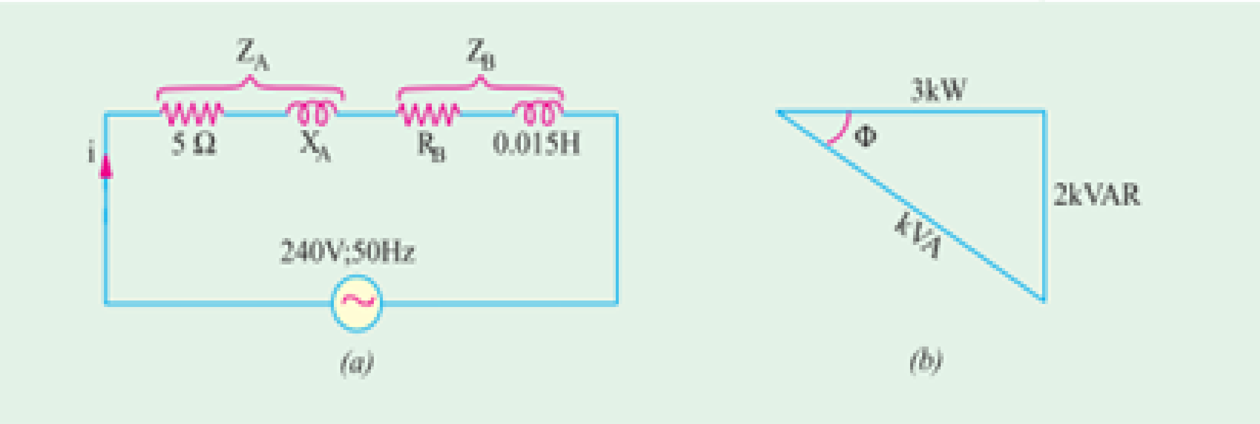
$I_m = \sqrt{2} \times 20 = 28.28; \quad \therefore\ i = 28.28 \sin(\omega t - 53.1°)$

(*i*) $P = VI \cos \phi = 100 \times 20 \times \cos 53.1° =$ **1200 W.**

(*ii*) p.f. $= \cos \phi = \cos 53.1° =$ **0.6.**

Example 26: **Two coils A and B are connected in series across a 240V, 50Hz supply .the resistance of coil A is 5Ω and the inductance of coil B is 0.015H .if the input from the**

supply is 3KW and 2KVAR , find the inductance of A and resistance of B , calculate the voltage across each coil .



Solution :

The power traingle in figure b

$$S = \sqrt{P^2 + Q^2} = \sqrt{3^2 + 2^2} = 3 \cdot 606KVA$$

$$S = VI \Rightarrow I = \frac{S}{V} = \frac{3606}{240} = 15 \cdot 03A$$

$$P = I^2R = I^2 \times (R_A + R_B) \Rightarrow (R_A + R_B) = \frac{P}{I^2} = 13 \cdot 3\Omega$$

$$(R_A + R_B) = 13 \cdot 3 \Rightarrow R_B = 13 \cdot 3 - 5 = 8 \cdot 3\Omega$$

$$Z = \frac{V}{I} = \frac{240}{15 \cdot 3} = 15 \cdot 97\Omega$$

$$(X_A + X_B) = \sqrt{Z^2 - (R_A + R_B)^2} = \sqrt{15 \cdot 97^2 - 13 \cdot 3^2} = 8 \cdot 84\Omega$$

$$(X_A + X_B) = 8 \cdot 84\Omega$$

$$X_B = 2\pi f L_B = 2 \times \pi \times 50 \times 0 \cdot 015 = 4 \cdot 713\Omega$$

$$\therefore X_A = 8 \cdot 84 - 4 \cdot 713 = 4 \cdot 13\Omega$$

$$X_A = 2\pi f L_A \Rightarrow L_A = \frac{X_A}{2 \times \pi \times 50} = 0 \cdot 0132\Omega$$

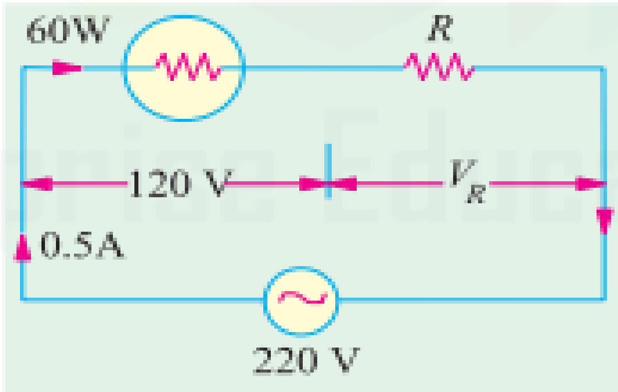
$$Z_A = \sqrt{({R_A}^2 + {X_A}^2)} = \sqrt{5^2 + 4 \cdot 13^2} = 6 \cdot 485\Omega$$

$$Z_B = \sqrt{({R_B}^2 + {X_B}^2)} = \sqrt{8\cdot3^2 + 4\cdot713^2} = 9\cdot545\Omega$$

P.D across coil A = $I\,Z_A = 15\cdot05 \times 6\cdot485 = 97\cdot5V$

P.D across coilB = $I\,Z_B = 15\cdot05 \times 9\cdot545 = 143\cdot5V$

Example 27: ***A 120-V ,60 W lamp is be operated on 220v ,50Hz supply mains .culculate what value of (R) would be required in order that lamp is run on correct voltage .***



(a)

Solution:

