وزارة التعليم العالي والبحث العلمي
الجامعة التقنية الجنوبية
المعهد التقني العمارة
قسم التقنيات الميكانيكية

الحقيبة التدريسية لمادة

الميكانيك الهندسي

الصف الاول

تدريسي المادة

م. احمد هاشم كريم

الفصل الدراسي
الاول

وزارة التعليم العالي والبحث العلمي
الجامعة التقنية الجنوبية
التخصصات / التكنولوجية
القسم الميكانيك الفرع / الإنتاج (مستمر)

الساعات الأسبوعية			السنة الدراسية	اسم المادة
total المجموع	pra. عملي	th. نظري	1st.stage الأولى	الميكانيك الهندسي (علم السكون) Engineering Static Mechanics
5	3	2		

Theoretical Subjects

Week No.	Subject Topics
1	Static, fundamental concepts , Force , Scalars and , Vectors , Units , Force polygon , Cartesian Components .
2	Analysis of Forces
3	Resultant of Concrrent , Coplanar Force system (2-D)
4	Moments
5	Moments
6	Couples, the transformation of the Couple and the force
7	Equilibrium, free body diagram (F.B.D.)
8	Equilibrium Conditions (2-D)
9	Equilibrium Conditions (2-D)
10	Friction, type of friction, Dry Friction
11	Center of Gravity, Centroid (length, area), Centroid of Simple area
12	Centroids of Composite areas.
13	Centroids of Composite areas.
14	Moment of inertia (Simple and Composite areas).
15	Moment of inertia (Simple and Composite areas).

الساعات الأسبوعية			السنة الدراسى	اسم المادة
totalالمجموع	pra.عملى	th.نظرى	1st.stage الأولى	الميكانيك الهندسي (علم الحركة) Engineering Dynamic Mechanics
5	3	2		

Theoretical Subjects

Week No.	Subject Topics
1	Newton's Second Law
2	Type of motion, Linear motion with constant speed.
3	Linear motion with Constant acceleration.
4	Curvilinear motion
5	Angular motion, Relative Motion
6	Work, Energy, Power
7	Strength of material: Fundamental concept
8	Loads, Stress, Strain, Elasticity, Plasticity, and Deformation.
9	Hook's Law, Stress -strain curve, type of stress.
10	Normal stress due to an axial load on 1-Uniformam Cross section area 2- Variable cross section area .
11	Shear Stress
12	Torsional Stress
13	Thermal Stress
14	Beams, types of loads, types of beams
15	Shear force (S.F.) & bending moment (B.M.) of Simple supported beam under an –axial load .

الهدف من دراسة مادة :

الهدف من دراسة الميكانيك الهندسي هو تمكين المهندسين من تحليل وتصميم وتصنيع وتطوير الأنظمة والآلات والمعدات الميكانيكية المختلفة .يركز هذا المجال على تطبيق مبادئ الفيزياء، وخاصة قوانين الحركة والقوة والطاقة، لفهم كيفية عمل الأشياء وكيفية تحسينها .يهدف المهندسون الميكانيكيون إلى إنشاء تقنيات تلبي الاحتياجات البشرية، من خلال تصميم وتصنيع منتجات وخدمات مبتكرة وفعالة في مختلف المجالات مثل الطاقة، والنقل، والرعاية الصحية، وغيرها.

الفئة المستهدفة

طلبة الصف الاول /قسم التقنيات الميكانيكية

التقنيات التربوية المستخدمة :

1- سبورة واقلام
2- السبورة التفاعلية
3- عارض البيانات Data show
4- جهاز حاسوب محمول Laptop

Ministry of Higher Education
And Scientific Research
Southern Technical University
Technical Institute Of Amara

Engineering Mechanics

The References

- **Engineering-Mechanics-Statics-R.C.-Hibbeler**
- **Singer, "Engineering-Mechanics"**
- **Hidgon and Stile "Engineering-Mechanics"**

Mechanics define:-

Mechanics is the physical science that deals with the behavior of bodies under the influence of forces.

Mechanics can be divided into:

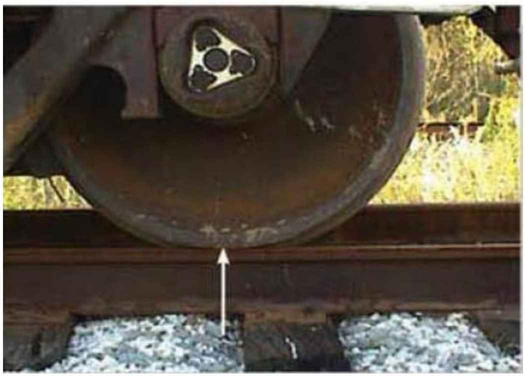
1. Rigid-body Mechanics
2. Deformable-body Mechanics
3. Fluid

Rigid-body Mechanics deals with

- Statics – Equilibrium of bodies; at rest or moving with constant velocity
- Dynamics – Accelerated motion of bodies.

Basic Quantities

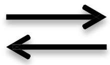
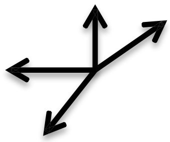
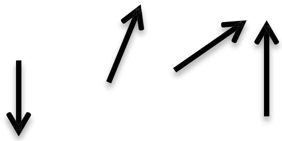
- Length - locate the position of a point in space
- Mass - measure of a quantity of matter
- Time - succession of events
- Force - any action which change or try to change the shape ,volume or the motion of a body.
- Particle - has a mass and size can be neglected
- Rigid Body - a combination of a large number of particles
- Concentrated Force - the effect of a loading



Physical Quantities is classified to:-

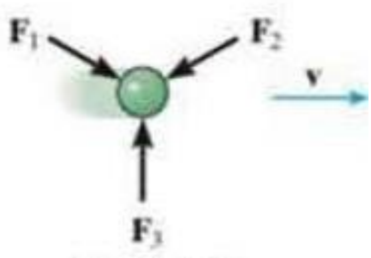
1. Scalar quantities :have only magnitude(mass ,volume)
2. Vector quantities :have both magnitude and direction(couple,force)

Classification of forces :

1. Collinear 
2. Parallel forces 
3. Concurrent forces 
4. Non parallel , non -concurrent forces 

Newton's Laws of Motion

• First Law - A particle originally at rest, or moving in a straight line with constant velocity, will remain in this state provided that the particle is not subjected to an unbalanced force.



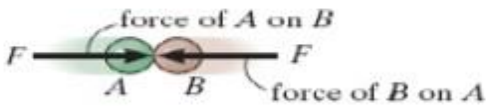
Equilibrium

• Second Law - A particle acted upon by an unbalanced force F experiences an acceleration a that has the same direction as the force and a magnitude that is directly proportional to the force.

$$F = ma$$

Accelerated motion

• Third Law - The mutual forces of action and reaction between two particles are equal and, opposite and collinear.



Action – reaction

Unit Measurement

1- SI

The International System of Units (abbreviated as SI, from the French System international) is the modern form of the metric system, and is the most widely used system of measurement. ($g = 9.81\ m/s^2$)

2- U.S customary

United States customary units are a system of measurements commonly used in the United States. The United States customary system developed from English units which were in use in the British Empire before the U.S. became an independent country. ($g = 32.2\ ft/s^2$)

TABLE 1–1 Systems of Units

Name	Length	Time	Mass	Force
International System of Units SI	meter m	second s	kilogram kg	newton* N $\left(\frac{kg \cdot m}{s^2}\right)$
U.S. Customary FPS	foot ft	second s	slug* $\left(\frac{lb \cdot s^2}{ft}\right)$	pound lb

*Derived unit.

TABLE 1–2 Conversion Factors

Quantity	Unit of Measurement (FPS)	Equals	Unit of Measurement (SI)
Force	lb		4.448 N
Mass	slug		14.59 kg
Length	ft		0.304 8 m

❖ Resultant the vector :

The resultant force is the force which can replace the original system without changing its external effects on rigid bodies .There are two methods for founding the resultant force:-

1- Parallelogram law.

The parallelogram of forces is a method for solving the results of applying two forces to an object.

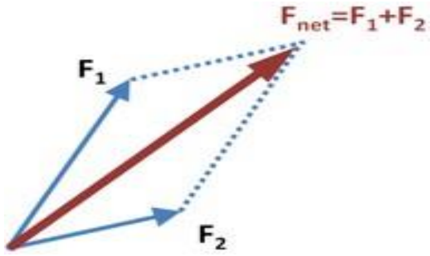


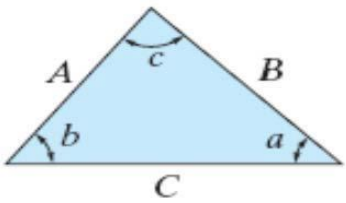
Figure 1: Parallelogram construction for adding vectors

2- Trigonometry.

Triangle law of forces states that, If two forces acting at a point are represented in magnitude and direction by the two adjacent sides of a triangle taken in order, then the closing side of the triangle taken in the reversed order represents the resultant of the forces in magnitude and direction.

Procedure for Analysis

- Redraw half portion of the parallelogram
- Magnitude of the resultant force can be determined by the law of cosine
- Direction if the resultant force can be determined by the law of sine
- Magnitude of the two components can be determined by the law of sine



Cosine law:-

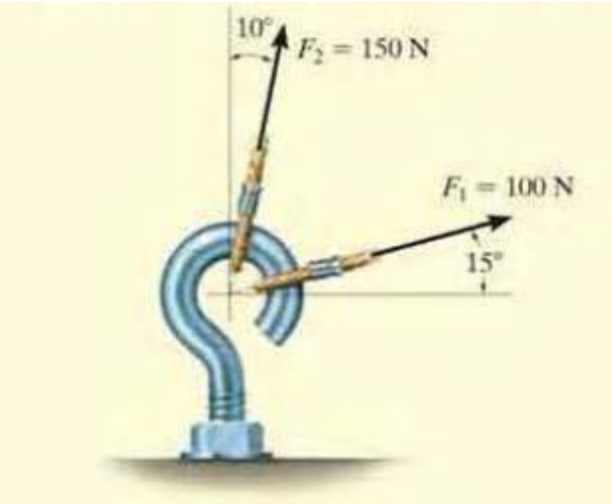
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:-

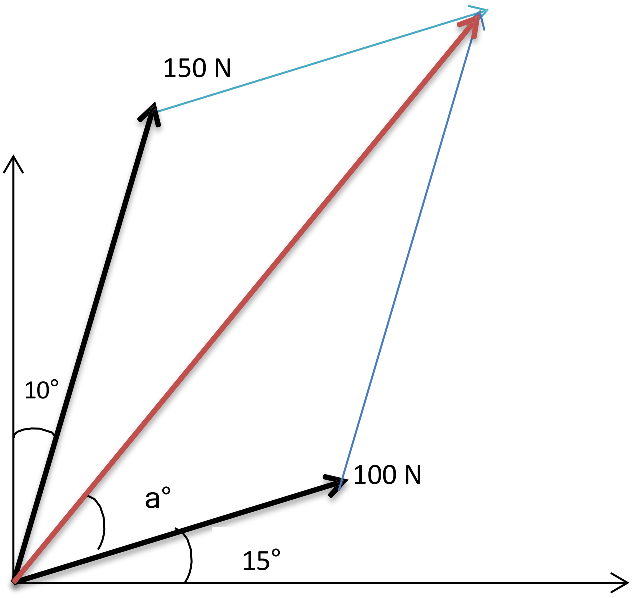
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Ex1

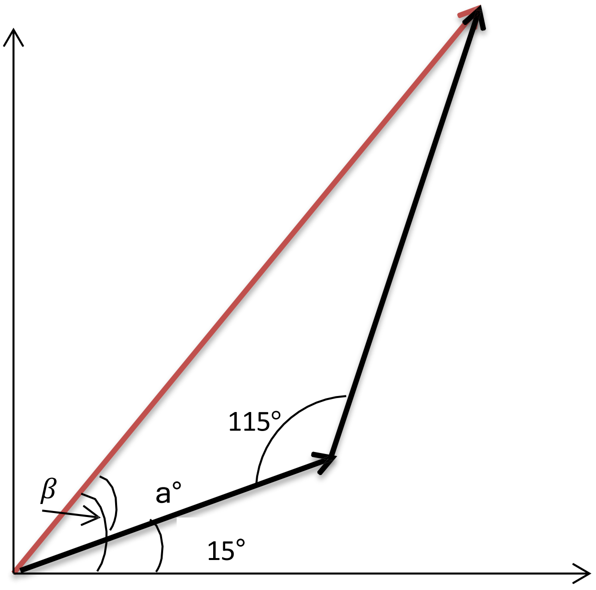
The screw eye in Fig. below is subjected to two forces, F_1 and F_2. Determine the magnitude and direction of the resultant force.



Solution:-



a- Parallelogram



b- Trigonometry

$$R = \sqrt{A^2 + B^2 - 2AB\cos c}$$

$$R = \sqrt{100^2 + 150^2 - 2*100*150\cos(115°)}$$

R= 213 N

Sine law:-

$$\frac{A}{\sin a} = \frac{R}{\sin c} \longrightarrow \frac{150}{\sin a} = \frac{213}{\sin(115°)}$$

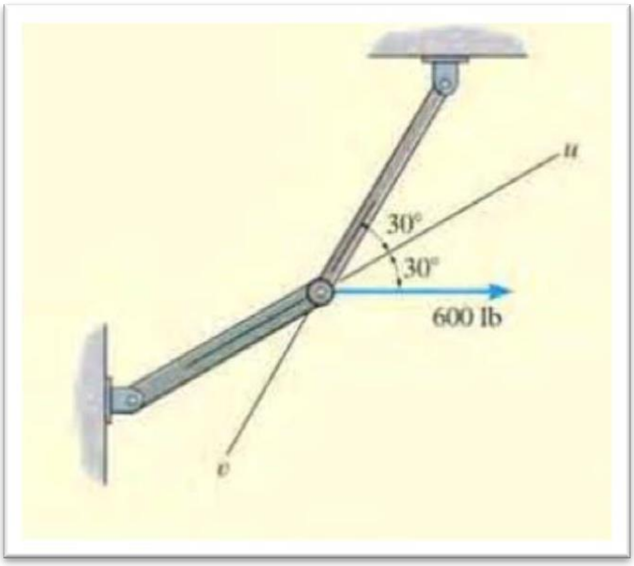
a° = 40

direction R measured from the horizontal. Is

β° =40° + 15° = 55°

Ex 2

Resolve the horizontal 600-lb force in Fig below into two components acting along the u and v axes and determine the magnitude of these components.



Ex 2

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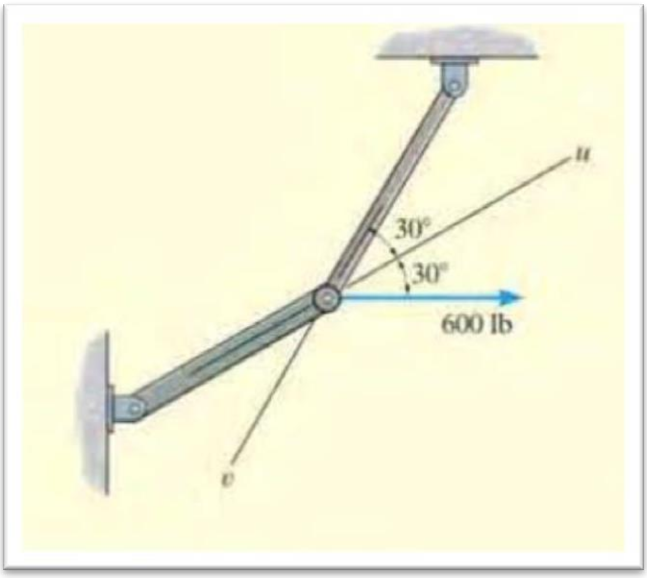
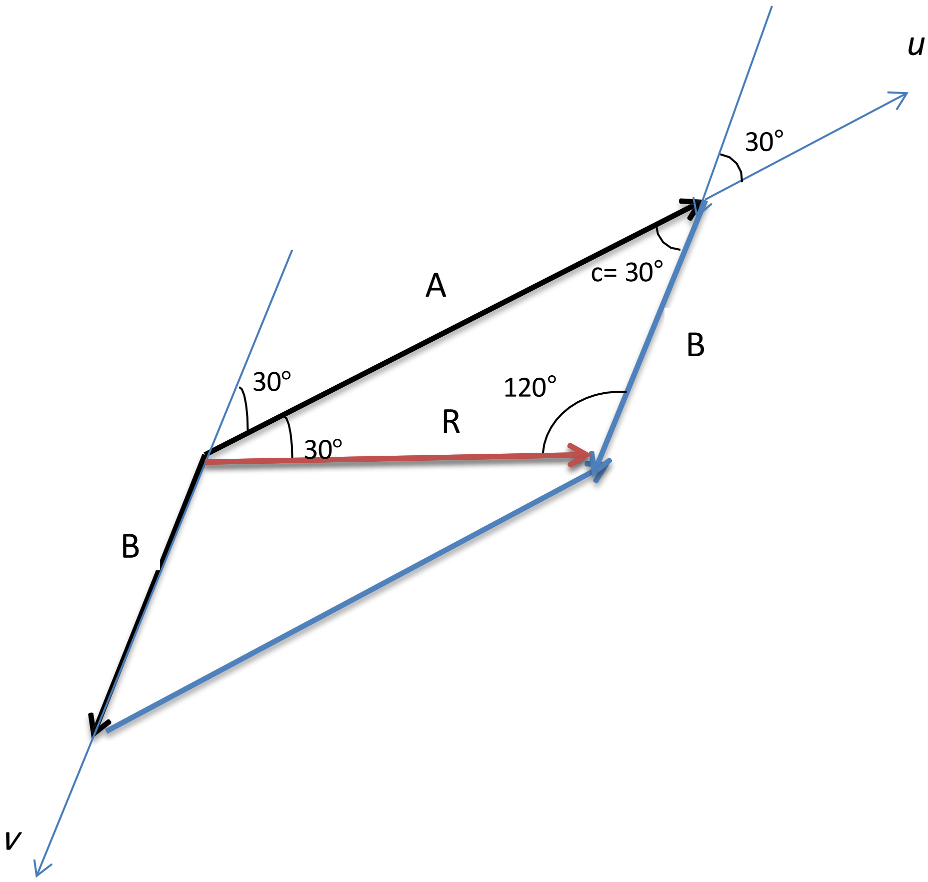


SOL:-



Sine law:-

$$\frac{A}{\sin\ a} = \frac{R}{\sin c} \longrightarrow \frac{A}{\sin(120°)} = \frac{600\ Ib}{\sin(30°)}$$

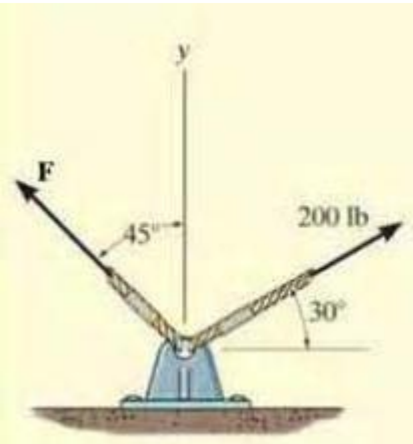
A = 1039 Ib

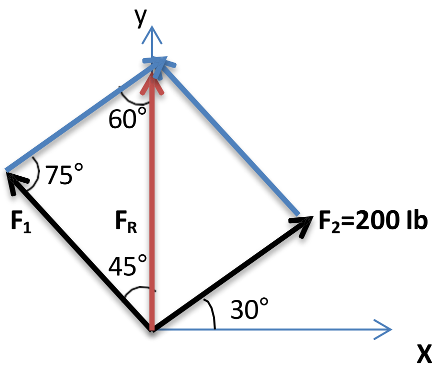
$$\frac{B}{\sin\ a} = \frac{R}{\sin c} \longrightarrow \frac{B}{\sin(30°)} = \frac{600\ Ib}{\sin(30°)}$$

A = 600 Ib

Ex 3

Determine the magnitude of the component force **F** in Fig. below and the magnitude of the resultant force **FR** if **FR** is directed along the positive y axis.





SOL:-

Sine law:-

$$\frac{F_1}{\sin a} = \frac{F_2}{\sin b} \longrightarrow \frac{F_1}{\sin(60°)} = \frac{200}{\sin(45°)}$$

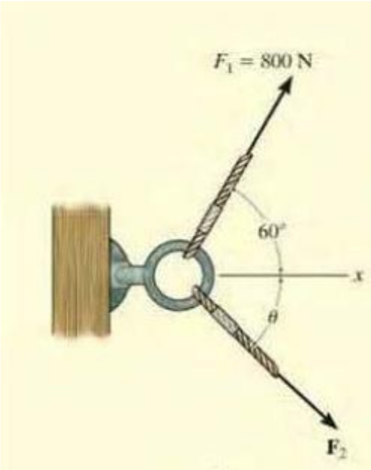
F_1 = 244 Ib

$$\frac{F_2}{\sin b} = \frac{F_R}{\sin c} \longrightarrow \frac{200}{\sin(45°)} = \frac{F_R}{\sin(75°)}$$

F_R = 273 Ib

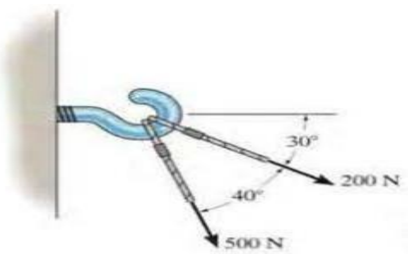
H.W

1- It is required that the resultant force acting on the eyebolt in Fig. below be directed along the positive x axis and that F_2 have a minimum magnitude. Determine this magnitude the angle θ, and the corresponding resultants force.

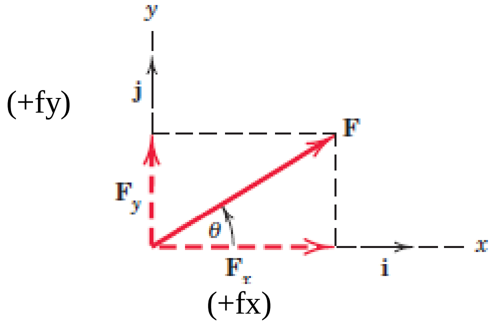


2-Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.

3-Two forces act on the hook. Determine the magnitude of the resultant force.



Rectangular Components



$fx = F \sin \theta°$

$fy = F \text{ cosine } \theta°$

$F = \sqrt{fx^2 + fy^2}$

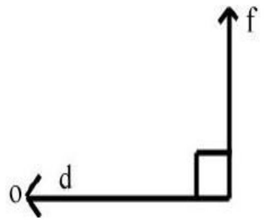
$\theta = \tan^{-1} \left(\frac{fy}{fx}\right)$

note :- fy = ↑ + ↓ -   fx = → + ← -

Moment of force

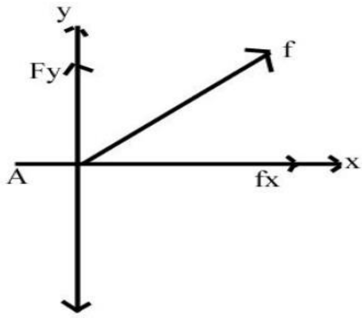
Define :- the product of the magnitude of the force by the perpendicular distance (arm) from the point to the action line of the force. It's units are N.m, lb. ft, ect.

$M_o = f \cdot d$



Varignon's theorem:-

that the moment of a resultant of two concurrent forces about any point is equal to the algebraic sum of the moments of its components about the same point.

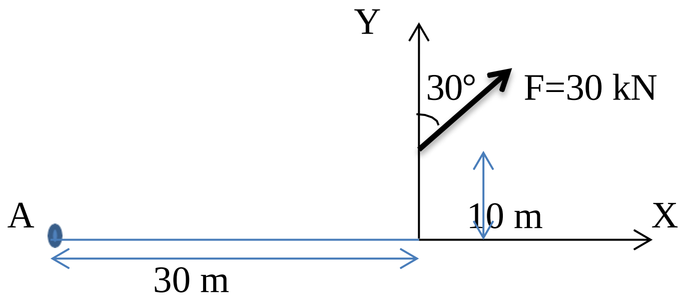


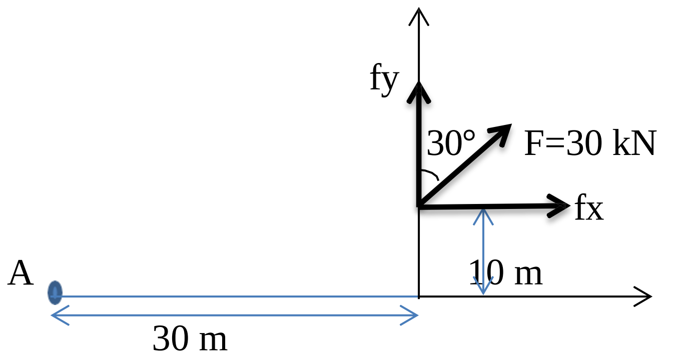
$MA = MA^{fx} + MA^{fy}$

Note:- moment its **(-)** value ,if the force rotates **clockwise**.

moment its **(+)**value ,if the force rotates **counter clockwise**

<u>Ex :-</u> Find the moment of force (f) about point (A) as shown in the following figure below.





Sol:-

fx = F sin 30°

= 30 sin 30°

= 15 kN

fy = F cosin 30°

= 30 cosin 30°

= 25.9 kN

$MA = MA^{fx} + MA^{fy}$

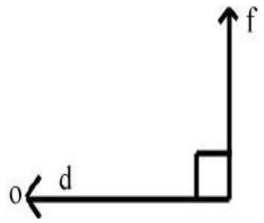
= fx * ry + fy * rx
= - (15 * 10) + 25.9 * 30
= 627 kN.m

Another method?

Moment of force

Define :- the product of the magnitude of the force by the perpendicular distance (arm) from the point to the action line of the force. It's units are N.m, lb. ft, ect.

$$M_o = f \cdot d$$



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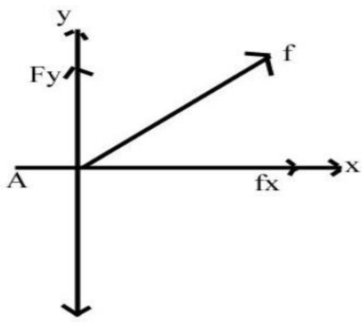

$$MA = MA^{fx} + MA^{fy}$$

Note:- moment its **(-)** value ,if the force rotates **clockwise**.

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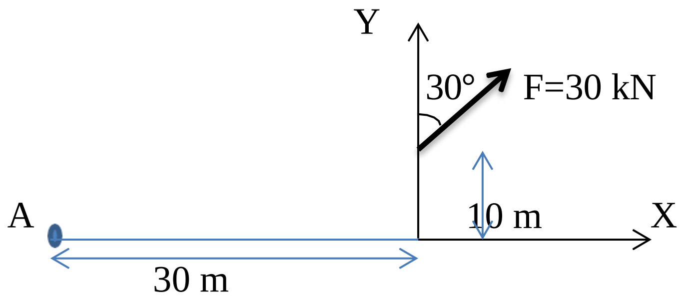
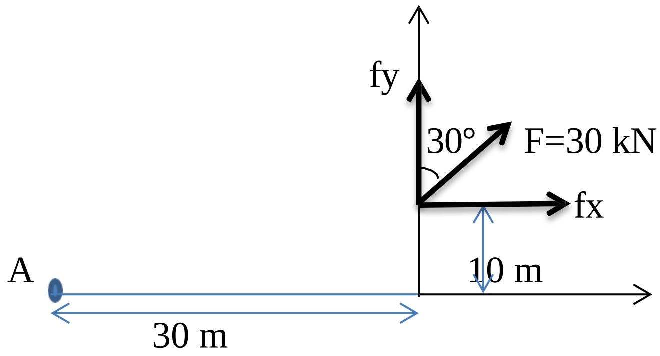
Ex 1:- Find the moment of force (f) about point (A) as shown in the following figure below.



Sol:-



fx = F sin 30°

= 30 sin 30°

= 15 kN

fy = F cosin 30°

= 30 cosin 30°

= 25.9 kN

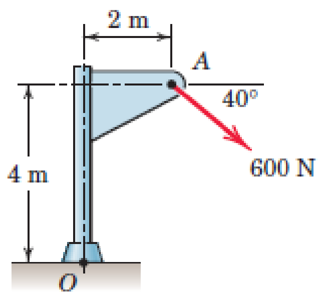
$MA = MA^{fx} + MA^{fy}$

= fx * ry + fy * rx
= - (15 * 10) + 25.9 * 30
= 627 kN.m

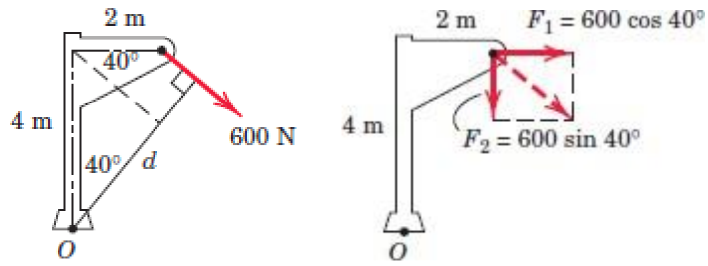
Another method?

Ex 2

Calculate the magnitude of the moment about the base point O of the 600-N force.



sol :-



Replace the force by its rectangular components at A,

F_1 = 600 cos (40) = 460 N,

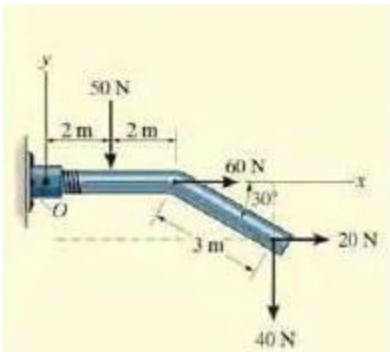
F_2 = 600 sin (40) = 386 N

By Varignon's theorem, the moment becomes

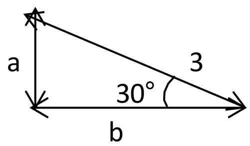
*MO = 460 *(4) + 386 *(2) = 2610 N.m*

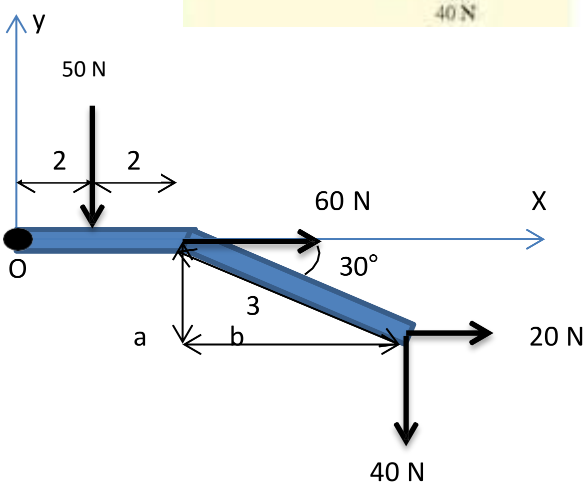
Ex 3

Determine the rcsu1tant moment of the four forces acting on the rod shown in Fig. below about point *0*.



,sol :-





Mo = -(50 * 2) + (60 * (0)) + (20 * a) - (40 * (b + 4))

$\cos 30 = \frac{b}{3}$ ⟶ b = 2.59

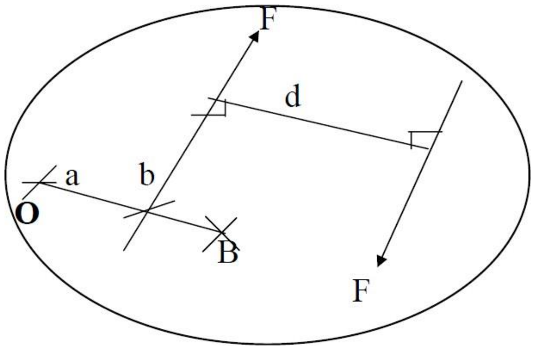
$\sin 30 = \frac{a}{3}$ ⟶ a = 1.5

Mo = -(50 * 2) + (60 * (0)) + (20 * 1.5) - (40 * (2.59 + 4))

Mo = - 333.6 N.m

Moment of a Couple
(عزم الازدواج)

A Couple is defined as two parallel forces that have the same magnitude. but opposite directions. and arc separated by a perpendicular distance (d). Fig. below.



M_o = -F * a + F (a+ d)

= -F*a + F*a + F * d

= F * d

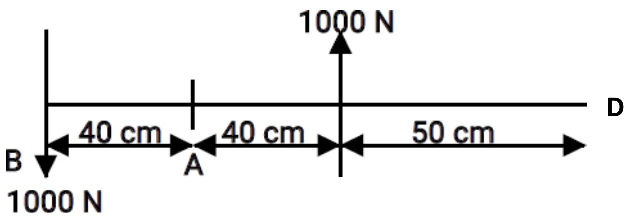
M_B = F * d + F (d – b)

= F * b + F * d – F * b

= F * d ⟶ M_c = F*d

Ex1:-

Determine the moment of the couple shown in figure about the axis through Points A,B,D.



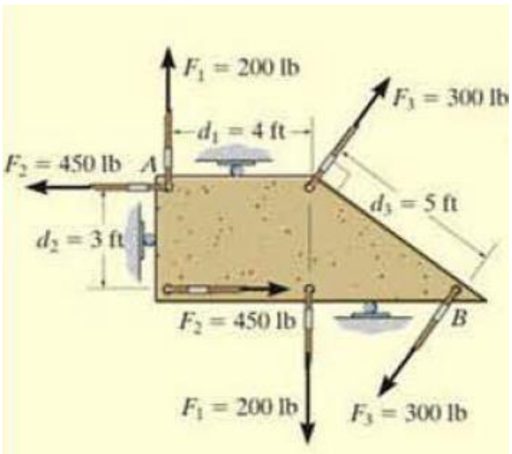
Sol:-

Mc(A)=1000× 40+1000× 40 = 80000 N.Cm

Mc(B)=1000× (40+40) = 80000 N.Cm

Mc(D)=1000× (40+40+50)-1000× 50 = 80000 N.Cm

Ex 2 Determine the resultant couple moment of the three couples acting on the plate in Fig. below.



Sol:-

MR = - M1 + M2 - M3

= -(200 * 4) + (450 * 3) – (300 * 5)

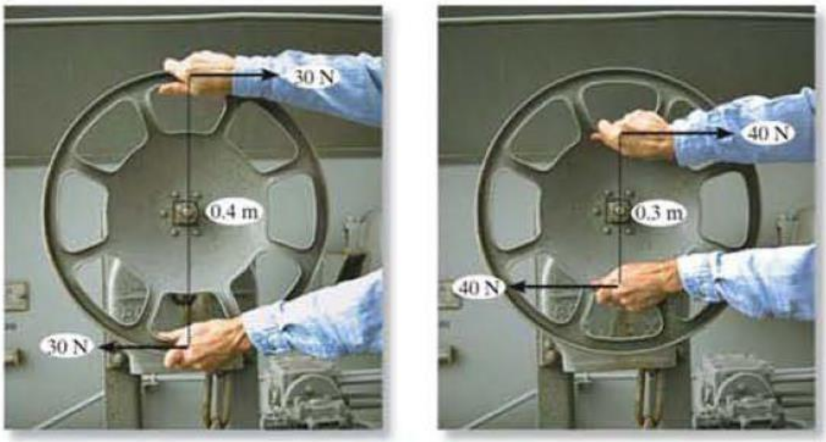
= -800+1350-1500

MR = -950 Ib.ft (clockwise)

Equivalent Couples.

(عزم الازدواج المتكافئ)

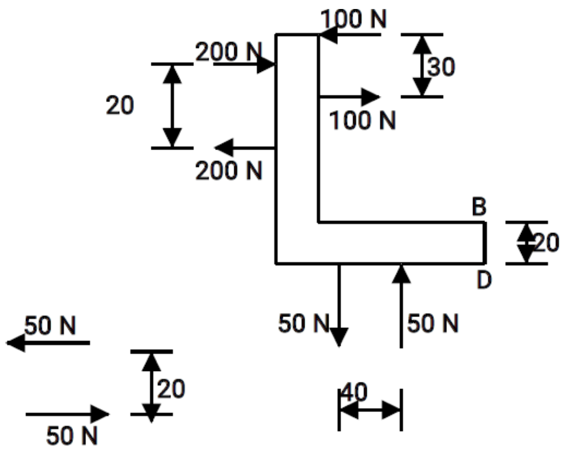
If two couples produce a moment with the same magnitude and direction then these two couples are Equivalent For example. The two couples shown in Fig, below are Equivalent because each couple moment has a magnitude of $M = 30\ N * 0.4\ m = 40\ N * 0.3\ m = 12\ N\cdot m$



Ex 4:

Replace the following couples shown in figure by a single couple its forces effects horizontally at points B,D.