Resistor is connected in series with lamp

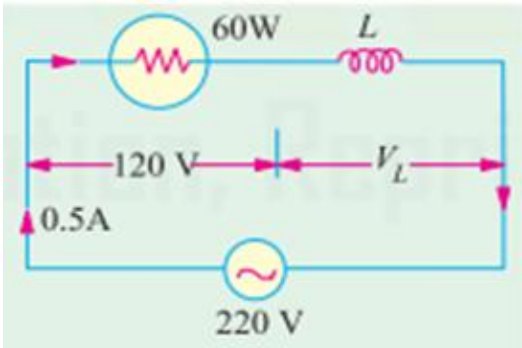
The current passing through lamp to run on correct voltage

$$I = \frac{P_{lamp}}{V_{lamp}} = \frac{60}{120} = 0\cdot5A$$

P.D across R is V_R $\quad V_R = 220 - 120 = 100\,V$

$$\therefore R = \frac{V_R}{I} = \frac{100}{0\cdot5} = 200\Omega$$

Example28 ***A 120-V ,60 W lamp is be operated on 220v ,50Hz supply mains .culculate what value of (L) would be required in order that lamp is run on correct voltage .***



(b)

Solution :

Inductance is connected in series with lamp

The current passing through lamp to run on correct voltage

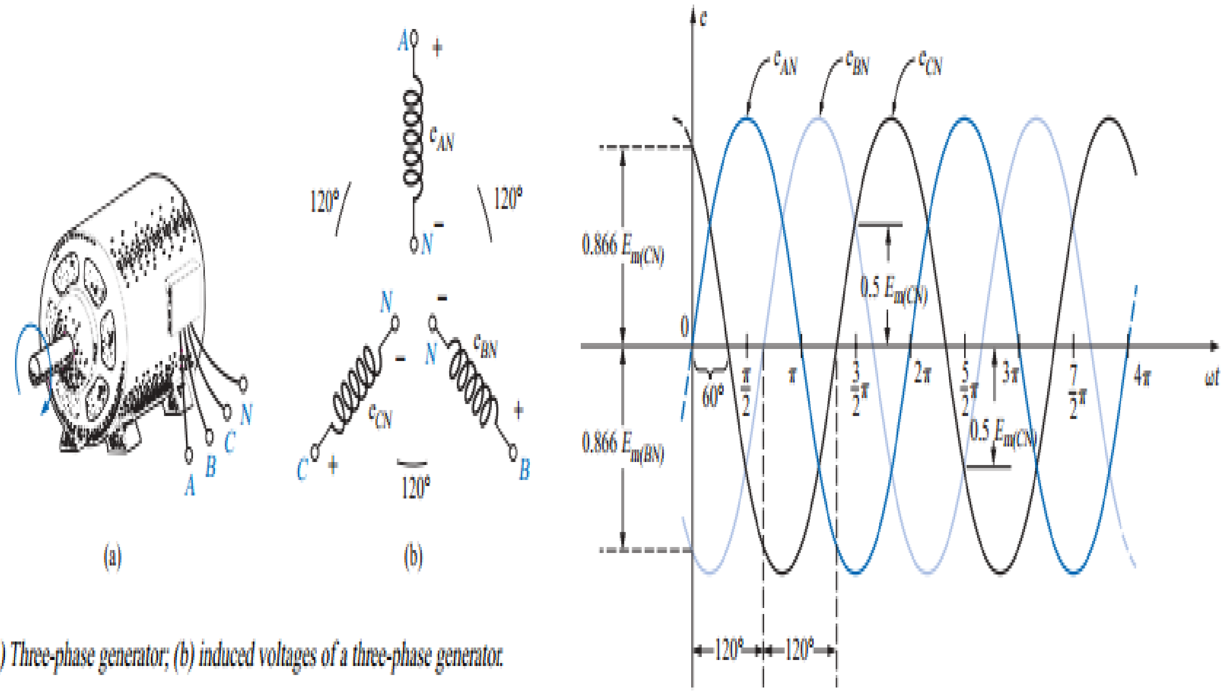
$$I = \frac{P_{lamp}}{V_{lamp}} = \frac{60}{120} = 0\cdot5A$$

P.D across L is V_L $\quad V_L == \sqrt{(220^2 + 120^2)} = 184\cdot4V$

$$\therefore X_L = \frac{V_L}{I} = \frac{184\cdot4}{0\cdot5} = 368\cdot8\Omega \Rightarrow L = \frac{X_L}{2\times\pi\times50} = \frac{368\cdot8}{314} = 1\cdot14H$$

Three phase generator:

Large scale generation of power is achieved by generating three phase e.m.f. using three separate windings insulated from each other. They are placed on the rotor of the alternator. The windings are displaced at angle of 120º with each other as shown in figure. When the rotor is rotated e.m.f. will be induced in the three coils (phases).



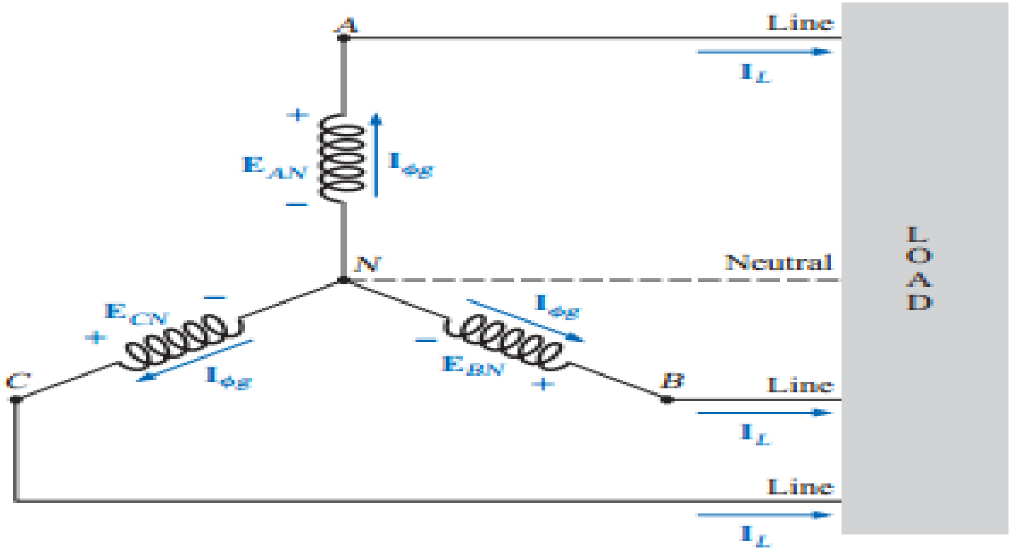
(a) Three-phase generator; (b) induced voltages of a three-phase generator.