Sol:-



Mc=-200× 20+100× 30+50× 40

=1000N.Cm

Mc=F . d

1000=F × 20

F=50N

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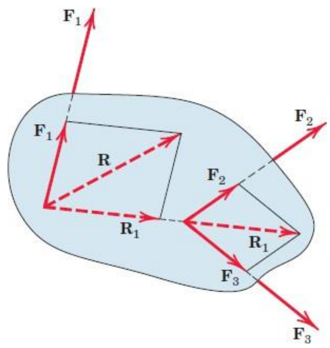
Resultant Non-Concurrent Coplanar Forces

(محصلة القوى الغير متلاقية الواقعة في نفس المستوى)



1- Resolve the forces in to x and y compent

2- $R_x = \sum F_x$

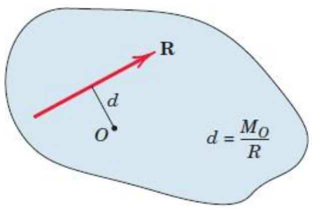
3- $R_y = \sum F_y$

4- $R = \sqrt{R_x^2 + R_y^2}$

5- $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

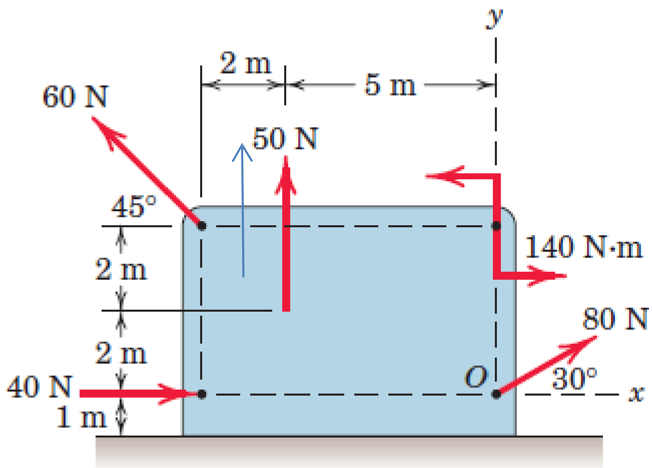
6- $M_O = \sum M$

7- $M_O = R * d$ → $d = \frac{M_O}{R}$



Ex1 :-

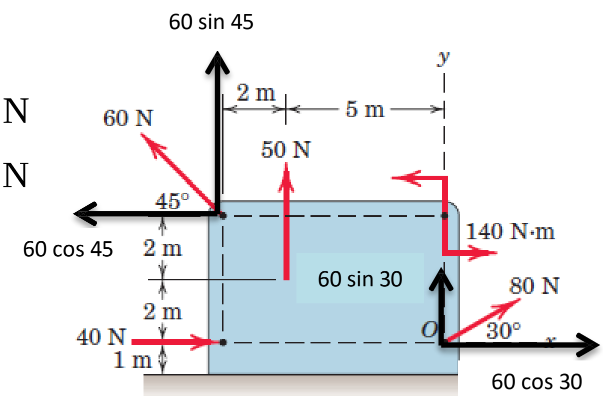
Determine the resultant of the four forces and one couple which act on the plate shown.



sol :-

$R_x = \sum F_x = 40 + 80\cos(30) - 60\cos(45) = 66.9$ N

$R_y = \sum F_y = 50 + 80\sin(30) + 60\sin(45) = 132.4$ N



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$$R = \sqrt{Rx^2 + Ry^2}$$

$$R = \sqrt{66.9^2 + 132.4^2} = 148.3 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{Ry}{Rx}\right)$$

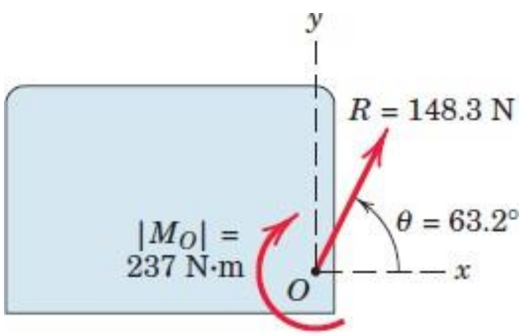
$$\theta = \tan^{-1}\left(\frac{132.4}{66.9}\right) \longrightarrow \theta = 63.2°$$

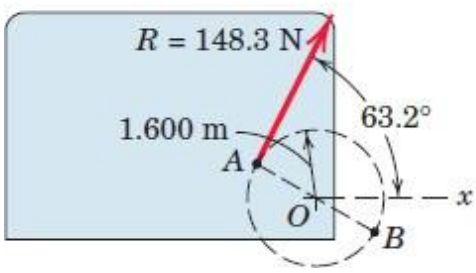
$$M_O = \sum M$$

$$M_O = 140 - 50*5 + 60\cos(45)*4 - 60\sin(45)*7 = -237 \text{ N.m}$$

$$M_O = R * d$$

$$237 = 148.3 * d \longrightarrow d = \frac{237}{148.3} = 1.6 \text{ m}$$





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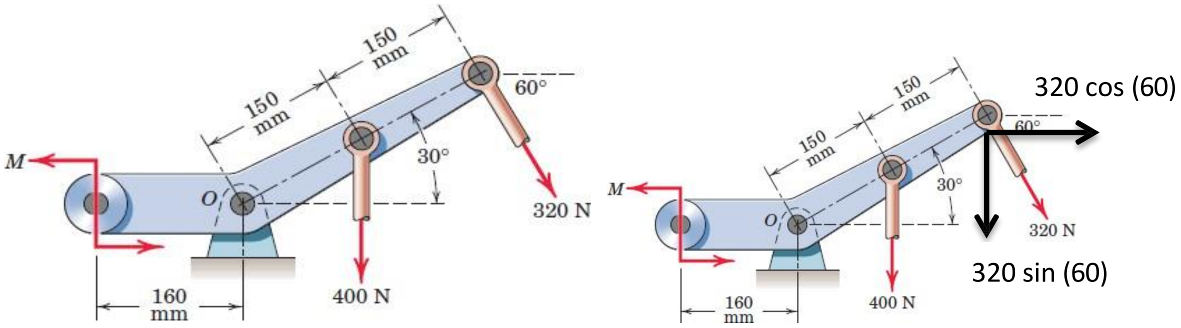
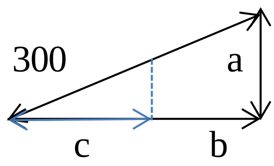
Ex 2:-

If the resultant of the two forces and couple *M* passes through point O, determine the resultant ***(R)*** and *M*.



Sol :-



$\sin 30 = \frac{a}{300}$ ⟶ a = 300 * sin 30 = 150 mm

$\cos 30 = \frac{b}{300}$ ⟶ b = 300 * cos 30 = 259.8 mm

$\cos 30 = \frac{c}{300}$ c = 300 * cos 30 = 129.9 mm

$R_x = \sum F_x = 320 \cos(60) = 160$ N

$R_y = \sum F_y = -400 - 320 \sin(60) = -677$ N

$$R = \sqrt{R_x^2 + R_y^2}$$

$$R = \sqrt{160^2 + (-677)^2} = 695.65 \text{ N}$$

For the resultant to pass through O the moment about O must be zero.

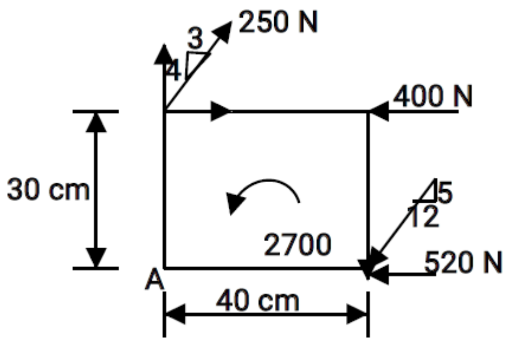
$\sum M_O = 0$

$M_O = M - 400 * 129.9 - 320 \cos(60) * 150 - 320 \sin(60) * 259.8 = 0$

$M = -147957$ N.mm ⟶ = 148 N.m

Ex 3:-

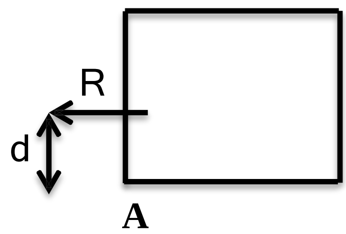
Determine the resultant of the forces and the couple shown in figure and locate it with respect to point (A).



Sol :-

Rx =250 * 3/5 - 520 * 12/13 - 400

=-730 N=730N ←



Ry=250 * 4/5 – 520 * 5/13 = 0

$R = \sqrt{Rx^2 + Ry^2}$

$R = \sqrt{(-730)^2 + (0)^2} = -730\ N$ ←

R=730 N

R * d= ΣMa

730 * d=-250 * 3/5 * 30+400 * 30 - 520 * 5/13 * 40+ 2700

d =3 Cm

Equilibrium

(التوازن)

When a body is in equilibrium, the resultant of all forces acting on it is zero. Thus, the resultant forces Rx, Ry and the resultant moment M are both zero, and we have the equilibrium equations.

Three equation equilibrium :-

$\sum Fx = 0$

$\sum Fy = 0$

$\sum Mo = 0$

Support reaction

A reaction force is a force that acts in the opposite direction to an action force. Friction is the reaction force resulting from surface interaction and adhesion during sliding. Reaction forces and reaction moment are usually the result of the actions of applied forces.

Figures shows types support reaction

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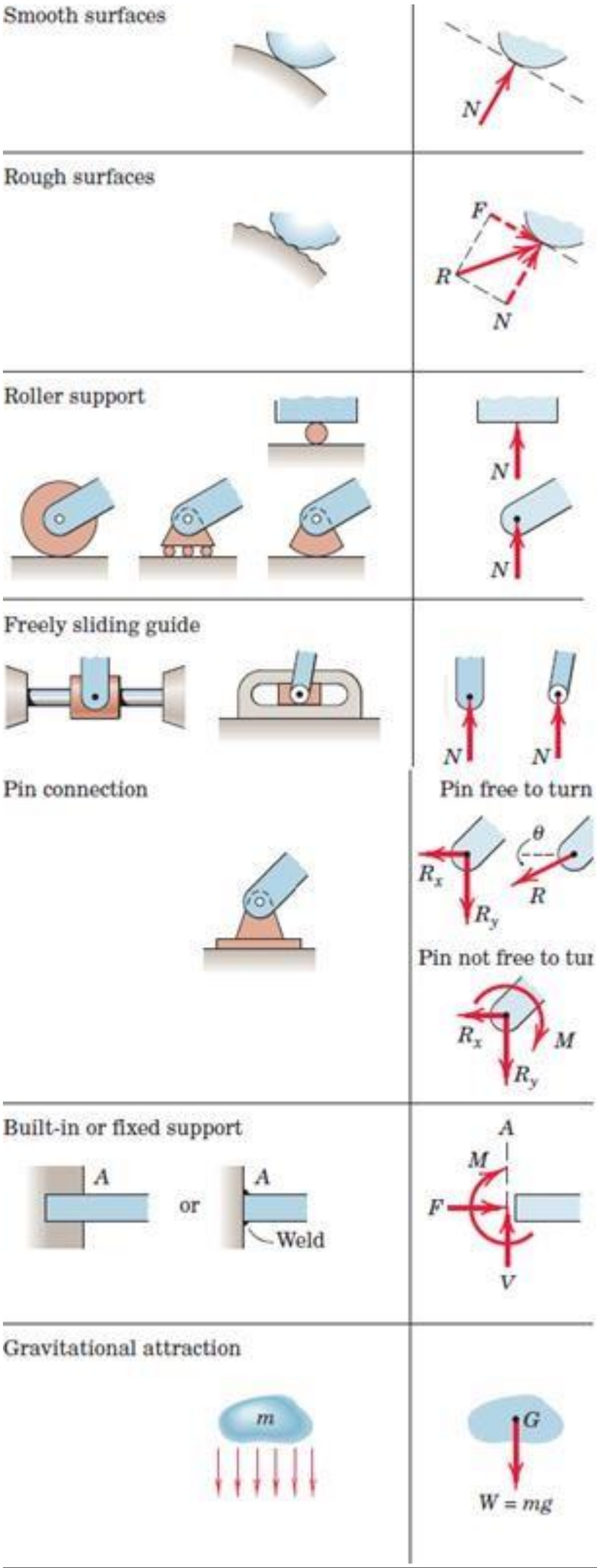
- **Type of support**




Rollor reaction

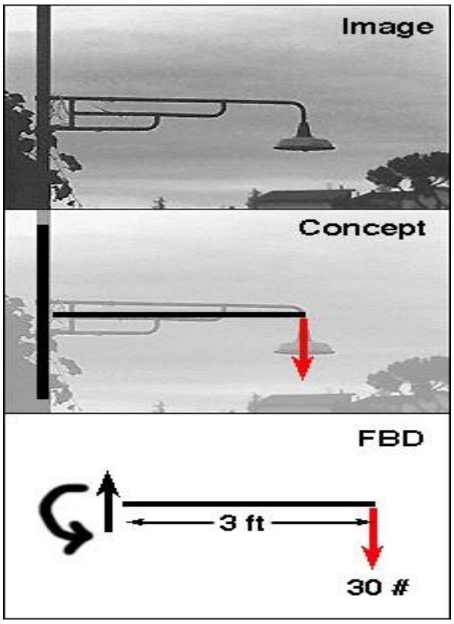

Hinge reaction

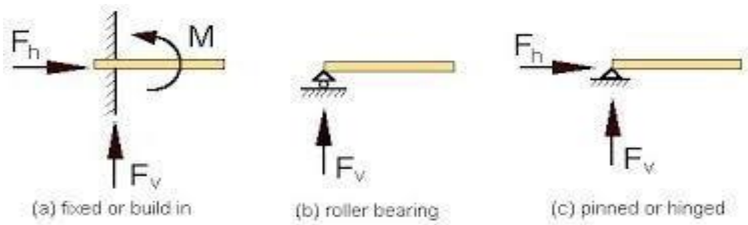

Fixed reaction

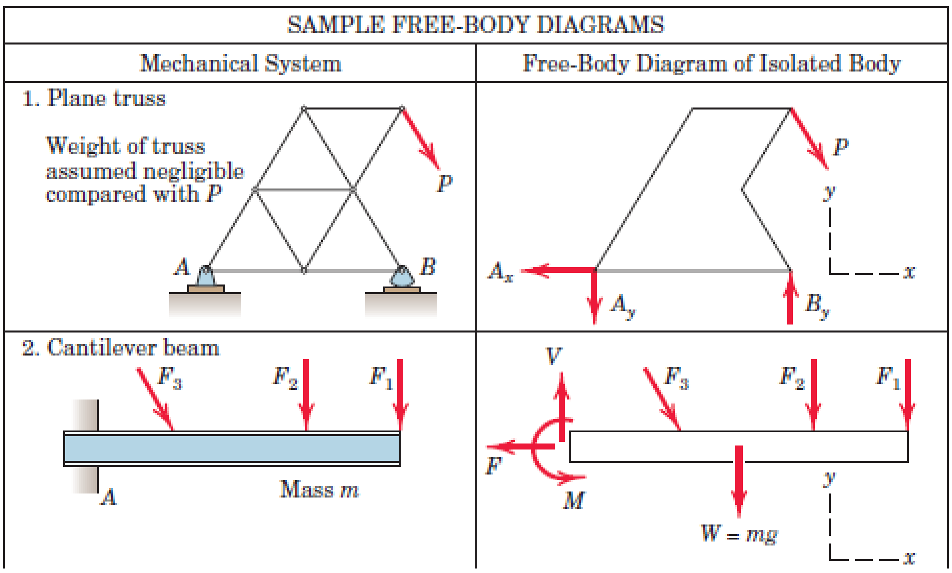
Free body diagram (FBD) (مخطط الجسم الحر)

The diagram shows all forces applied to objects and forces of reaction to the body after removal of supports.

(ويمثل الرسن البياني لجميع القوى المسلطة علي الاجسام و قوى رد الفعل للجسمن بعد ازالة الاسناد)







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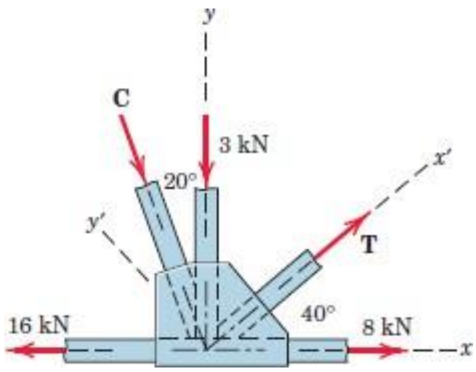
Ex 1

Determine the magnitudes of the forces C and T, which, along with the other three forces shown, act on the bridge-truss joint.

[∑Fx = 0]
[ΣFy = 0]
[ΣMo = 0]



SOL:-

[∑Fx = 0]

$8 + T\cos 40 + C\sin(20) - 16 = 0$

$0.766T + 0.342C = 8 \longrightarrow$ (a)

[ΣFy = 0]

$T\sin 40 - C\cos 20 - 3 = 0$

$0.643T - 0.940C = 3 \longrightarrow$ (b)

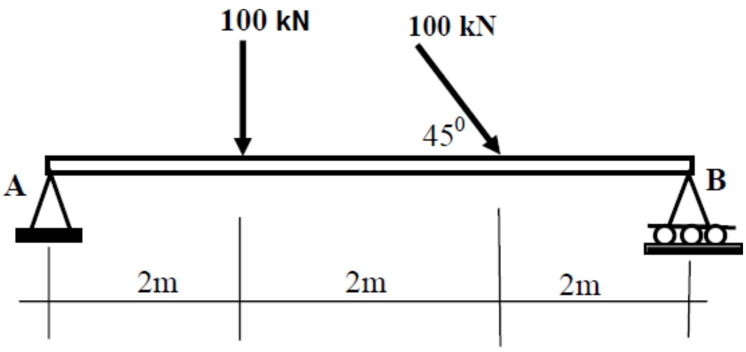
From (a) and (b)

T = 9.09 kN

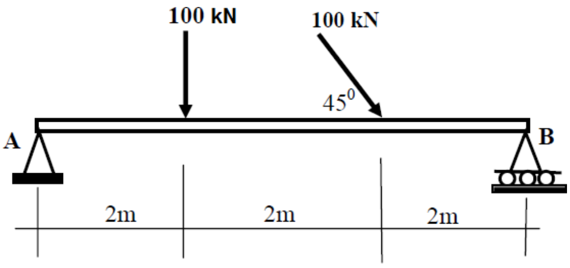
C = 3.03 kN

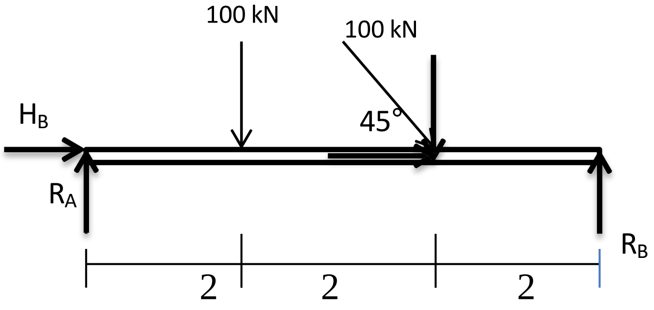
Ex 2

Find supports reaction at support (A , B) for the beam shown below.



Sol:-





F.B.D

SOL:-

$[\sum Fx = 0]$, $[\Sigma Fy = 0]$, $[\Sigma Mo = 0]$

$[\sum Fx = 0]$

$H_B + 100 \cos(45) = 0$

$H_B = -70.71$ kN $\longleftarrow$

$\Sigma Fy = 0$

$R_A + R_B - 100 - 100 \sin(45) = 0$

$R_B = 170.71 - R_A$ eq (1)

$[\Sigma M_A = 0]$

$R_A * 6 - (100 \sin 45) * 4 - 100 * 2 = 0$

$R_{A} = 80.47$ kN

By solve (1)

$R_{A} = 90.23$ kN

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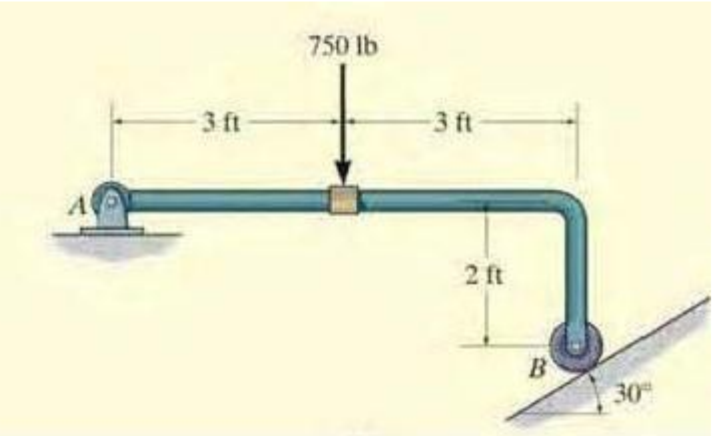
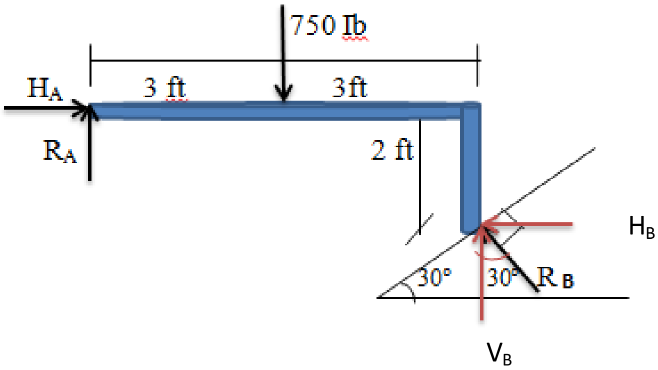
Ex 3

Determaine the horizontal and verlical components of reaction on the member at the pin ***A***, and the normlal rcaction at the roller ***B*** in figure below.



SOL:-



$H_B = R_B \sin 30$

$V_B = R_B \cos 30$

$[\Sigma M_A = 0]$

$(R_B \cos30)*6 - (R_B \sin 30)*2 - 750*3 = 0$

$R_B = 536.2$ Ib

$[\sum Fx = 0]$

$H_A - H_B = 0 \longrightarrow H_A - 536.2 \sin 30 = 0$

$H_A = 268$ Ib

$\Sigma Fy = 0$

$R_A + V_B - 750 = 0$

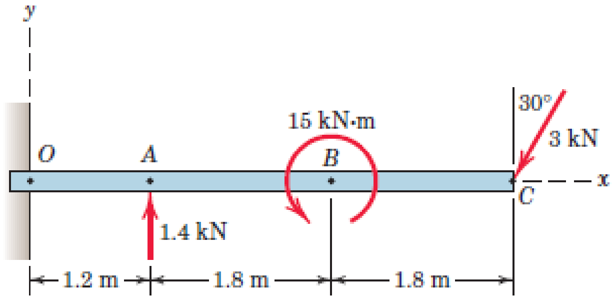
$R_A + 536.2 \cos(30) - 750 = 0$

$R_A = 285.6$ Ib

H . W

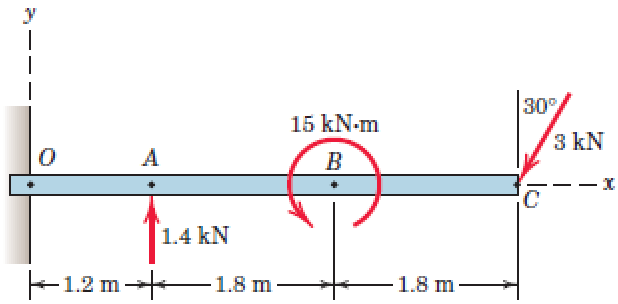
Q1

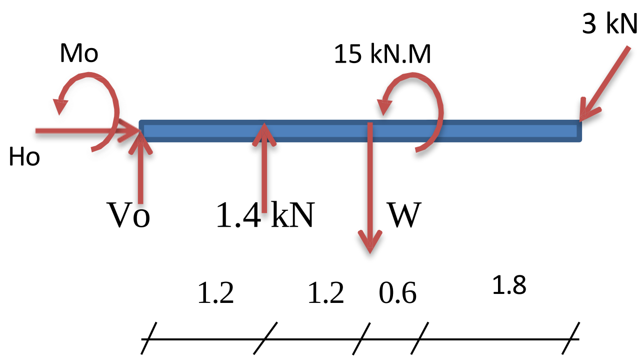
The 500-kg uniform beam is subjected to the three external loads shown. Determine the reactions at the support point O. The x-y plane is vertical.



Ex1

The 500-kg uniform beam is subjected to the three external loads shown. Determine the reactions at the support point O. The x-y plane is vertical.





F.B.D

sol :-

$W = m.g$

$= 500 * 9.81 = 4905\ N = 4.905\ kN$

$[\Sigma M_o = Mo]$

$-[(3\cos 30) * 4.8] - [(4.9) * 2.4] + 1.4 * 1.2 + 15 = Mo$

$Mo = -7.55$ kN.m (counter clockwise)

$[\sum Fx = 0]$

$H_o - 3\sin(30) = 0$

$H_o = 1.5\ kN$

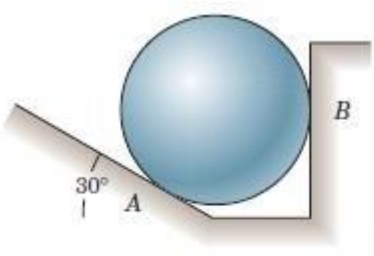
$\Sigma Fy = 0$

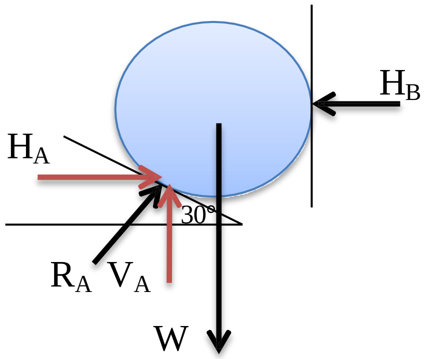
$V_O + 1.4 - 4.9 - 3\cos(30) = 0$

$V_O = 6.1\ kN$

Ex2

The 50-kg homogeneous smooth sphere rests on the incline ***A*** and bears against the smooth vertical wall ***B.*** Calculate the contact forces at ***A*** and ***B.***





Sol :-

$W = m.g$

$= 50 * 9.81 = 490.5$ N

$\Sigma Fy = 0$

$V_A - W = 0$

$V_A - 490.5 = 0$

$V_A = 490.5$ N

$V_A = R_A \cos 30$

$490.5 = R_A \cos 30$

$R_A = 566.38$ N

$H_A = R_A \sin(30)$

$= 566.38 \sin(30) = 283.2$

$[\sum Fx = 0]$

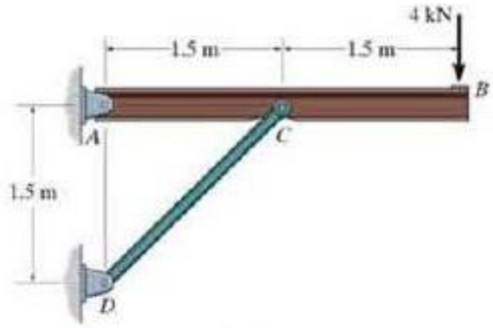
$H_A - H_B = 0$

$283.2 - H_B = 0$

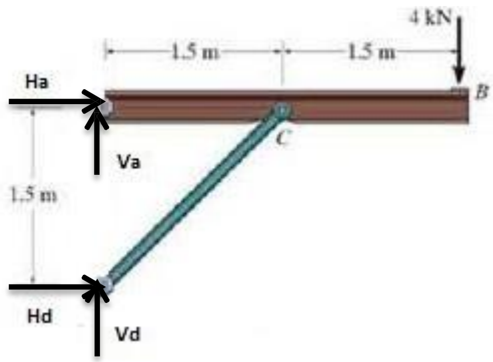
$H_B = 283.2$ N

Ex3

Determine the horizonal and vertiacl reaction at the pin A and the reaction on the beam at C.



Sol :-



F.B.D

$[\Sigma M_A = 0]$

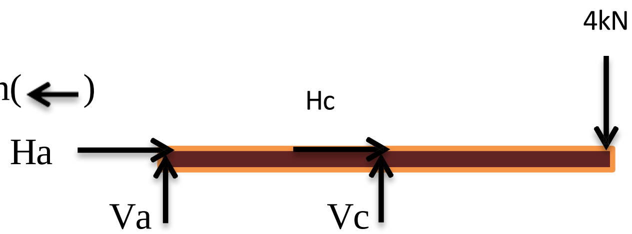
$[H_A * 1.5] - [4 * 3] = 0$

$H_A = 8$ kN

$[\sum Fx = 0]$

$H_A + H_D = 0$

$8 + H_D = 0 \longrightarrow H_D = -8$ Kn(←)



$[\Sigma M_A = 0]$

$[Vc * 1.5] - [4 * 3] = 0 \longrightarrow Vc = 8$ kN

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$\Sigma Fy = 0$

$V_a + V_c - 4 = 0$

$V_a + 8 - 4 = 0$

$V_a = -4$ kN ↓

$[\sum Fx = 0]$

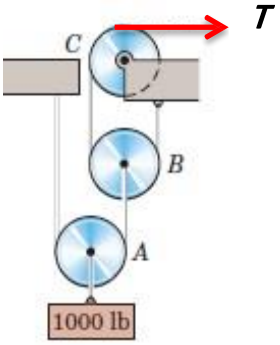
$H_A + H_c = 0$

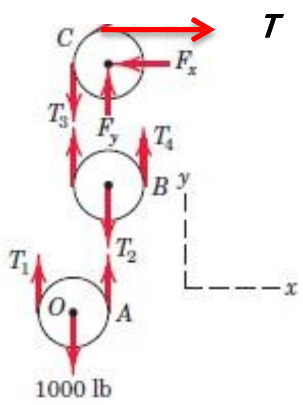
$8 + H_c = 0 \longrightarrow H_c = -8$ kN ←

Equilibrium of pulley

Ex1

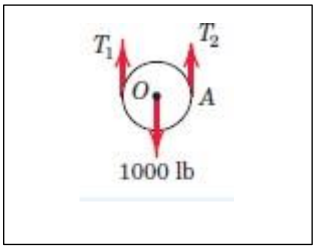
Calculate the tension *T* in the cable which supports the 1000-lb load with the pulley arrangement shown. Each pulley is free to rotate about its bearing, Find the magnitude of the total force on the bearing of pulley *C*.





F.B.D

Sol :-



Pulley A

$[\Sigma M_O = 0]$

$T_1 * r - T_2 * r = 0 \longrightarrow T_1 = T_2$

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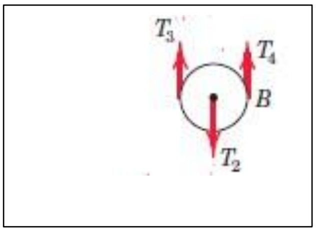
$\Sigma Fy = 0$

$T_1 + T_2 - 1000 = 0 \longrightarrow 2T_1 = 1000$

$T_1 = 500$ Ib

$T_1 = T_2 = 500$ Ib

Pulley B



$[\Sigma M_B = 0]$

$T_3 * r - T_4 * r = 0 \longrightarrow T_3 = T_4$

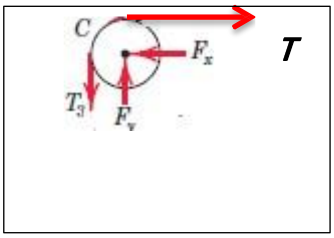
$\Sigma Fy = 0$

$T_3 + T_4 - T_2 = 0 \longrightarrow 2T_3 = 500$

$T_3 = 250$ Ib

$T_3 = T_4 = 250$ Ib

Pulley C



$[\Sigma M_C = 0]$

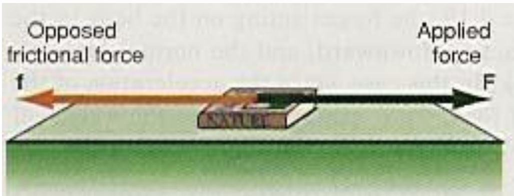
$T_3 * r - T * r = 0 \longrightarrow T_3 = T = 250$ Ib

The Friction

الاحتكاك

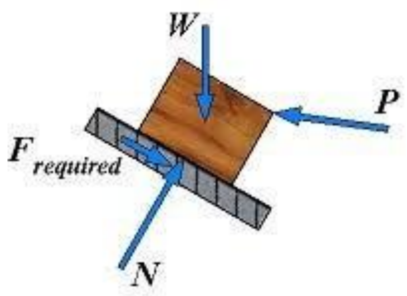
❖ Definition :-

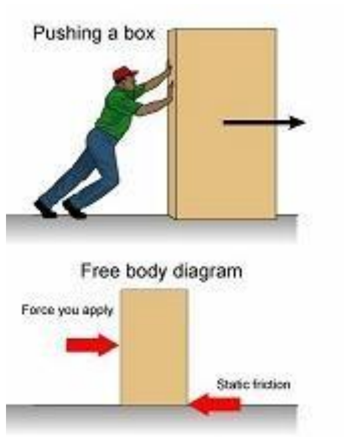
Whenever we try to slide one body over another body there is a force that opposes that motion. This opposing force is called the force of ***friction.*** For example, if this book is placed on the desk and a force is exerted on the book toward the right, there is a force of friction acting on the book toward the left opposing the applied force, as shown in figure below.



❖ Types of Friction

(a) Dry Friction. Dry friction occurs when the not oiled surfaces of two solids are in contact under a condition of sliding or a slope to slide. A friction force tangent to the surfaces of contact occurs both during the interval leading up to impending slippage and while slippage takes place.





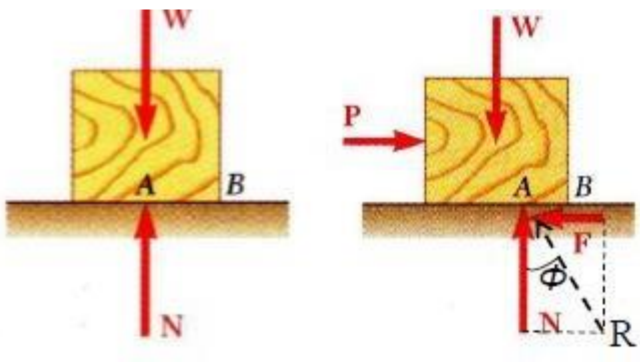
(b) Fluid Friction. Fluid friction occurs when adjacent layers in a fluid (liquid or gas) are moving at different velocities. This motion causes frictional forces between fluid elements, and these forces depend on the relative velocity between layers. When there is no relative velocity, there is no fluid friction.

Static Friction

As P increases, static-friction force F increases as well until it reaches a maximum value F.

$$F = \mu * N$$

$$\mu = \frac{F}{N}$$



Where:

F = friction force

N =Normal reaction

μ = Coefficient of friction

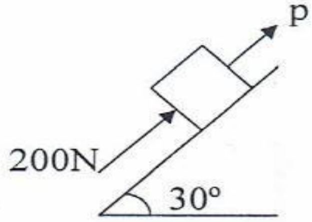
Angle of friction (Ø):

It's the angle between the total reaction (R) and its normal component, when limiting friction.

The tangent of this angle is equal to the coefficient of Friction (μ).

$$\tan Ø = \frac{F}{N} \longrightarrow \mu = \tan Ø$$

Example: Calculate the force (P) required to move the (500N) block weight up the inclined surface shown in figure ,if the block is subjected to (200N)force assume (μ=0.5) .



Solution:

Wx=500× Sin30=250N

Wy=500× Cos30=433N

∑Fy=0

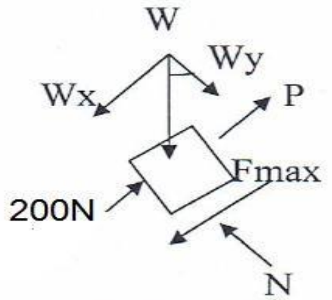
N-433=0 ⟹ N=433N

Fmax.=μ*N=0.5×433=216.5N

∑Fx =0

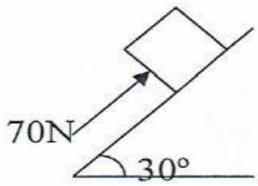
200+p-250-216.5=0

P.=266.5N



Example: Determine the frictional force exerted on the (200N) block weight by the Inclined surface shown in figure if the block is subjected to (70N) force (μ=0.2) .

Solution:



Wx=200× Sin30=100N

Wy=200× Cos30=173.2N

Assume the block will move upward

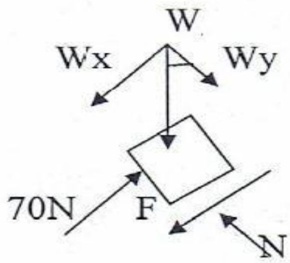
∑Fx=0

70-100-F=0

F=-30N

That means the block is try to move downward

(F) must be equal or less than(Fmax.)



Fmax.=μ*N

∑Fy=0

N - 173.2 = 0 ⟹ N = 173.2 N

Fmax.=0.2× 173.2=34.64N ▯ 30N

F=30N

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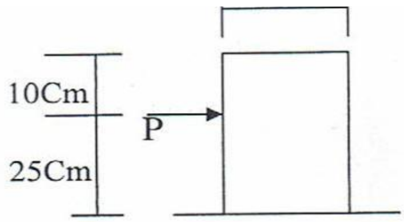
Example: Explain if the block (400 N) weight turns or slides if pushed by force **P** ($\mu = 0.34$)

Solution:

The block is either slides or overturn



1-the block is slides

From (F.B.D 1)

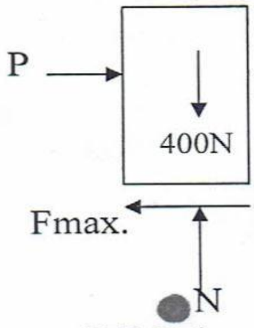
$\sum Fx = 0$

P = Fmax.

$\sum Fy = 0 \Rightarrow N=400N$

Fmax.= μ*N=0.34× 400=136N

P=136N



F.B.D 1

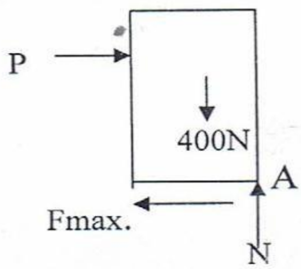
2-the block is overturn

From (F.B.D 2)

$\sum MA = 0$

$25 \times p - 400 \times 10 = 0$

P=160N



F.B.D 2

The block is slides and P= 136N

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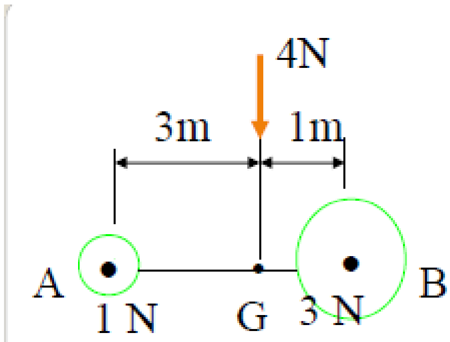
Center of Gravity , Center of Mass And Center of body

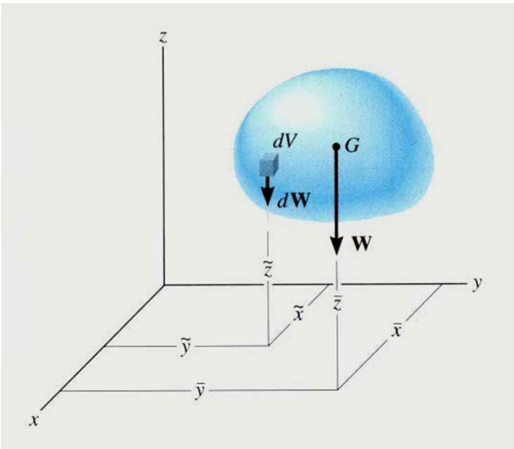
مركز ثقل الاجسام ومركز ثقل الكتلة, و مركز ثقل الجسم

❖ **Definition** :-

The center of gravity (G):- is a point which locates the resultant weight of a system of particles or body.



From the definition of a resultant force, the sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at G. For the figure above, try taking moments about A and B.



Similarly, **the center of mass** is a point which locates the resultant mass of a system of particles or body. Generally, its location is the same as that of **G**.

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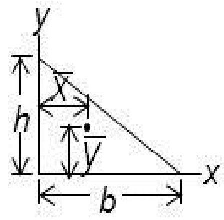
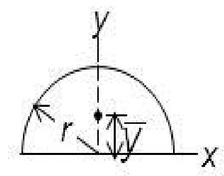
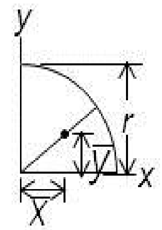
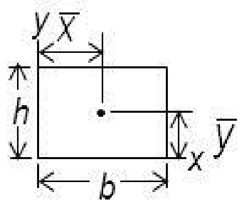
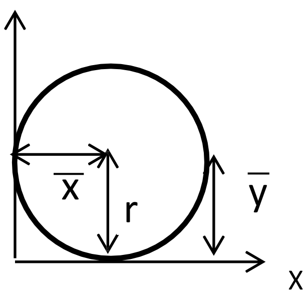
❖ Centroid of simple shapes:-

	Shape	$\bar{x}$	$\bar{y}$	Area A
1. Triangle		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{1}{2}bh$
2. Semicircle		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
3. Quarter circle		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
4. Rectangle		$\frac{b}{2}$	$\frac{h}{2}$	bh
5- Circle		r	r	πr^2

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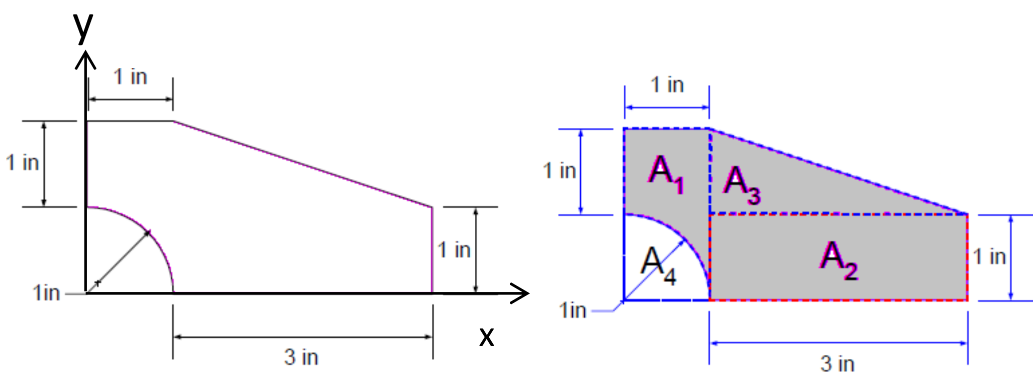
Centroid of complex shapes:- When a body or figure can be conveniently divided into several parts whose centroid are easily determined.