The instantaneous values of the three e.m.f. will be given by curves of figure

$$e_{AN} = E_{m(AN)} \sin \omega t$$
$$e_{BN} = E_{m(BN)} \sin(\omega t - 120°)$$
$$e_{CN} = E_{m(CN)} \sin(\omega t - 240°) = E_{m(CN)} \sin(\omega t + 120°)$$

$$E_{AN} = 0.707 E_{m(AN)}$$
$$E_{BN} = 0.707 E_{m(BN)}$$
$$E_{CN} = 0.707 E_{m(CN)}$$
$$\mathbf{E}_{AN} = E_{AN} \angle 0°$$
$$\mathbf{E}_{BN} = E_{BN} \angle -120°$$
$$\mathbf{E}_{CN} = E_{CN} \angle +120°$$

THE Y-CONNECTED GENERATOR



line current = phase current

$$\mathbf{I}_L = \mathbf{I}_{\phi g}$$

$$E_{PH} = E_{AN} = E_{BN} = E_{CN}$$

$$E_L = E_{AB} = E_{BC} = E_{CA}$$

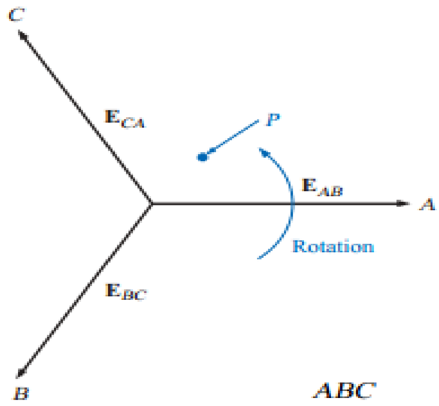
$$E_L = \sqrt{3}\, E_{\emptyset} = \sqrt{3} E_{ph}$$

$$I_{PH} = I_L$$

Numbering of phase

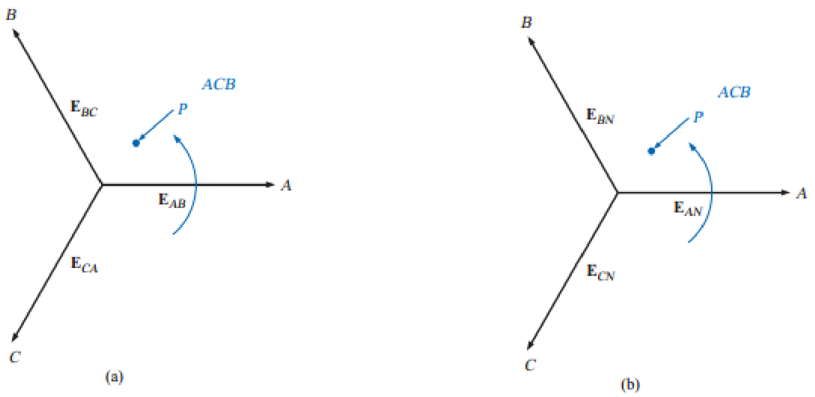
The three phases may be numbered 1; 2; 3 or a; b; c or as is customary they may be given three colour. The colours used commercially are red; yellow and blue. In this case the sequence is RYB.

PHASE SEQUENCE (Y-CONNECTED GENERATOR)



Line voltages

$$\begin{cases} \mathbf{E}_{AB} = E_{AB}\angle 0^\circ & \text{(reference)} \\ \mathbf{E}_{CA} = E_{CA}\angle -120^\circ \\ \mathbf{E}_{BC} = E_{BC}\angle +120^\circ \end{cases}$$

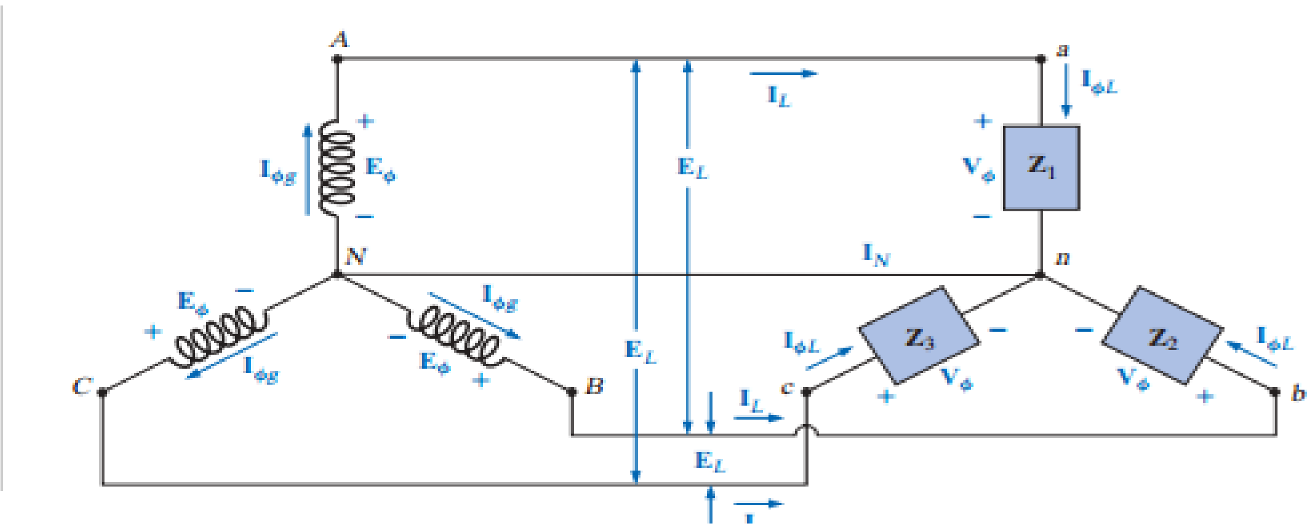


Phase voltages

$$\begin{cases} \mathbf{E}_{AN} = E_{AN}\angle 0^\circ & \text{(reference)} \\ \mathbf{E}_{CN} = E_{CN}\angle -120^\circ \\ \mathbf{E}_{BN} = E_{BN}\angle +120^\circ \end{cases}$$

Drawing the phasor diagram from the phase sequence.