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} \qquad \bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

Ex1 Find the centroid of the given area



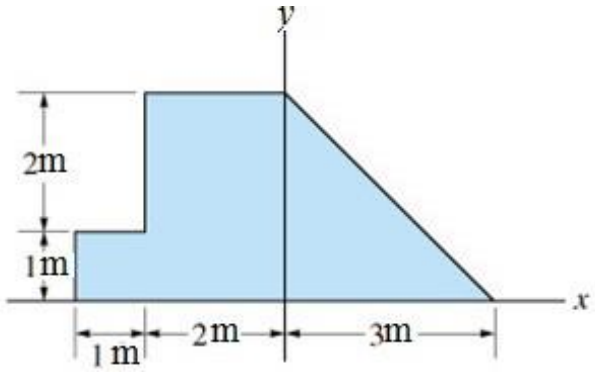
Soulstion:-

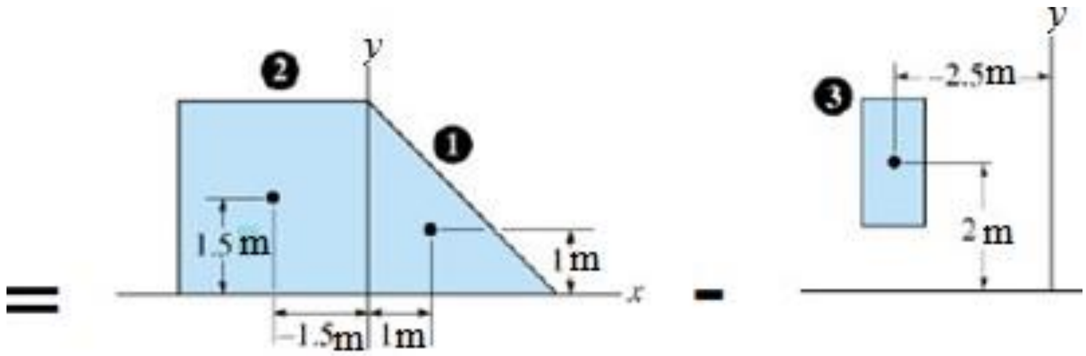
ID	Area	x_i	x_i*Area	y_i	y_i*Area
	(in^2)	(in)	(in^3)	(in)	(in^3)
A_1	2	0.5	1	1	2
A_2	3	2.5	7.5	0.5	1.5
A_3	1.5	2	3	1.333333	2
A_4	-0.7854	0.42441	-0.33333	0.42441	-0.33333
	5.714602		11.16667		5.166667

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{11.16}{5.71} = 1.95 \text{ in}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{5.16}{5.71} = 0.904 \text{ in}$$

Ex2 Locate the centroid of the plate area shown in Figure below.





Segment	A (m²)	$\tilde{x}$(m)	$\tilde{y}$(m)	$\tilde{x}A$(m³)	$\tilde{y}A$(m³)
1	$0.5 \times 3 \times 3 = 4.5$	1	1	4.5	4.5
2	$3 \times 3 = 9$	-1.5	1.5	- 13.5	13.5
3	$-2 \times 1 = -2$	-2.5	2	5	-4
Σ	$\Sigma A = 11.5$			$\Sigma\tilde{x}A = -4$	$\Sigma\tilde{y}A = 14$

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{-4}{11.5} = -0.348\ m$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{14}{11.5} = 1.22\ m$$

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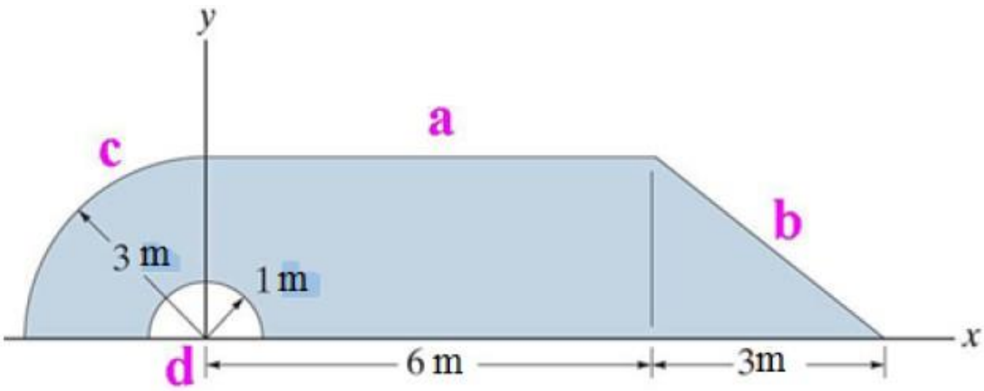
Ex2 Find the centroid of the part.



Solution:

1. This body can be divided into the following pieces:

rectangle (a) + triangle (b) + quarter circular (c)—semicircular area (d).
(Note the negative sign on the hole!)

Steps 2 & 3: Make up and fill the table using parts a, b, c, and d.

Segment	A (m^2)	$\tilde{x}$(m)	$\tilde{y}$(m)	$\tilde{x}A$(m^3)	$\tilde{y}A$(m^3)
Rectangle	18	3	1.5	54	27
Triangle	4.5	7	1	31.5	4.5
Q. Circle	$9\pi/4$	$-4\times3/3\pi$	$4\times3/3\pi$	-9	9
Semi-Circle	$-\pi/2$	0	$-4\times1/3\pi$	0	-2/3
Σ	28.0			$\Sigma\tilde{x}A$ =76.5	$\Sigma\tilde{y}A$ =39.83

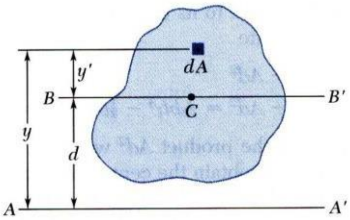
$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{76.5}{28.0} = 2.73\ m$$

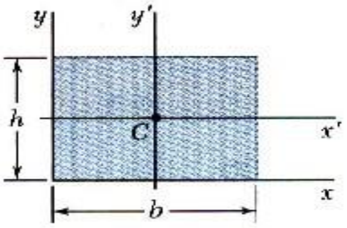
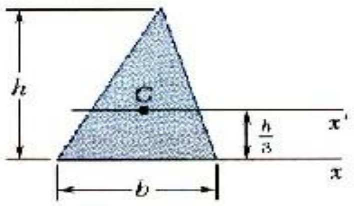
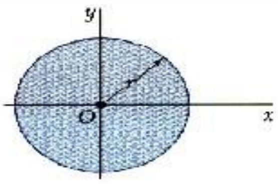
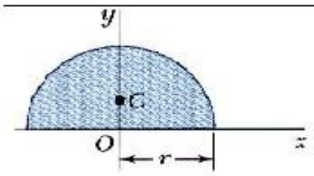
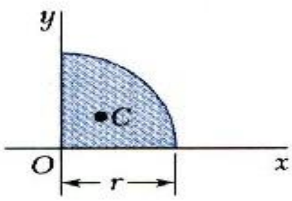
$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{39.83}{28.0} = 1.42\ m$$

Moment of Inertia (عزم القصور الذاتي)

- It is a measure of an object's resistance to changes to its rotation.
- Also defined as the capacity of a cross-section to resist bending.
- The sum of the products of the mass of each particle in the body and the square of its perpendicular distance from the axis of rotation I.

$$I = \bar{I} + Ad^2$$



shape	Moment of Inertia at C	Moment of Inertia at base
	$I_{x'} = \frac{1}{12} bh^3$	$I_x = \frac{1}{3} bh^3$
	$I_{x'} = \frac{1}{36} bh^3$	$I_x = \frac{1}{12} bh^3$
	$I_x' = \frac{1}{4} \pi r^4$	$I_0 = \frac{1}{2} \pi r^4$
	$I_x' = I_y' = \frac{1}{8} \pi r^4$	$I_0 = \frac{1}{4} \pi r^4$
	$I_x' = I_y' = \frac{1}{16} \pi r^4$	$I_0 = \frac{1}{8} \pi r^4$

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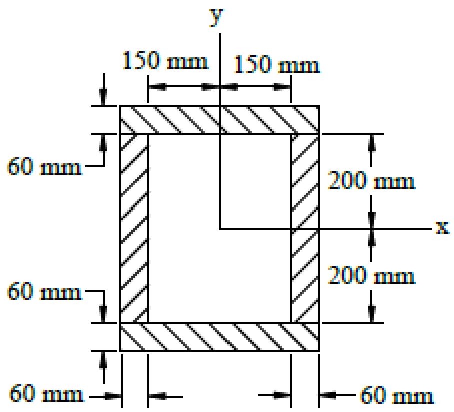
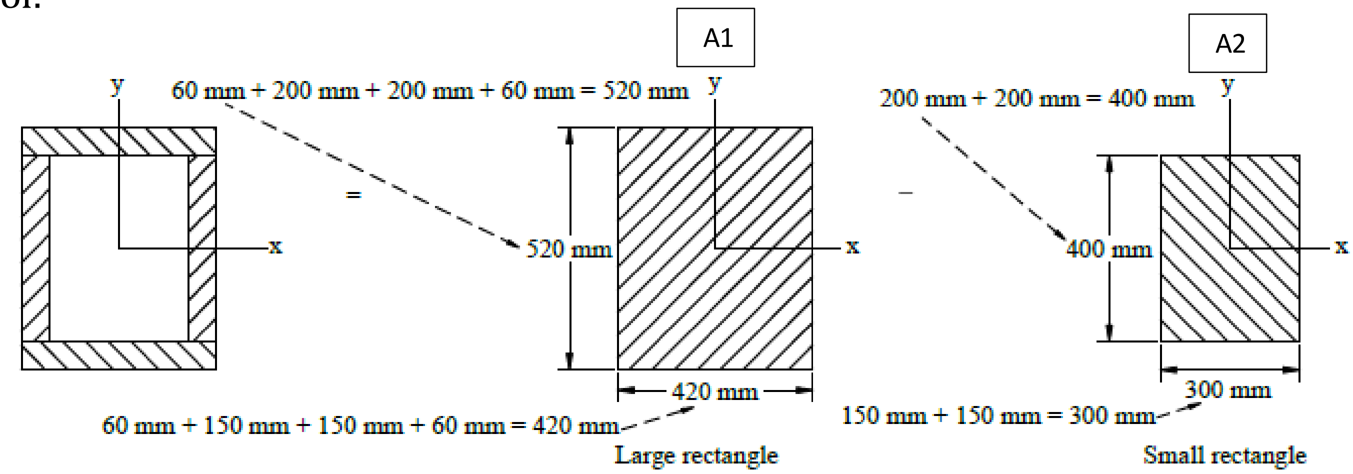
EX 1

The figure shows the cross section of a beam made by gluing four planks together. Determine the moment of inertia of the cross section about the x axis.



Sol:-

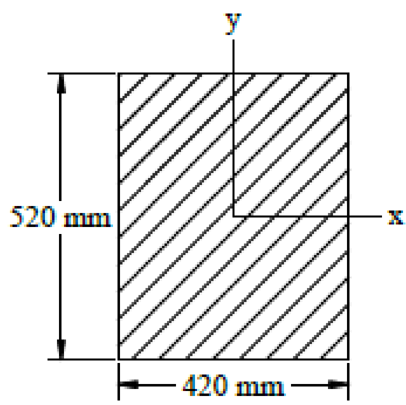


1- A1

Large rectangle $I_x = \dfrac{bh^3}{12}$

$$= \frac{(420\text{ mm})(520\text{ mm})^3}{12}$$

$$= 4.9213 \times 10^9\text{ mm}^4$$



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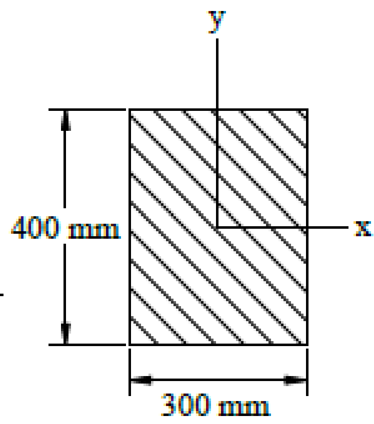
2- A 2

Small rectangle $I_x = \dfrac{bh^3}{12}$

$$= \frac{(300\text{ mm})(400\text{ mm})^3}{12}$$

$$= 1.6000 \times 10^9\text{ mm}^4$$



For the composite region, subtracting gives

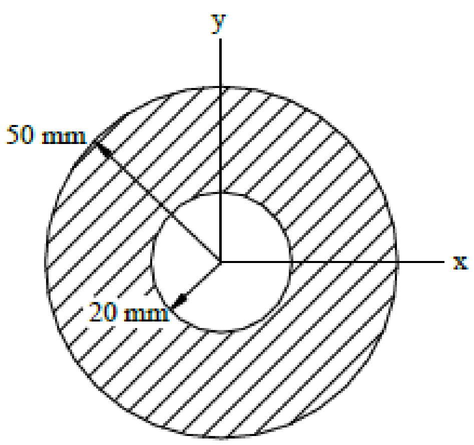
$$I_x = \text{Large rectangle } I_x - \text{Small rectangle } I_x$$

$$= 4.9213 \times 10^9\text{ mm}^4 - 1.6000 \times 10^9\text{ mm}^4$$

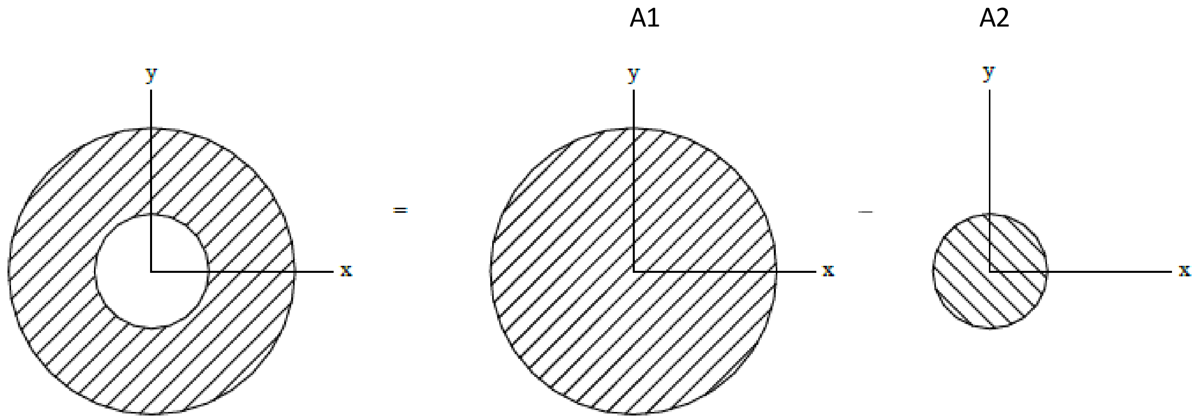
$$= 3.32 \times 10^9\text{ mm}^4 \qquad \leftarrow \text{Ans.}$$

EX 2

Determine the moment of inertia of the cross section about the x axis.



Sol;-



A1 Large circle $I_x = \frac{1}{4}\pi(50\ mm)^4$

$= 4.9087 \times 10^6\ mm^4$

A2 Small circle $I_x = \frac{1}{4}\pi(20\ mm)^4$

$= 0.1257 \times 10^6\ mm^4$

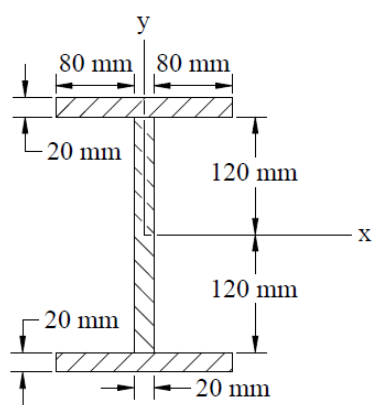
For the composite region, subtracting gives

I_x = Large circle I_x − Small circle I_x

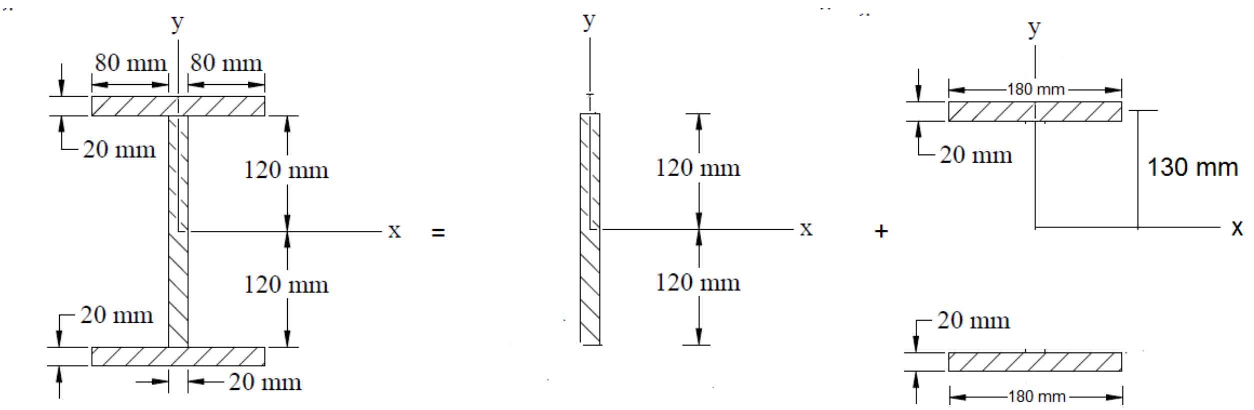
$= 4.9087 \times 10^6\ mm^4 - 0.1257 \times 10^6\ mm^4$

$= 4.78 \times 10^6\ mm^4$ ←Ans.

Ex3 Determine the moment of inertia of the beam cross section about the x centroid axis.