The Y- connected generated with A Y-connected load



Y-connected generator with a Y-connected load.

If the load is balanced, the **neutral connection** can be removed without affecting the circuit

$$Z_1 = Z_2 = Z_3$$

$$I_{PHg} = I_L = I_{phL}$$

$$E_L = \sqrt{3}\, E_{PH}$$

$$V_{PH} = E_{PH}$$

For the balanced load I_N=0

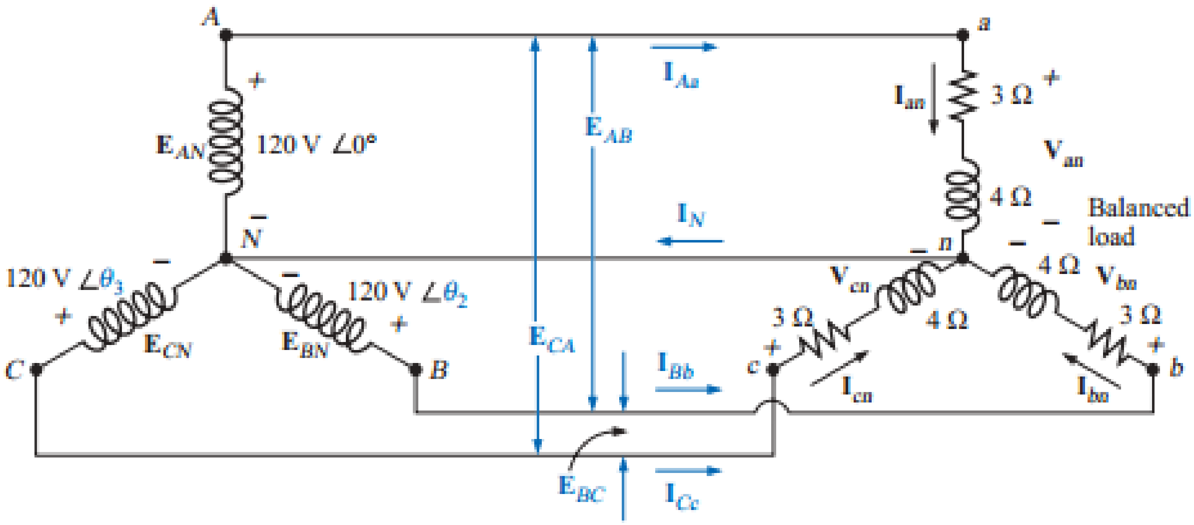
Example ( ):The phase sequence of the Y – connected generator is ABC

١- **Find the phase angles Θ_2 and Θ_3**

٢- **Find the magnitude of the line voltages**

٣- **Find the line currents**

٤- **Prove I_N=0**



Solution :

1-for the A B C phase sequrnce

Θ_2= -120 and Θ_3=120

2- $E_L = \sqrt{3}\, E_{ph} = (1 \cdot 73)(120V) = 208V$

Therefor $E_{AB} = E_{BC} = E_{CA} = 208V$

3- $V_{ph} = E_{ph}$ **Therefore**

$$V_{an} = E_{AN} \qquad V_{bn} = E_{BN} \qquad V_{cn} = E_{CN}$$

$$I_L = I_{an} = \frac{V_{an}}{Z_{an}} = \frac{120\angle 0°}{3 + j4} = \frac{120\angle 0°}{5\angle 53 \cdot 13°} = 24\angle - 53 \cdot 13° A$$

$$I_L = I_{bn} = \frac{V_{bn}}{Z_{bn}} = \frac{120\angle - 120°}{3 + j4} = \frac{120\angle 0°}{5\angle 53 \cdot 13°} = 24\angle - 173 \cdot 13° A$$

$$I_L = I_{cn} = \frac{V_{cn}}{Z_{cn}} = \frac{120\angle 120°}{3 + j4} = \frac{120\angle 0°}{5\angle 53 \cdot 13°} = 24\angle 66 \cdot 87° A$$

$$I_L = I_{ph}$$

$$I_{Aa} = I_{an} = 24\angle - 53 \cdot 13° A$$

$$I_{Ba} = I_{bn} = 24\angle - 173 \cdot 13° A$$

$$I_{Cc} = I_{cn} = 24\angle 66 \cdot 87° A$$

4-applying K.C.L

$$I_N = I_{Aa} + I_{Bb} + I_{Cc} = 24\angle - 53 \cdot 13° + 24\angle - 173 \cdot 13° + 24\angle 66 \cdot 87° = 0$$

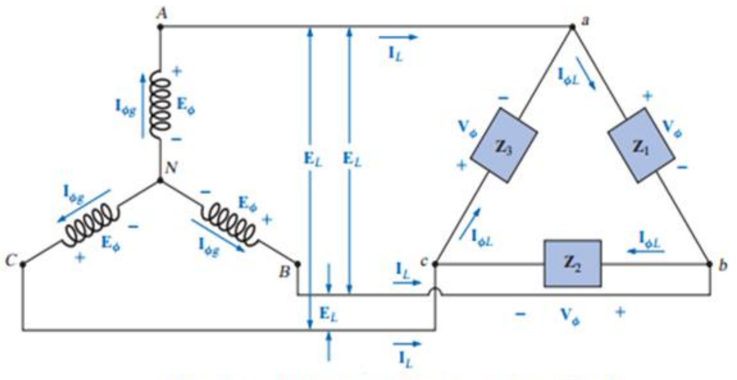
The Y – Δ System

There is no neutral

For the balanced load $Z_1 = Z_2 = Z_3$

$$V_{ph} = E_L$$

$$I_L = \sqrt{3} I_{ph}$$



Y-connected generator with a Δ-connected load.