SOL:-



A1

A2

1- A1

$$I_x = \frac{bh^3}{12}$$

$$= \frac{(20\ \text{mm})(240\ \text{mm})^3}{12}$$

$$= 2.304 \times 10^7\ \text{mm}^4$$

2- A2

$$I_x = I_{xc'} + d^2A$$

$$I_x' = \frac{bh^3}{12}$$

$$= \frac{(180\ \text{mm})(20\ \text{mm})^3}{12}$$

$$= 1.2 \times 10^5\ \text{mm}^4$$

$$I_x = 1.2 \times 10^5 + d^2A$$

$$= 1.2 \times 10^5 + (130)^2 (180 \text{ X } 20)$$

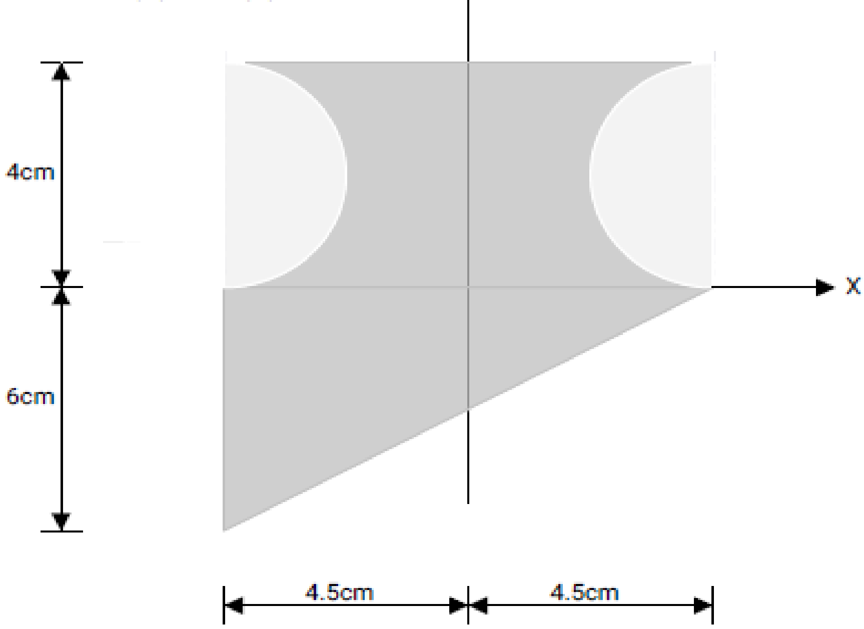
$$= 60960000\ \text{mm}^4$$

$$I_x \text{ (TOTAL)} = \text{A1} + 2\text{ A2}$$

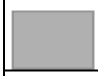
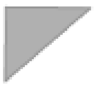
$$= 2.304 \text{ X } 10^7 + 2\,(6.096 \text{ X} 10^7) = 14.5 \text{ X } 10^8\ \text{mm}^4$$

Example : Determine the centroid of the shaded area shown in figure with respect to

(X) and (Y) axes .



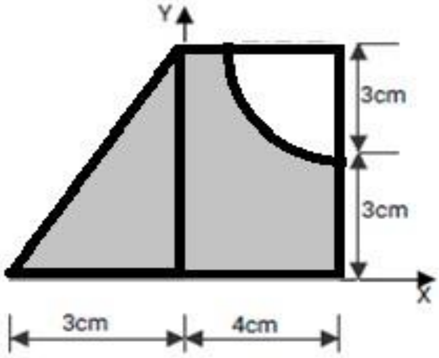
Solution:

Fig.	ai	xi	yi	aixi	aiyi
	4× 9=36	0	2	0	72
	1/2× 6× 9=27	-1.5	-2	-40.5	-54
	-π(2)²/2=-6.283	-(4.5-0.424×2) =-3.652	2	22.945	-12.566
	-6.283	3.652	2	- 22.945	-12.566
	50.434			-40.5	-7.132

X=-40.5/50.434=-0.803Cm Y=-7.132/50.434=-0.141Cm

Example: Determine the centroid of the shaded area shown in figure with respect to

(X) and (Y) axes .



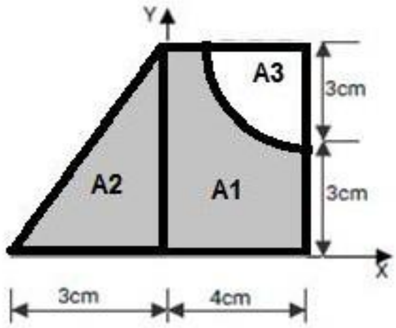
Solution:

Fig.	ai	xi	yi	aixi	aiyi
	4× 6=24	2	3	48	72
	1/2× 3× 6=9	-1	2	-9	18
	$-\pi(3)^2/4=-$ -7.069	4-(0.424×3) =2.728	6-(0.424× 3) =4.728	-19.27	-33.4

Σ 25.931 19.73 56.6

X=19.73/25.931=0.76Cm Y=56.6/25.931=2.18Cm

Example: Determine the moment of inertia of the shaded area shown in figure with respect to(x) axis .



Solution:

$A1=4\times 6=24Cm^2$

$A2=1/2\times 3\times 6=9Cm^2$

$A3=\pi(3)^2/4=7.06Cm^2$

For(A1) :

$Ix=bh^3/12+Ad^2=4*(6)^3/12+24*(3)^2=288Cm^4$ (+)

For(A2) :

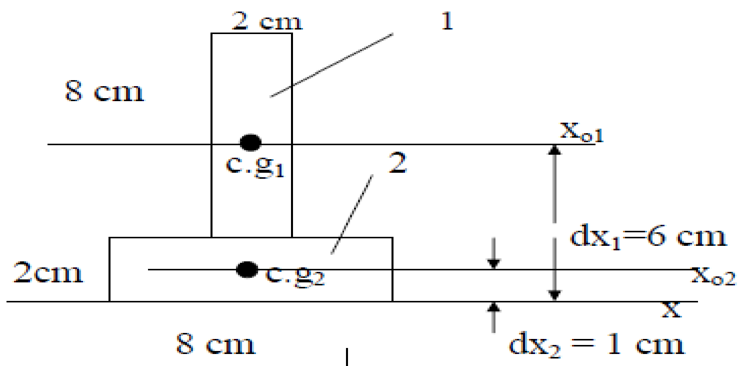
$Ix=bh^3/36+Ad^2=3*(6)^3/36+9*(2)^2=54Cm^4$ (+)

For(A3) :

$Ix=0.196(r)^4+Ad^2=0.196*(3)^4+7.06*(4.728)^2=173.72\ Cm^4$ (-)

$Ix(total)=288+54-173.42=168.27\ Cm^4$

Q/ Find the moment of inertia of T- section show in Fig. About x-axis .



$(Ix)_A = (Ix)_{a1} + (Ix)_{a2}$

1) $Ix_{o1}=\frac{2(8)^3}{12}=85.3\,cm^4$

$(Ix)_{a1}=I_{xo1}+a_1 d^2_{x1}$

$(Ix)_{a1}=85.3+(8*2)(6)^2$

$(Ix)_{a1}=661.3\,cm^4$

2) $Ix_{o2}=\frac{8(2)}{12}=5.3\,cm4$

$(Ix)_{a2}=I_{xo2}+a_2 d^2_{x2}$

$(Ix)_{a2}=5.3+(8*2)*(1)^2$

$(Ix)_{a2}=21.3cm^4$

3) $(Ix)_A=(Ix)_{a1}+(Ix)_{a2}$

$(Ix)_A=661.3+21.3$

$(Ix)_A=682.6\,cm^4$

Strength of Materials

Strength of materials : مقاومة المواد
Is the resistance of materials to the external forces القوى الخارجية acting المؤثرة on it عليها .

Stress : الإجهاد

The resistance per unit area to deformation the symbol of stress is (σ) the unit وحدات of stress is unit of force divide by مقسومة على unit of area (N/ m^2) .

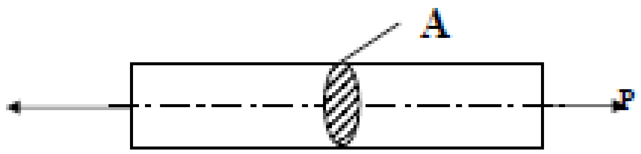
$$\sigma = \frac{P}{A}$$

σ = stress الاجهاد
P = Force القوة
A = Area المساحة

Type of stress أنواع الإجهاد

1. Tensile stress (σ_T) إجهاد الشد

$$\sigma_T = \frac{P}{A}$$

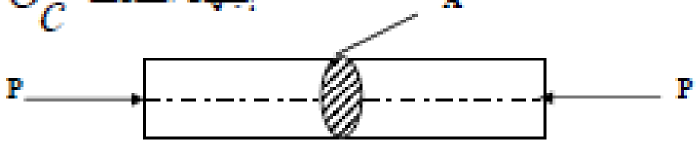


P

P = tensile force قوة شد
A = cross sectional Area مساحة المقطع

ملاحظة : (يجب ان تكون قوى الشد على محور واحد ويجب أخذ المساحة للمقطع عمودية على القوة)

2. Compression stress σ_C إجهاد الضغط

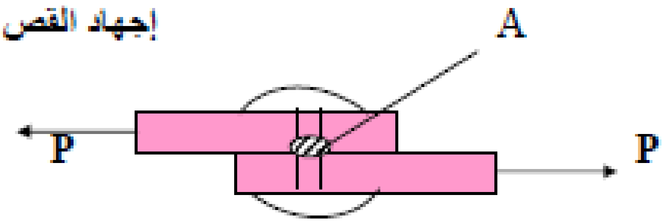


$$\sigma_C = \frac{P}{A}$$

P = Compression force قوة ضغط
A = cross sectional Area مساحة المقطع

ملاحظة : (يجب ان تكون قوى الضغط على محور واحد ويجب أخذ المساحة عمودية على القوة) .

3.Shear stress (σ_s) إجهاد القص



$$\sigma_s = \frac{P}{A}$$

P = shear force قوة قص
A = cross sectional Area مساحة المقطع

ملاحظة : (يجب ان تكون قوى القص على محاور متوازية ويجب أخذ المساحة للمقطع موازية للقوة)

Strain الانفعال

When a force acting عندما تؤثر قوة **on a body it under**يحدث **goes some deformation** تغيير **the deformation per unit length** لوحدة الطول **is known as strain .**
Strain has no unit الانفعال بدون وحدات
(عندما تسلط قوة على جسم يحصل تغيير وهذا التغيير اذا قسم على وحدات الطول نحصل على الانفعال) .

The sample of strain is (ε)

$$\varepsilon = \frac{\Delta L}{L}$$

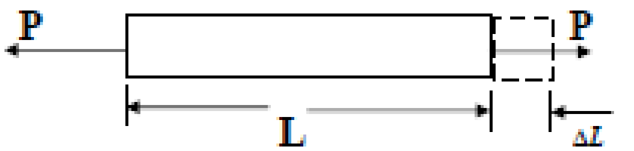
ε = strain الانفعال

ΔL = deformation التغيير في الطول

L = Length of body طول الجسم

Type of strain أنواع الانفعال

1. Tensile strain (ε_T)

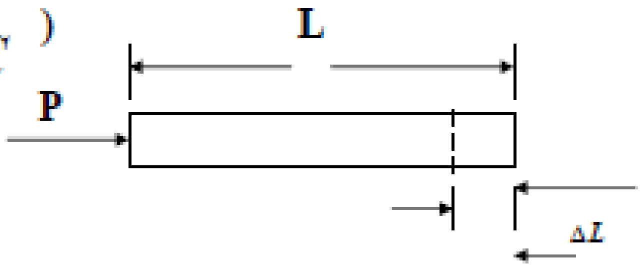


$$\varepsilon_T = \frac{\Delta L}{L}$$

ΔL = Tensile deformation الزيادة في الطول

L = Length of body طول الجسم

2. Compressive strain (ε_C)



P

$$\varepsilon_C = \frac{\Delta L}{L}$$

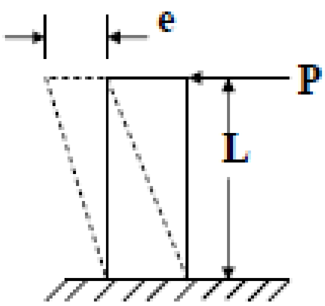
ΔL = Compressive deformation النقصان بالطول

L = Length of body طول الجسم

3. Shear strain (ε_s) انفعال القص

$$\varepsilon_s = \frac{e}{L}$$

e = shear de formation مقدار الحيود بالطول



L = Length of body طول الجسم

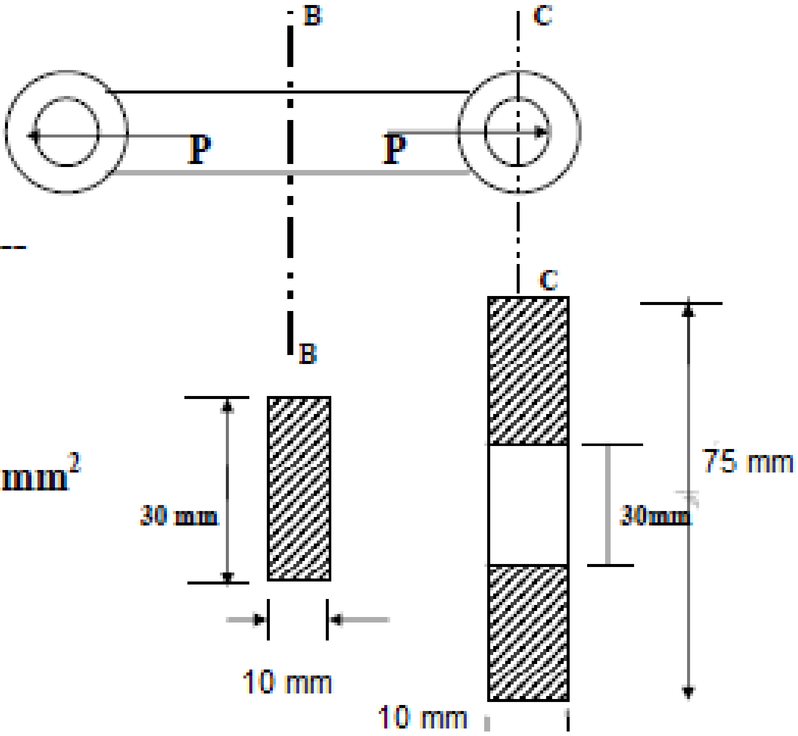
Elasticity المرونــــــــــــة

Is the property صفة of material المعدن of returning الرجوع back to their original position الموقع الأصلي after بعد removing إزالة the external force القوة الخارجية .

Elastic limit حدود المرونة

When the external force القوة الخارجية is removed تزال the force of resistance قوة المقاومة also vanishes يختفي and the body spring يقفز back to its original position الوضع الأصلي . This thing happens يحدث in Elastic limits حدود المرونة only فقط .

EX 1/ In Fig find the tensile stress in the section مقطع BB , CC if P = 14 400 N .



Section BB المقطع

$$\sigma_T = \frac{P}{A}$$

$A = 10 * 30 = 300\ mm^2$

$$\sigma_T = \frac{14400}{300}$$

$$\boxed{\sigma_T = 48 N/mm^2}$$

Section CC المقطع

$$\sigma_T = \frac{P}{A}$$

A = 75 * 10 – 30 * 10
A = 750 – 300
A = 450 mm^2

$$\sigma_T = \frac{14400}{450}$$

$$\boxed{\sigma_T = 32 N/mm^2}$$

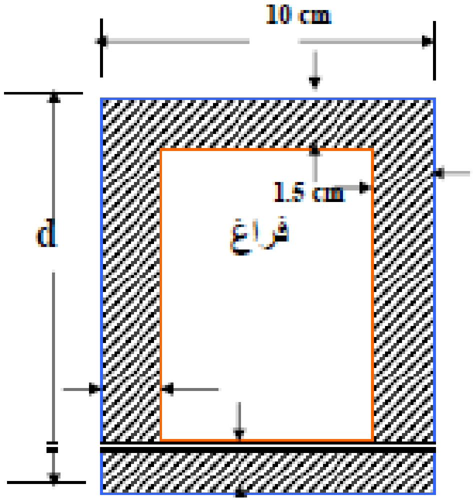
-- EX 2

The section shown in fig . is subjected to compression force of (820 000 N) .IF the compression stress is (12 000 N / cm^2) Find the dimension (d) ?

A = (10 * d) – 7 (d – 3)

A = 10 d – 7d + 21

A = 3 d + 21 ——— 1



$$\sigma_C = \frac{P}{A}$$

$$A = \frac{P}{\sigma_C} = \frac{820000}{12000}$$

1.5cm

A = 68.33 cm²

68.33 = 3d + 21
3d = 63.33 – 21
3d = 47 .33

$$d = \frac{47.33}{3}$$

d = 15 .76 cm

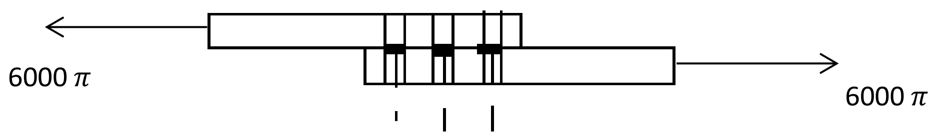
Q / In Fig . Two plates صفيحة are joint by three rivets برشام of (20 mm) diameter قطر . How much the shear stress in the material of rivet if (p = 6000 π N)

A = 3a
A = 3 (r^2 π)

مساحة الدائرة = (نصف القطر)2 * النسبة الثابتة

= = نق 2 * النسبة الثابتة

A = 3 ((10)^2 π)
A = 300 π mm²



$$\sigma_s = \frac{P}{A} = \frac{6000\pi}{300\pi}$$

$$\sigma_s = 20N/mm^2$$

Chapter 2: FORCE and MOTION

Linear Motion

Linear motion is the movement of an object along a straight line.

Distance

The **distance** traveled by an object is the **total length** that is traveled by that object.
Unit: **metre (m)**
Type of Quantity: **Scalar quantity**

Displacement

Displacement of an object from a point of reference, O is the **shortest distance** of the object from point O in a **specific direction**.
Unit: **metre (m)**
Type of Quantity: **Vector quantity**

Distance vs Displacement

Distance travelled = 200m
Displacement = 120 m, in the direction of Northeast
Distance is a scalar quantity,
Displacement is a vector quantity

Speed

Speed is the **rate of change** in distance.
Formula:

$$v = \frac{d}{t}$$

v = speed
d = distance travelled
t = time taken

Unit: ms^{-1}
Type of quantity: **Scalar quantity**

Velocity

Velocity is the rate of change in displacement.
Formula:

$$v = \frac{s}{t}$$

v = velocity
s = displacement
t = time taken

Unit: ms^{-1}
Type of quantity: **Vector quantity**

Acceleration

Acceleration is the **rate of velocity change.** Acceleration is a vector quantity
Formula:

$$a = \frac{v - u}{t}$$

a = acceleration
v = final velocity
u = initial velocity
t = time taken

Unit: ms^{-2}
Type of quantity: **Vector quantity**

Notes - Acceleration

- An object moves with a **constant velocity** if the **magnitude** and **direction** of the motion is always constant.
- An object experiences changes in velocity if
    - the **magnitude** of velocity changes
    - the **direction** of the motion changes.
- An object that experiences **changes in velocity** is said to have **acceleration**.
- An object traveling with a constant acceleration, ***a,*** if the velocity changes at a constant rate.

4. Equations of Uniform Acceleration

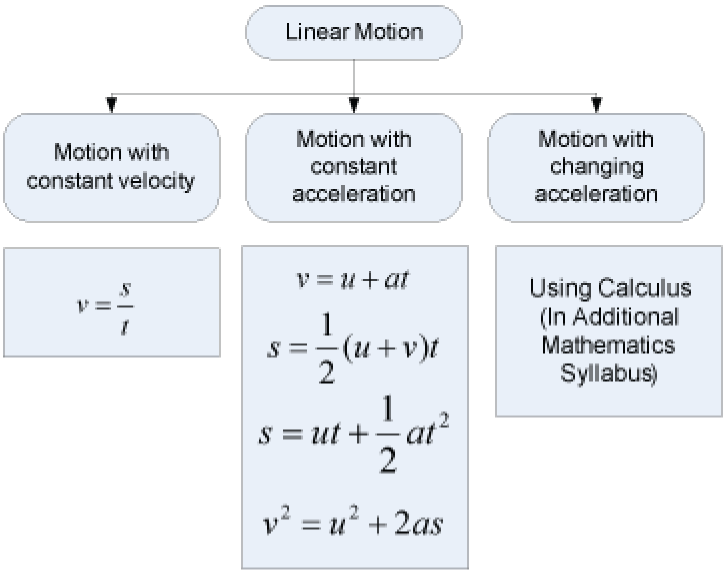
$$v = u + at \qquad s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t \qquad v^2 = u^2 + 2as$$

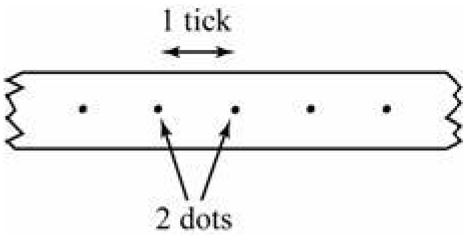
a = acceleration
v = final velocity
u = initial velocity
t = time taken
s = displacement

The above equation is for solving numerical problems involving uniform acceleration.