Example ( ):The three phase system of figure

١- Find the phase angles Θ_2 and Θ_3

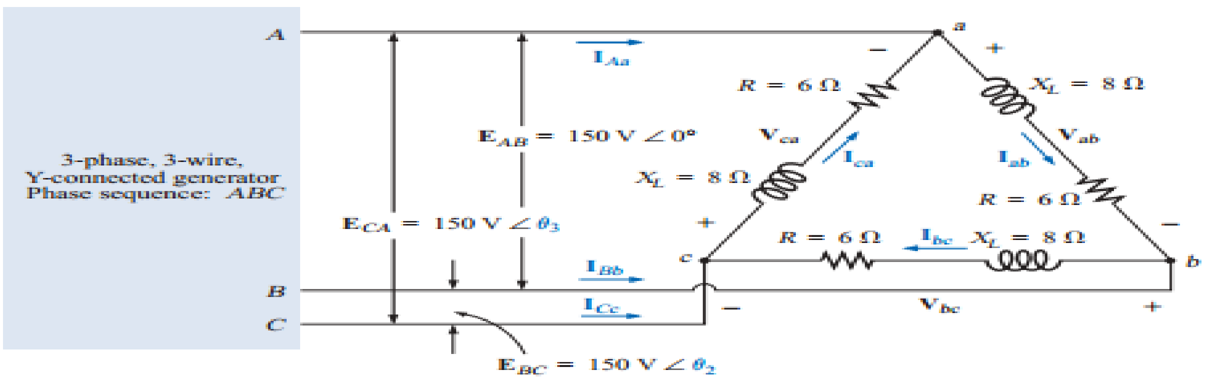
٢- Find the current in each phase of the load

٣- Find the magnitude of the line currents

Solution :

1-for an A B C sequence

Θ_2 = -120° and Θ_3=120°



2-

$$V_{ph} = V_L$$

$$V_{ab} = E_{AB} \qquad V_{ac} = E_{AC} \qquad V_{bc} = E_{BC}$$

The phase currents

$$I_{ab}=\frac{V_{ab}}{Z_{ab}}=\frac{150\angle 0^\circ}{6+j8}=\frac{150\angle 0^\circ}{10\angle 53\cdot 13^\circ}=15\angle -53\cdot 13^\circ A$$

$$I_{bc}=\frac{V_{bc}}{Z_{bc}}=\frac{150\angle -120^\circ}{6+j8}=\frac{150\angle -120^\circ}{10\angle 53\cdot 13^\circ}=15\angle -173\cdot 13^\circ A$$

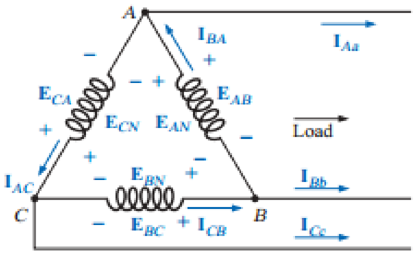
$$I_{ca}=\frac{V_{ca}}{Z_{ca}}=\frac{150\angle 120^\circ}{6+j8}=\frac{120\angle 120^\circ}{10\angle 53\cdot 13^\circ}=15\angle 66\cdot 87^\circ A$$

3-

$$I_L=\sqrt{3}I_{ph}=1\cdot 73\times 15=25\cdot 95A$$

$$I_{Aa}=I_{Bb}=I_{Cc}=25\cdot 95A$$

The Δ connected generator



(b)

Δ-connected generator.

$$I_{phg} = \frac{I_L}{\sqrt{3}}$$

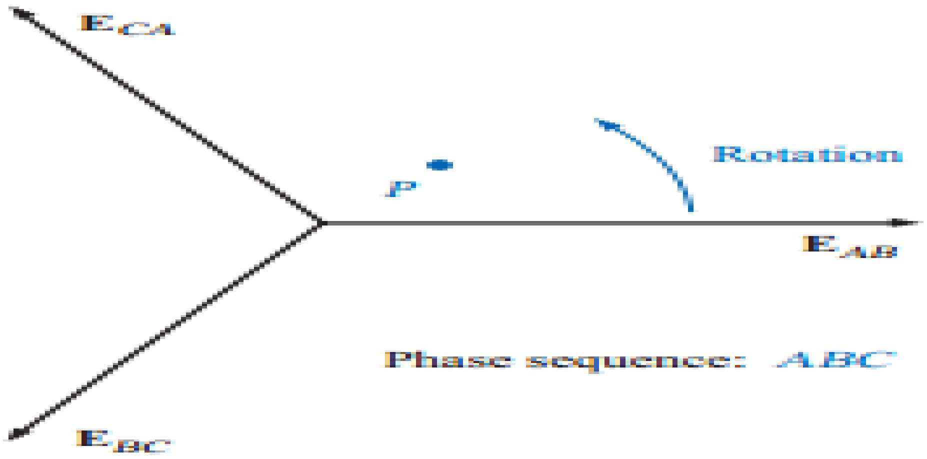
$$E_{AB} = E_{BC} = E_{CA} = E_{PHg} = E_L$$

Phase sequence (Δ – connected generator)

$$E_{AB} = E_{AB}\angle 0°$$

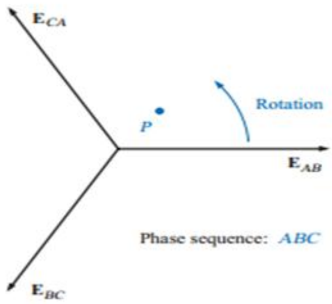
$$E_{BC} = E_{BC}\angle -120°$$

$$E_{CA} = E_{CA}\angle 120°$$



PHASE SEQUENCE (Δ-CONNECTED GENERATOR)

$\mathbf{E}_{AB} = E_{AB}\angle 0°$
$\mathbf{E}_{BC} = E_{BC}\angle -120°$
$\mathbf{E}_{CA} = E_{CA}\angle 120°$



Determining the phase sequence for a Δ-connected, three-phase generator.

THE Δ-Δ, Δ-Y THREE-PHASE SYSTEMS

The basic equations necessary to analyze either of the two systems (Δ-Δ, Δ-Y) have been presented at least once in this chapter. We will therefore proceed directly to two descriptive examples, one with a Δ-connected load and one with a Y-connected load.

EXAMPLE For the Δ-Δ system shown in Fig.

a. Find the phase angles θ_2 and θ_3 for the specified phase sequence.
b. Find the current in each phase of the load.
c. Find the magnitude of the line currents.

The Δ-Δ, Δ-Y system

Example ( ): for The Δ-Δ system shown in figure

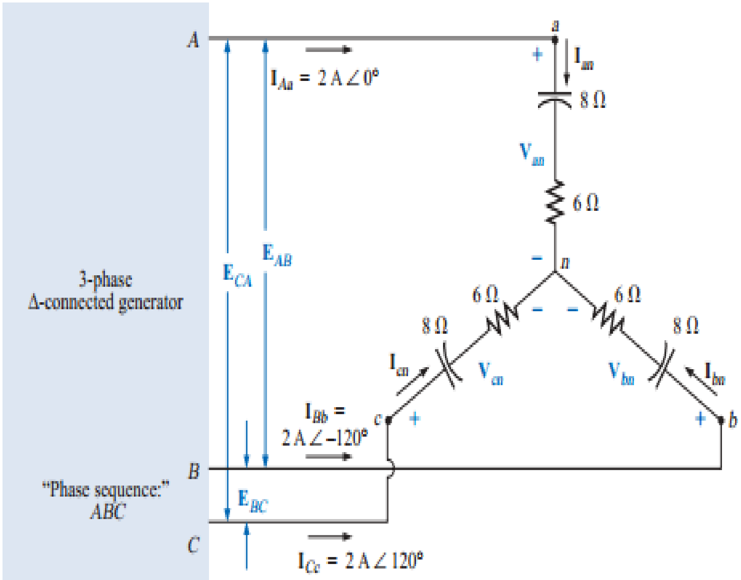
١- Find the phase angles Θ_2 and Θ_3

٢- Find the current in each phase of the load

٣- Find the magnitude of the line currents

EXAMPLE For the Δ-Y system shown in Fig

Find the voltage across each phase of the load.
Find the magnitude of the line voltages.



Solutions:

a. $\mathbf{I}_{\phi L} = \mathbf{I}_L$. Therefore,

$\mathbf{I}_{an} = \mathbf{I}_{Aa} = 2\,A\angle 0°$
$\mathbf{I}_{bn} = \mathbf{I}_{Bb} = 2\,A\angle -120°$
$\mathbf{I}_{cn} = \mathbf{I}_{Cc} = 2\,A\angle 120°$

The phase voltages are

$\mathbf{V}_{an} = \mathbf{I}_{an}\mathbf{Z}_{an} = (2\,A\angle 0°)(10\,\Omega\angle -53.13°) = \mathbf{20\,V\angle -53.13°}$
$\mathbf{V}_{bn} = \mathbf{I}_{bn}\mathbf{Z}_{bn} = (2\,A\angle -120°)(10\,\Omega\angle -53.13°) = \mathbf{20\,V\angle -173.13°}$
$\mathbf{V}_{cn} = \mathbf{I}_{cn}\mathbf{Z}_{cn} = (2\,A\angle 120°)(10\,\Omega\angle -53.13°) = \mathbf{20\,V\angle 66.87°}$

b. $E_L = \sqrt{3}V_\phi = (1.73)(20\,V) = 34.6\,V$. Therefore,

$$E_{BA} = E_{CB} = E_{AC} = \mathbf{34.6\,V}$$

Example ( ): for The Δ-Y system shown in figure

١- Find the voltage across each phase of the losd

٢- Find the magnitude of the line voltages

Solution

$\boldsymbol{I_{ph} = I_L}$ **therefore**

$$V_{an} = I_{an}Z_{an} = (2\angle 0°)(10\angle -53\cdot 13°) = 20\angle -53\cdot 53°V$$

$$V_{bn} = I_{bn}Z_{bn} = (2\angle -120°)(10\angle -53\cdot 13°) = 20\angle -173\cdot 53°V$$

$$V_{cn} = I_{cn}Z_{cn} = (2\angle 120°)(10\angle -53\cdot 13°) = 20\angle 66\cdot 87°V$$

$$V_L = \sqrt{3}V_{ph} = (1\cdot 73)(20) = 34\cdot 6V$$

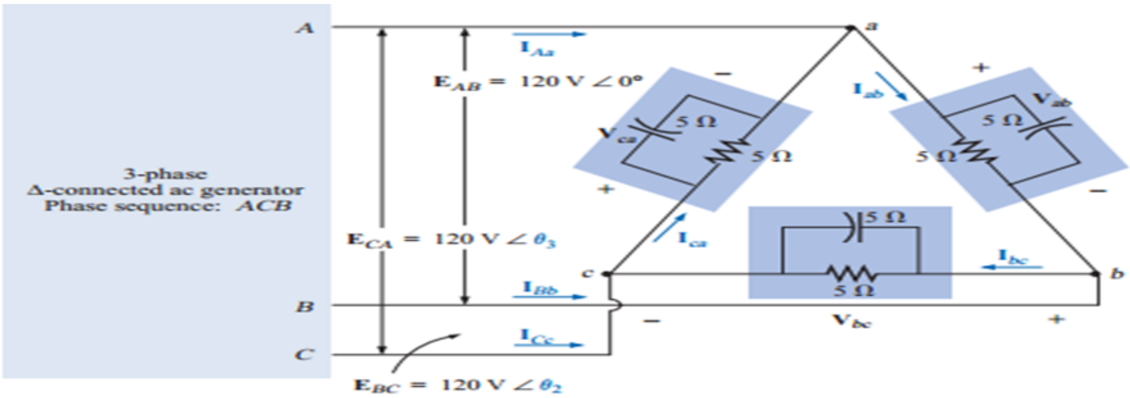
$$E_{AB} = E_{BC} = E_{CA} = 34\cdot 6$$