Summary

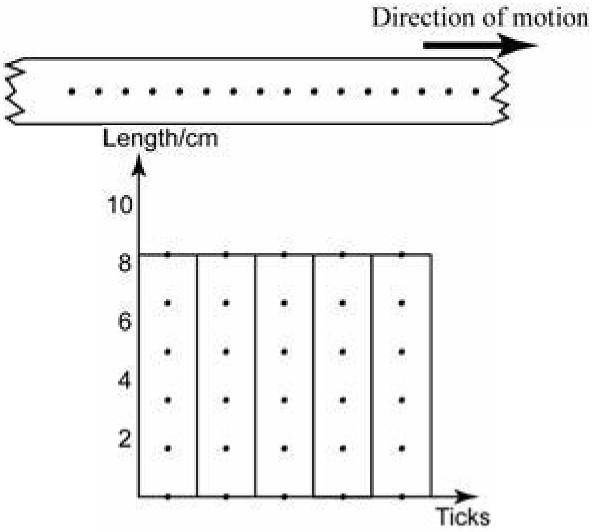


Ticker Timer



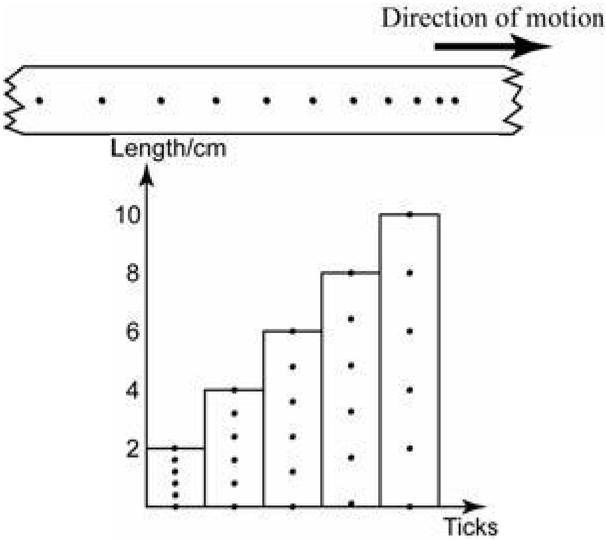
- A ticker-timer consists of an electrical vibrator which vibrates 50 times per second.
- This enables it to make 50 dots per second on a ticker-tape being pulled through it.
- The time interval between two adjacent dots on the ticker-tape is called one tick.
- One tick is equal to 1/50 s or 0.02 s.

Uniform Velocity



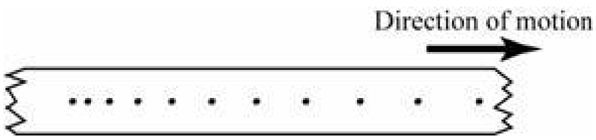
- The distance of the dots is equally distributed.
- All lengths of tape in the chart are of equal length.
- The object is moving at a uniform velocity.

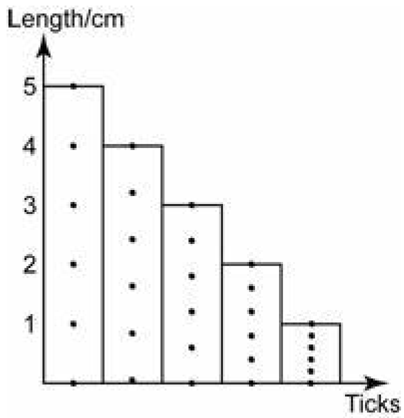
Uniform Acceleration



- The distance between the dots increases uniformly.
- The length of the strips of tape in the chart increase uniformly.
- The velocity of the object is increasing uniformly, i.e. the object is moving at a constant acceleration.

Uniform Deceleration





- The distance between the dots decreases uniformly.
- The length of the strips of tape in the chart decreases uniformly.
- The velocity of the object is decreasing uniformly, i.e. the object is decelerating uniformly.

Finding Velocity

Velocity of a motion can be determined by using ticker tape through the following equation:

$$v = \frac{s}{t}$$

v = velocity
s = displacement
t = time taken

Caution!!!
t is time taken from the first dot to the last dot of the distance measured.

Example 1

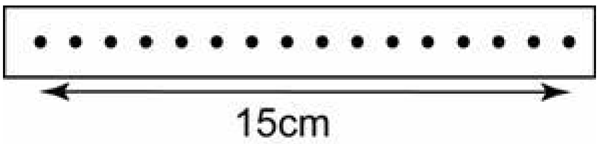


Diagram 2.4 shows a strip of ticker tape that was pulled through a ticker tape timer that vibrated at 50 times a second. What is the

a. time taken from the first dot to the last dot?

b. average velocity of the object that is represented by the ticker tape?

Answer

a. There are 15 ticks from the first dot to the last dot, hence
Time taken = 15 × 0.02s = 0.3s

b. Distance travelled = 15cm

Finding Acceleration

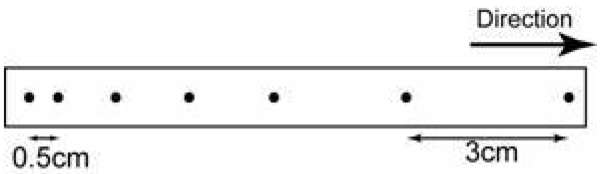
Acceleration of a motion can be determined by using ticker tape through the following equation:

$$a = \frac{v-u}{t}$$

a = acceleration
v = final velocity
u = initial velocity
t = time taken

Caution!!!
t is time taken from the initial velocity to the **final velocity**.

Example 2



The ticker-tape in figure above was produced by a toy car moving down a tilted runway. If the ticker-tape timer produced 50 dots per second, find the acceleration of the toy car.

Answer
In order to find the acceleration, we need to determine the initial velocity, the final velocity and the time taken for the velocity change.
Initial velocity,

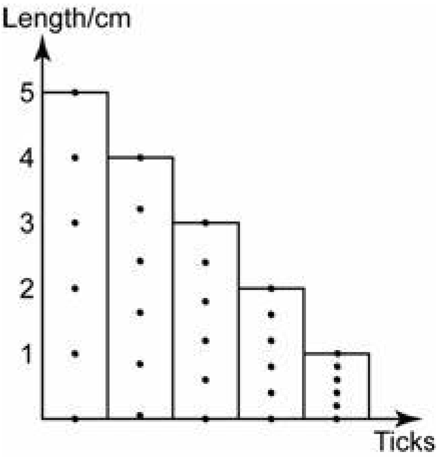
$$u = \frac{s}{t} = \frac{3cm}{0.02s} = 150cms^{-1}$$

$$v = \frac{s}{t} = \frac{0.5cm}{0.02s} = 25cms^{-1}$$

Time taken for the velocity change,
t = (0.5 + 4 + 0.5) ticks = 5 ticks
t = 5 × 0.02s = 0.1s

Acceleration, a =

Example 3



A trolley is pushed up a slope. Diagram above shows ticker tape chart that show the movement of the trolley. Every section of the tape contains 5 ticks. If the ticker-tape timer produced 50 dots per second, determine the acceleration of the trolley.

Answer

In order to find the acceleration, we need to determine the initial velocity, the final velocity and the time taken for the velocity change.

Initial velocity,

$u = \frac{s}{t} = \frac{5cm}{5[?]0.02s} = 50cms^{-1}$

$v = \frac{s}{t} = \frac{1cm}{5[?][?]0.02s} = 10cms^{-1}$

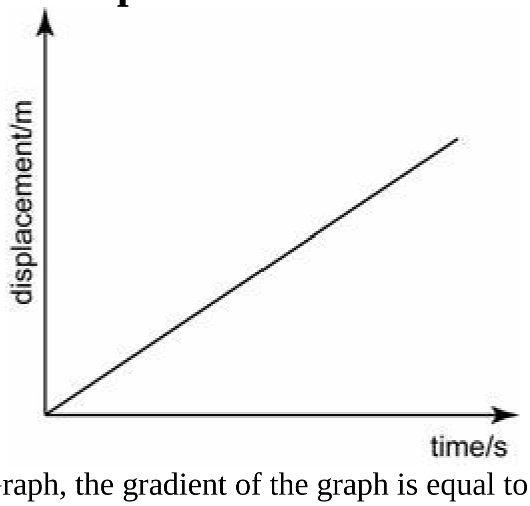
Time taken for the velocity change,

t = (2.5 + 5 + 5 + 5 + 2.5) ticks = 40 ticks

t = 40 × 0.02s = 0.8s

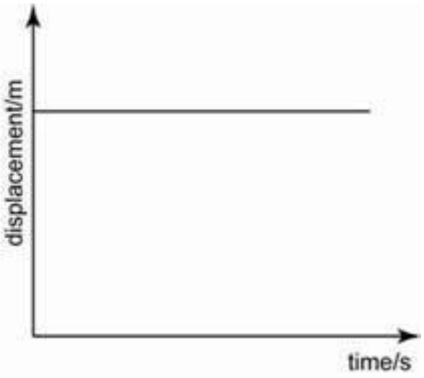
Acceleration, a:

Displacement - Time Graph

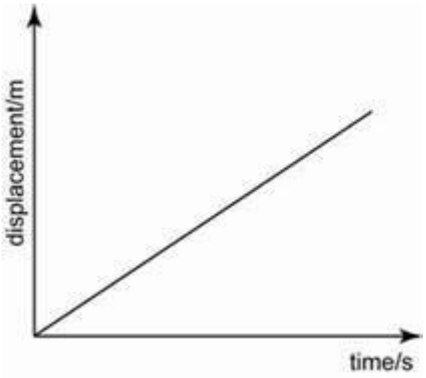


In a Displacement-Time Graph, the gradient of the graph is equal to the velocity of motion.

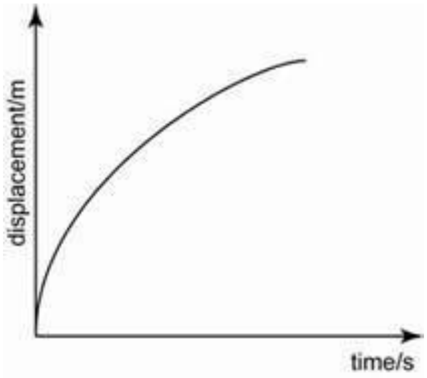
Analysing Displacement - Time Graph



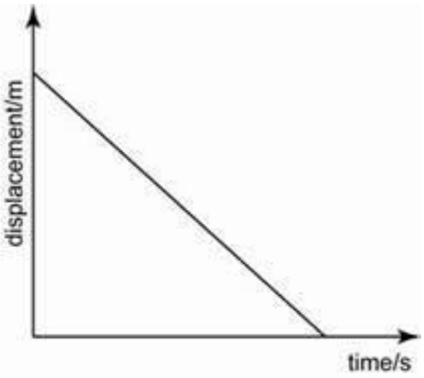
Gradient = 0
Hence, velocity = 0



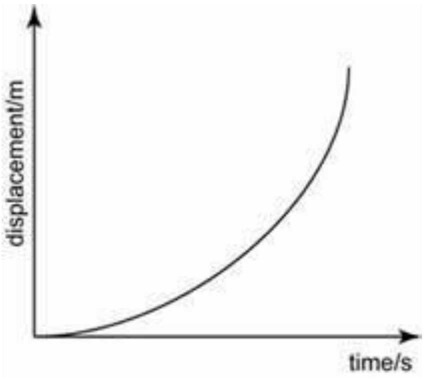
Gradient is constant,
hence, velocity is Uniform



Gradient is decreasing, hence velocity is decreasing.

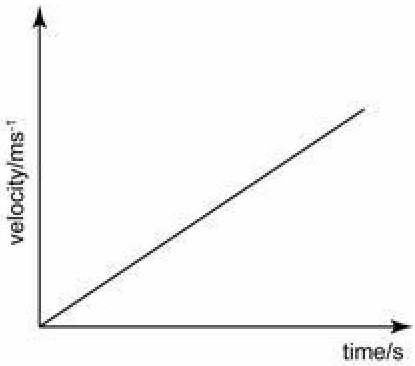


Gradient is negative and constant, hence velocity is uniform and in opposite direction



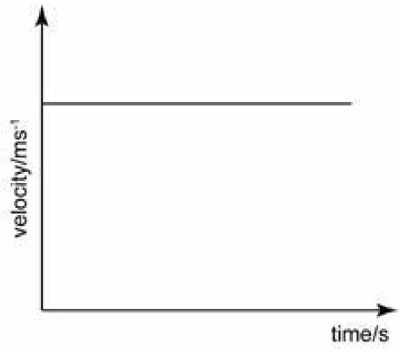
Gradient is increasing, hence velocity is increasing.

Velocity - Time Graph

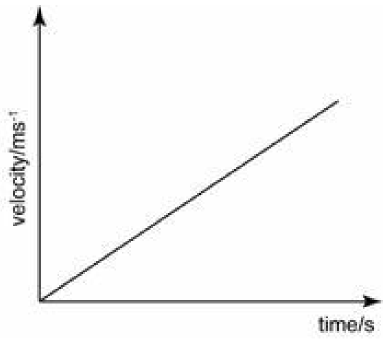


- The gradient of the velocity-time gradient gives a value of the changing rate in velocity, which is the acceleration of the object.
- The area below the velocity-time graph gives a value of the object's displacement.

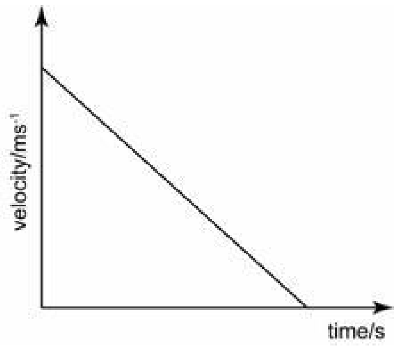
Analysing Velocity - Time Graph



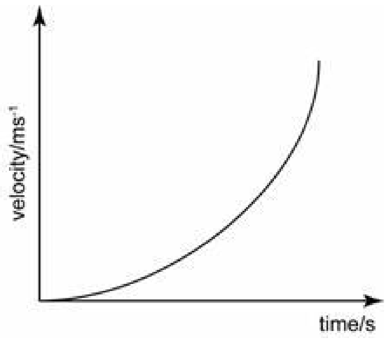
Uniform velocity



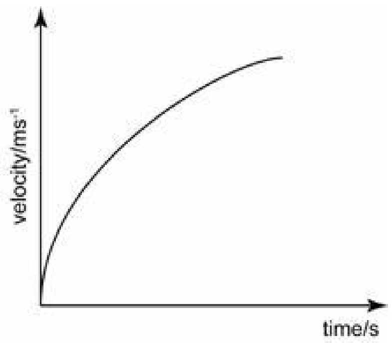
Uniform acceleration



Uniform deceleration



Increasing acceleration

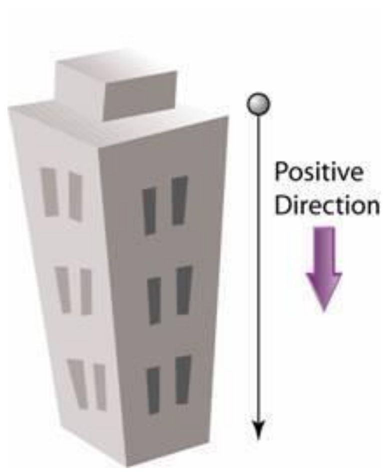
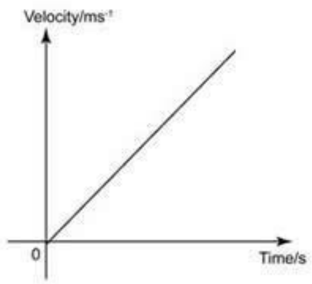
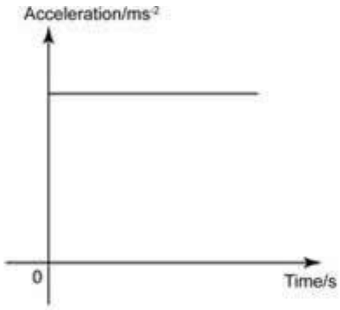


Increasing deceleration

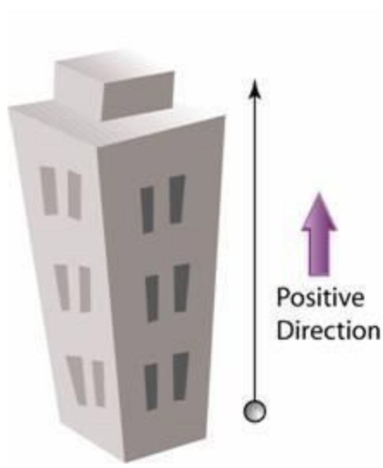
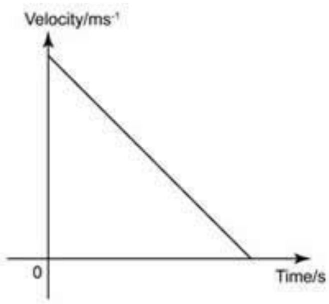
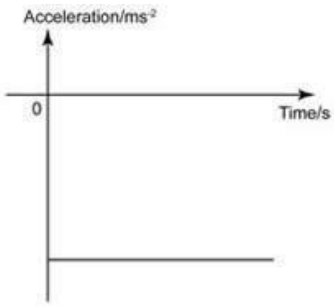
Converting a Velocity-Time graph to Acceleration-Time graph

In order to convert a velocity-time graph to acceleration time graph, we need to find the gradient of the velocity time graph and plot it in the acceleration-time graph.