Example ( ): for The Δ-Δsystem shown in figure

١ - find the phase angles Θ_2 and Θ_3

2-Find the current in each phase of the load

٣- Find the magnitude of the line voltages



Solutions:

a. For an ACB phase sequence,

$$\theta_2 = 120° \quad \text{and} \quad \theta_3 = -120°$$

b. $V_\phi = E_L$. Therefore,

$$V_{ab} = E_{AB} \qquad V_{ca} = E_{CA} \qquad V_{bc} = E_{BC}$$

The phase currents are

$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{120\text{ V} \angle 0°}{\frac{(5\ \Omega \angle 0°)(5\ \Omega \angle -90°)}{5\ \Omega - j\,5\ \Omega}} = \frac{120\text{ V} \angle 0°}{\frac{25\ \Omega \angle -90°}{7.071 \angle -45°}}$$

$$= \frac{120\text{ V} \angle 0°}{3.54\ \Omega \angle -45°} = 33.9\text{ A} \angle 45°$$

$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{120\text{ V} \angle 120°}{3.54\ \Omega \angle -45°} = 33.9\text{ A} \angle 165°$$

$$I_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{120\text{ V} \angle -120°}{3.54\ \Omega \angle -45°} = 33.9\text{ A} \angle -75°$$

c. $I_L = \sqrt{3} I_\phi = (1.73)(34\text{ A}) = 58.82\text{ A}$. Therefore,

$$I_{Aa} = I_{Bb} = I_{Cc} = 58.82\text{ A}$$

Solution :

a- For an A C B phase sequence

Θ_2 = 120° and Θ_3=-120°

b-

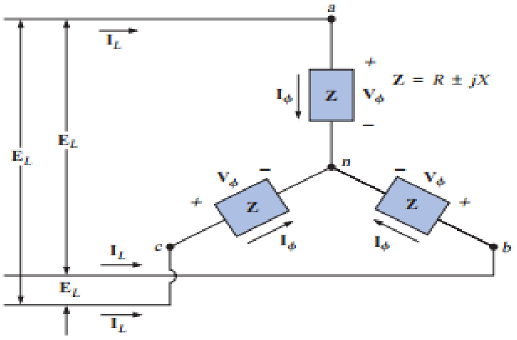
$$V_{ph} = V_L$$

$$V_{ab} = E_{AB} \qquad V_{ac} = E_{AC} \qquad V_{bc} = E_{BC}$$

The phase currents

POWER

Y-Connected Balanced Load



Y-connected balanced load.

Average Power The average power delivered to each phase can be determined

$$P_\phi = V_\phi I_\phi \cos\theta_{I_\phi}^{V_\phi} = I_\phi^2 R_\phi = \frac{V_R^2}{R_\phi} \qquad \text{(watts, W)}$$

where $\theta_{I_\phi}^{V_\phi}$ indicates that θ is the phase angle between V_ϕ and I_ϕ. The total power to the balanced load is

$$P_T = 3P_\phi \qquad \text{(W)}$$

or, since $V_\phi = \frac{E_L}{\sqrt{3}}$ and $I_\phi = I_L$

then $P_T = 3\frac{E_L}{\sqrt{3}} I_L \cos\theta_{I_\phi}^{V_\phi}$

Therefore,

$$P_T = \sqrt{3}E_L I_L \cos\theta_{I_\phi}^{V_\phi} = 3I_L^2 R_\phi \qquad \text{(W)}$$

Reactive Power The reactive power of each phase (in volt-amperes reactive) is

$$Q_\phi = V_\phi I_\phi \sin\theta_{I_\phi}^{V_\phi} = I_\phi^2 X_\phi = \frac{V_X^2}{X_\phi} \qquad \text{(VAR)}$$

The total reactive power of the load is

$$Q_T = 3Q_\phi \qquad \text{(VAR)}$$

$$Q_T = \sqrt{3}E_L I_L \sin\theta_{I_\phi}^{V_\phi} = 3I_L^2 X_\phi \qquad \text{(VAR)}$$

Apparent Power The apparent power of each phase is

$$S_\phi = V_\phi I_\phi \qquad \text{(VA)}$$

The total apparent power of the load is

$$S_T = 3S_\phi \qquad \text{(VA)}$$

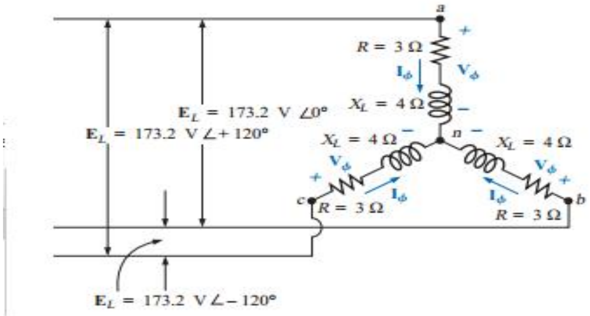
or

$$S_T = \sqrt{3}E_L I_L \qquad \text{(VA)}$$

Power Factor The power factor of the system is given by

$$F_p = \frac{P_T}{S_T} = \cos\theta_{I_\phi}^{V_\phi} \qquad \text{(leading or lagging)}$$

EXAMPLE For the Y-connected load of Fig.



a. Find the average power to each phase and the total load.
b. Determine the reactive power to each phase and the total reactive power.
c. Find the apparent power to each phase and the total apparent power.
d. Find the power factor of the load.

Solutions:

a. The *average power* is

$$P_\phi = V_\phi I_\phi \cos\theta_{I_\phi}^{V_\phi} = (100\text{ V})(20\text{ A})\cos 53.13° = (2000)(0.6) = \mathbf{1200\ W}$$

$$P_\phi = I_\phi^2 R_\phi = (20\text{ A})^2(3\ \Omega) = (400)(3) = \mathbf{1200\ W}$$

$$P_\phi = \frac{V_R^2}{R_\phi} = \frac{(60\text{ V})^2}{3\ \Omega} = \frac{3600}{3} = \mathbf{1200\ W}$$

$$P_T = 3P_\phi = (3)(1200\text{ W}) = \mathbf{3600\ W}$$

or

$$P_T = \sqrt{3}E_L I_L \cos\theta_{I_\phi}^{V_\phi} = (1.732)(173.2\text{ V})(20\text{ A})(0.6) = \mathbf{3600\ W}$$

b. The *reactive power* is

$$Q_\phi = V_\phi I_\phi \sin\theta_{I_\phi}^{V_\phi} = (100\text{ V})(20\text{ A})\sin 53.13° = (2000)(0.8) = \mathbf{1600\ VAR}$$

or

$$Q_\phi = I_\phi^2 X_\phi = (20\text{ A})^2(4\ \Omega) = (400)(4) = \mathbf{1600\ VAR}$$

$$Q_T = 3Q_\phi = (3)(1600\text{ VAR}) = \mathbf{4800\ VAR}$$

or

$$Q_T = \sqrt{3}E_L I_L \sin\theta_{I_\phi}^{V_\phi} = (1.732)(173.2\text{ V})(20\text{ A})(0.8) = \mathbf{4800\ VAR}$$

c. The *apparent power* is

$$S_\phi = V_\phi I_\phi = (100\text{ V})(20\text{ A}) = \mathbf{2000\ VA}$$

$$S_T = 3S_\phi = (3)(2000\text{ VA}) = \mathbf{6000\ VA}$$

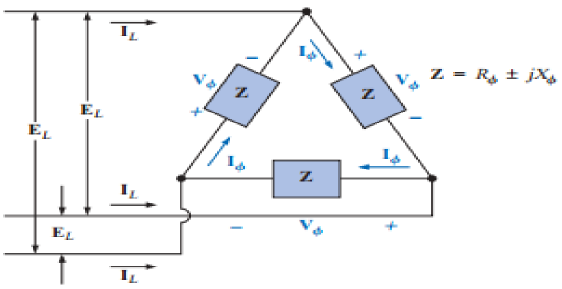
or

$$S_T = \sqrt{3}E_L I_L = (1.732)(173.2\text{ V})(20\text{ A}) = \mathbf{6000\ VA}$$

d. The *power factor* is

$$F_p = \frac{P_T}{S_T} = \frac{3600\text{ W}}{6000\text{ VA}} = \mathbf{0.6\ lagging}$$

Δ-Connected Balanced Load



Δ-connected balanced load.

Average Power

$$P_\phi = V_\phi I_\phi \cos\theta_{I_\phi}^{V_\phi} = I_\phi^2 R_\phi = \frac{V_R^2}{R_\phi} \quad \text{(W)}$$

$$P_T = 3P_\phi$$

Reactive Power

$$Q_\phi = V_\phi I_\phi \sin\theta_{I_\phi}^{V_\phi} = I_\phi^2 X_\phi = \frac{V_X^2}{X_\phi} \quad \text{(VAR)}$$

$$Q_T = 3Q_\phi \quad \text{(VAR)}$$

Apparent Power

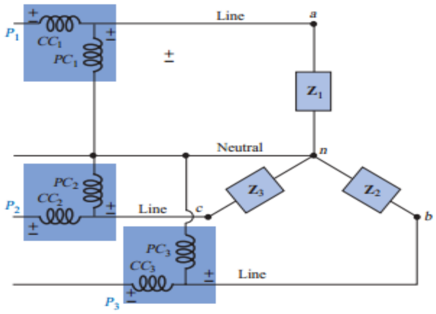
$$S_\phi = V_\phi I_\phi \quad \text{(VA)}$$

$$S_T = 3S_\phi = \sqrt{3}E_L I_L \quad \text{(VA)}$$

Power Factor

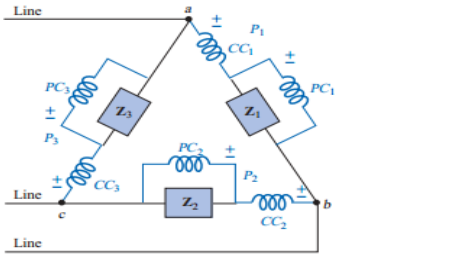
$$F_p = \frac{P_T}{S_T}$$

THE THREE-WATTMETER METHOD



Three-wattmeter method for a Y-connected load.

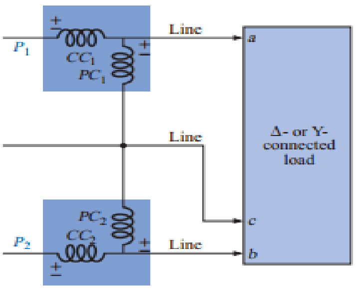
$$P_{T_Y} = P_1 + P_2 + P_3$$

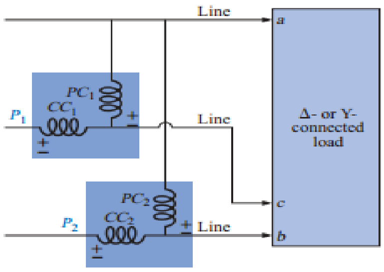


Three-wattmeter method for a Δ-connected load.

$$P_{T_\Delta} = P_1 + P_2 + P_3$$

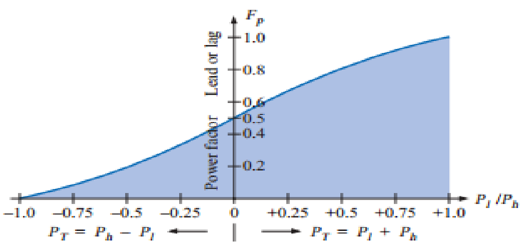
THE TWO-WATTMETER METHOD





$$P_T = P_h \pm P_l = \sqrt{3} E_L I_L \cos \theta_{I_\phi}^{V_\phi}$$

$$F_p = \cos \theta_{I_\phi}^{V_\phi} = \frac{P_h \pm P_l}{\sqrt{3} E_L I_L}$$



Determining whether the readings obtained using the two-wattmeter method should be added or subtracted.