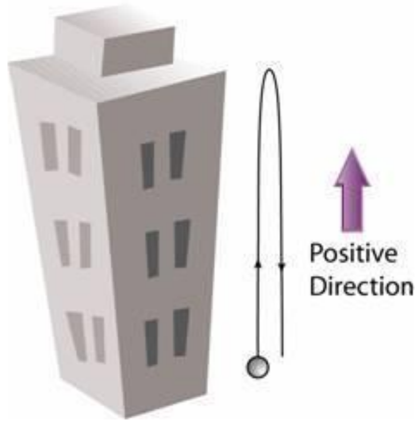
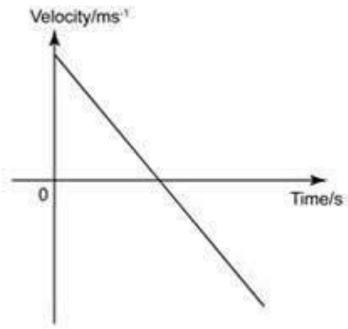
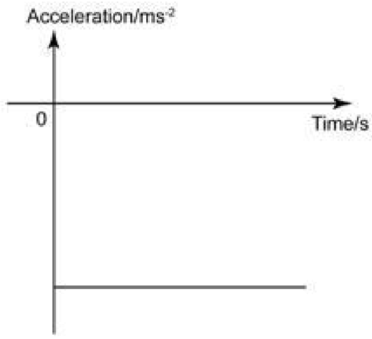
Dropping an object from high place

	Velocity - Time Graph	Acceleration - Time Graph
		

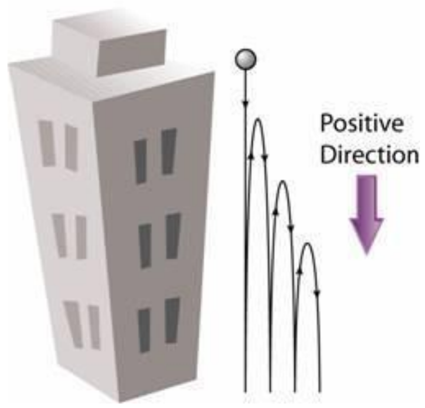
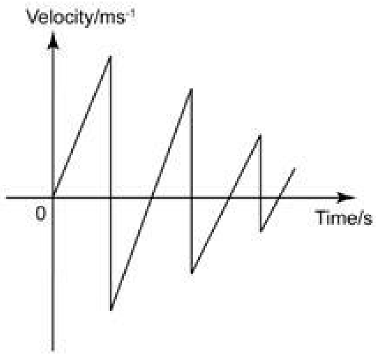
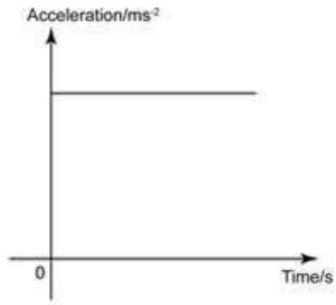
Launching Object Upward

	Velocity-Time Graph	Acceleration-Time Graph
		

Object moving upward and fall back to the ground

	Velocity-Time Graph	Acceleration-Time Graph
		

Object falling and bounces back

	Velocity-Time Graph	Acceleration-Time Graph
		

Mass

Mass is the amount of matter.
Unit: kilogram (kg)
Type of quantity: Scalar quantity

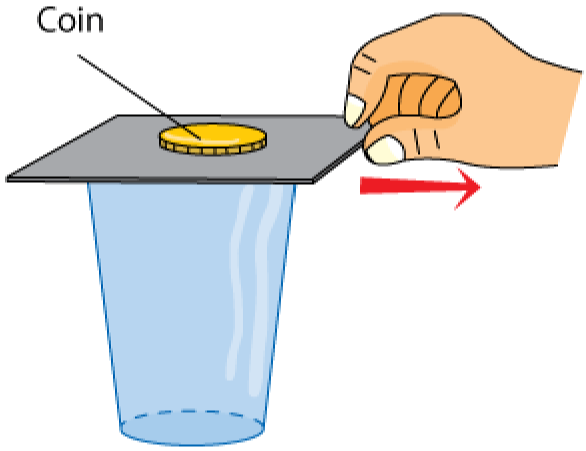
Inertia

Inertia is the property of a body that tends to maintain its state of motion.

Newton's First Law

In the absence of external forces, an object at rest **remains at rest** and an object in motion **continues in motion with a constant velocity** (that is, with a constant speed in a straight line).

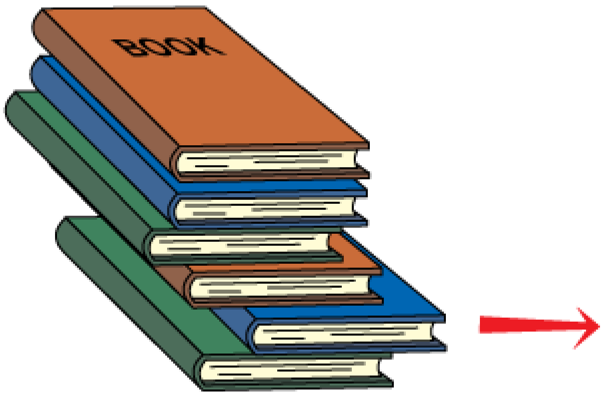
Jerking a Card



When the cardboard is jerked quickly, the coin will fall into the glass.
Explanation:

- The inertia of the coin resists the change of its initial state, which is stationary.
- As a result, the coin does not move with the cardboard and falls into the glass because of gravity.

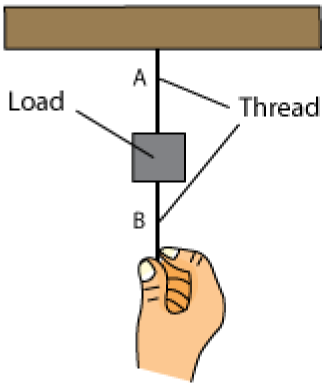
Pulling a Book



When the book is pulled out, the books on top will fall downwards.

Explanation
Inertia tries to oppose the change to the stationary situation, that is, when the book is pulled out, the books on top do not follow suit.

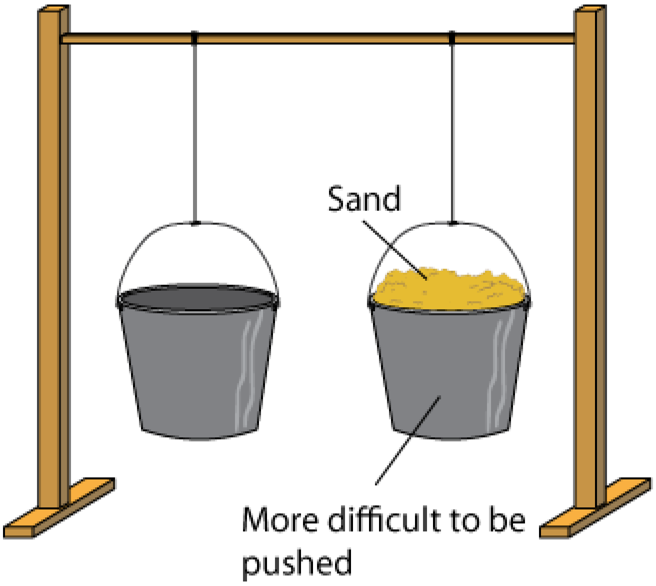
Pulling a Thread



Pull slowly - Thread A will snap.
Explanation:
Tension of thread A is higher than string B.
Tension at A = Weight of the load + Pulling Force
Yank quickly - Thread B will snap.

Explanation
The inertia of the load prevents the force from being transmitted to thread A, hence causing thread B to snap.

Larger Mass - Greater Inertia



Bucket filled with sand is more difficult to be moved. It's also **more difficult to be stopped** from swinging.

Explanation
Object with more mass offers a greater resistance to change from its state of motion.
Object with larger mass has larger inertia to resist the attempt to change the state of motion.

Empty cart is easier to be moved

An empty cart is easier to be moved compare with a cart full with load. This is because a cart with larger mass has larger inertia to resist the attempt to change the state of motion.

Momentum

Momentum is defined as the product of mass and velocity.
Formula:

$$p = mv$$

$$p = momentum$$
$$m = mass$$
$$v = velocity$$

Unit: kgms-1
Type of quantity: Vector

Example 1
A student releases a ball with mass of 2 kg from a height of 5 m from the ground. What would be the momentum of the ball just before it hits the ground?

Answer
In order to find the momentum, we need to know the mass and the velocity of the ball right before it hits the ground.
It's given that the mass, m = 2kg.
The velocity is not given directly. However, we can determine the velocity, v, by using the linear equation of uniform acceleration.
This is a free falling motion,
The initial velocity, u = 0
The acceleration, a = gravirational acceleration, g = 10ms-2
The dispacement, s = high = 50m.
The final velocity = ?
From the equation
v2 = u2 + 2as
v2 = (0)2 + 2(10)(5)
v = 10ms-1

The momentum,
p = mv =(2)(10) = 20 kgms-1

Principle of Conservation of Momentum

The principle of conservation of momentum states that **in a system** make out of objects that react (collide or explode), the total momentum is constant if **no external force** is acted upon the system.
Sum of Momentum Before Reaction = Sum of Momentum After Reaction

Formula

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

m_1 = mass of the 1st object

m_2 = mass of the 2nd object

u_1 = initial velocity of the 1st object

u_2 = initial velocity of the 2nd object

v_1 = final velocity of the 1st object

v_2 = final velocity of the 2nd object

Example 2: Both objects are in same direction before collision.
A Car A of mass 600 kg moving at 40 ms-1 collides with a car B of mass 800 kg moving at 20 ms-1 in the same direction. If car B moves forwards at 30 ms-1 by the impact, what is the velocity, v, of the car A immediately after the crash?

Answer
m1 = 600kg
m2 = 800kg
u1 = 40 ms-1
u2 = 20 ms-1
v1 = ?
v2 = 30 ms-1

According to the principle of conservation of momentum,
m1u1 + m2u2 = m1v1 + m2v2
(600)(40) + (800)(20) = (600)v1 + (800)(30)
40000 = 600v1 + 24000
600v1 = 16000
v1 = 26.67 ms-1

Example 3: <u>Both objects are in opposite direction before collision.</u>

A 0.50kg ball traveling at 6.0 ms-1 collides head-on with a 1.0 kg ball moving in the opposite direction at a
speed of 12.0 ms-1. The 0.50kg ball moves backward at 14.0 ms-1 after the collision. Find the velocity of the second ball after collision.
Answer:
m1 = 0.5 kg
m2 = 1.0 kg
u1 = 6.0 ms-1
u2 = -12.0 ms-1
v1 = -14.0 ms-1
v2 = ?

(**IMPORTANT**: **velocity is negative when the object move in opposite siredtion**)

According to the principle of conservation of momentum,
m1u1 + m2u2 = m1v1 + m2v2
(0.5)(6) + (1.0)(-12) = (0.5)(-14) + (1.0)v2
-9 = - 7 + 1v2
v2 = -2 ms-1

Elastic Collision

Elastic collision is the collision where the kinetic energy is conserved after the collision.
Total Kinetic Energy before Collision = Total Kinetic Energy after Collision

Additional notes:
-In an elastic collision, the 2 objects seperated right after the collision, and
-the momentum is conserved after the collision.

Inelastic Collision

Inelastic collision is the collision where the kinetic energy is not conserved after the collision.

Additional notes:
-In a perfectly elastic collision, the 2 objects attach together after the collision, and
-the momentum is also conserved after the collision.

Example 4: <u>Perfectly Inelastic Collision</u>
A lorry of mass 8000kg is moving with a velocity of 30 ms-1. The lorry is then accidentally collides with a car of mass 1500kg moving in the same direction with a velocity of 20 ms-1. After the collision, both the vehicles attach together and move with a speed of velocity v. Find the value of v.

Answer
(**IMPORTANT**: **When 2 object attach together, they move with same speed.**)
m1 = 8000kg
m2 = 1500kg
u1 = 30 ms-1
u2 = 20 ms-1

v1 = v
v2 = v

According to the principle of conservation of momentum,
m1u1 + m2u2 = m1v1 + m2v2
(8,000)(30) + (1,500)(20) = (8,000)v+ (1,500)v
270,000 = 9500v
v = 28.42 ms-1

Rocket

1. Mixture of hydrogen and oxygen fuels burn in the combustion chamber.
2. Hot gases are expelled through the exhausts at very high speed .
3. The high-speed hot gas produce a high momentum backwards.
4. By conservation of momentum, an equal and opposite momentum is produced and acted on the rocket, pushing the rocket upwards.

Jet Engine

1. Air is taken in from the front and is compressed by the compressor.
2. Fuel is injected and burnt with the compressed air in the combustion chamber.
3. The hot gas is forced through the engine to turn the turbine blade, which turns the compressor.
4. High-speed hot gases are ejected from the back with high momentum.
5. This produces an equal and opposite momentum to push the jet plane forward.

Newton's Second Law

The **rate of change of momentum** of a body is directly proportional to the resultant force acting on the body and is in the same direction.

Implication:
When there is resultant force acting on an object, the object will accelerate (moving faster, moving slower or change direction).

Force

- A force is push or pull exerted on an object.
- Force is a vector quantity that has magnitude and direction.
- The unit of force is Newton (or kgms-2).

Formula of Force

From Newton's Second Law, we can derived the equation

$$F = ma$$

(IMPORTANT: F Must be the net force)

F = Net force
m = mass
a = acceleration

Summary of Newton's 1st Law and 2nd Law

Newton's First Law:
When there is no net force acting on an object, the object is either **stationary** or move with **constant speed in a straight line**.

Newton's Second Law:
When there is a net force acting on an object, the object will accelerate.

Example 1
A box of mass 150kg is placed on a horizontal floor with a smooth surface; find the acceleration of the box when a 300N force is acting on the box horizontally.

Answer
F = ma
(300) = (150)a
a = 2 ms-2

Example 2
A object of mass 50kg is placed on a horizontal floor with a smooth surface. If the velocity of the object changes from stationary to 25.0 m/s in 5 seconds when is acted by a force, find the magnitude of the force that is acting?

Answer
We know that we can find the magnitude of a force by using the formula F = ma. The mass m is already given in the question, but the acceleration is not give directly.
We can determine the acceleration from the formula

From the formula
F = ma = (50)(5) = 250N
The force acting on the box is 250N.

Effects of Force

When a force acts on an object, the effect can change the

- size,
- shape,
- stationary state,
- speed and
- direction of the object.

Impulse

Impulse is defined as the product of the **force** (F) acting on an object and the **time** of action (t).
Impulse exerted on an object is equal to the momentum change of the object.
Impulse is a vector quantity.

Formula of impulse

Impulse is the product of force and time.

$$\textbf{Impulse} = \mathbf{F \times t}$$

Impulse = momentum change

$$\textbf{Impulse} = \mathbf{mv - mu}$$

Example 1

A car of mass 600kg is moving with velocity of 30m/s. A net force of 200N is applied on the car for 15s. Find the impulse exerted on the car and hence determine the final velocity of the car.

Answer

Impulse = F × t = (200) × (15) = 300oNs
Impulse = mv - mu
(3000) = 600v - 600(30)
600v = 3000 + 18000
v = 21000/600 = 35 m/s
[500,000N]

Impulsive Force

Impulsive force is defined as the rate of change of momentum in a reaction.
It is a force which acts on an object for a very short interval during a collision or explosion.

Example 2
A car of mass 1000kg is traveling with a velocity of 25 m/s. The car hits a street lamp and is stopped in0.05 seconds. What is the impulsive force acting on the car during the crash?

Answer:

Effects of impulse vs Force

A force determines the acceleration (rate of velocity change) of an object. A greater force produces a higher acceleration.
An impulse **determines the velocity change** of an object. A greater impulse yield a higher velocity change.

Examples Involving Impulsive Force

- Playing football
- Playing badminton
- Playing tennis
- Playing golf
- Playing baseball

Long Jump

- The long jump pit is filled with sand to increase the reaction time when athlete land on it.
- This is to reduce the impulsive force acts on the leg of the atlete because impulsive force is inversely proportional to the reaction time.

High Jump

- During a high jump, a high jumper will land on a thick, soft mattress after the jump.
- This is to increase the reaction time and hence reduces the impulsive force acting on the high jumper.

Jumping

A jumper bends his/her leg during landing. This is to increase the reaction time and hence reduce the impact of impulsive force acting on the leg of the jumper.

Crumble Zone

- The crumple zone increases the reaction time of collision during an accident.
- This causes the impulsive force to be reduced and hence reduces the risk of injuries.

Seat Belt

Prevent the driver and passengers from being flung forward or thrown out of the car during an emergency break.

Airbag

The inflated airbag during an accident acts as a cushion to lessen the impact when the driver flings forward hitting the steering wheel or dashboard.

Head Rest

Reduce neck injury when driver and passengers are thrown backwards when the car is banged from backward.

Windscreen

Shatter-proof glass is used so that it will not break into small pieces when broken. This may reduce injuries caused by scattered glass.

Padded Dashboard

Cover with soft material. This may increases the reaction time and hence reduce the impulsive force when passenger knocking on it in accident.

Collapsible Steering Columns
The steering will swing away from driver’s chest during collision. This may reduce the impulsive force acting on the driver.

Anti-lock Braking System (ABS)
Prevent the wheels from locking when brake applied suddenly by adjusting the pressure of the brake fluid. This can prevents the car from skidding.

Bumper
Made of elastic material so that it can increases the reaction time and hence reduces the impulsive force caused by collision.

Passanger Safety Cell
- The body of the car is made from strong, rigid stell cage.
- This may prevent the car from collapsing on the passengers during a car crash.

Example (1):- The car moves in a straight line such that for a short time is velocity is defined by $v = (3t^2 + 2t) ft/s$, where (t) is in seconds. Determine its position and acceleration when (t =3) sec. When t=0, s=0

مثال (1): - تتحرك السيارة في خط مستقيم بحيث يتم تحديد السرعة لفترة قصيرة من خلال $v = (3t^2 + 2t) ft/s$ (t) بالثواني. حدد موضعه وتسارعه عندما (t = 3) sec. عندما تكون t = 0 ، s = 0

Solution

$$V = 3t^2 + 2t$$

$$V = \frac{ds}{dt} \Rightarrow ds = V dt$$

$$\int_{s_0}^{s} ds = \int_0^t V\, dt$$

$$\int_{s_0}^{s} ds = \int_0^t \left(3t^2 + 2t\right) dt$$

$$S\Big]_0^s = \left[\frac{3t^3}{3} + \frac{2t^2}{2}\right]_0^t \Rightarrow S\Big]_0^s = \left[t^3 + t^2\right]_0^t$$

$$S - 0 = \left(t^3 + t^2\right) - \left(0 + 0\right)$$

$$S = \left(t^3 + t^2\right)$$

$$When \quad t = 3\sec$$

$$S = (3)^3 + (3)^2$$

$$S = 27 + 9$$

$$S = 36\ ft$$

$$V = 3t^2 + 2t$$

$$a = \frac{dv}{dt} \Rightarrow a = \frac{d}{dt}(V)$$

$$a = \frac{d}{dt}\left(3t^2 + 2t\right) \Rightarrow a = 6t + 2$$

$$at \quad t = 3\sec$$

$$a = 6*(3) + 2$$

$$a = 18 + 2$$

$$a = 20\ ft/s^2$$

Example (2):- Starting from rest, a particle moving in a straight line has an acceleration of $a = (2t - 6)\ m/s^2$, where (t) is in seconds. What is the particle's velocity when (t =6 sec) and what is its position when (t=11 sec)?

مثال (2): - بدءًا من السكون ، يتحرك الجسيم في خط مستقيم بعجلة مقدارها (t) بالثواني. ما سرعة الجسيم عندما (t = 6 sec) وما هو موضعه ومتى t=11sec

Solution :-

$V_o = 0 \quad , \quad S_o = 0$

a=f(t)

$a = 2t - 6$

$a = \frac{dv}{dt} \Rightarrow dv = a\,dt$

$\int_{v_o}^{v} dv = \int_0^t a\,dt$

$\int_{v_o}^{v} dv = \int_0^t (2t - 6)dt$

$\int_0^{v} dv = \int_0^t (2t - 6)dt$

$V \Big]_0^v = \left[\frac{2t^2}{2} - 6t\right]_0^t \Rightarrow V \Big]_0^v = \left[t^2 - 6t\right]_0^t$

$V - 0 = (t^2 - 6t) - (0 - 0)$

$V = t^2 - 6t$

$When \quad t = 6$

$V = 6^2 - 6*6$

$V = 36 - 36$

$V = 0 \ m/s$

$$V = \frac{ds}{dt}$$

$$ds = V\ dt$$

$$\int_{s_o}^{s} ds = \int_0^t V\ dt$$

$$\int_{s_o}^{s} ds = \int_0^t \left(t^2 - 6t\right) dt$$

$$\int_0^{s} ds = \int_0^t \left(t^2 - 6t\right) dt$$

$$S\Big]_0^s = \left[\frac{t^3}{3} - \frac{6t^2}{2}\right]_0^t$$

$$S\Big]_0^s = \left[\frac{t^3}{3} - 3t^2\right]_0^t \Rightarrow S - 0 = \left(\frac{t^3}{3} - 3t^2\right) - \left(0 - 0\right)$$

$$S = \frac{t^3}{3} - 3t^2$$

$$at \quad t = 11\ \sec$$

$$S = \frac{11^3}{3} - 3(11)^2 \Rightarrow S = 80.7\ m$$

Example (3):- A freight train travels at $V = 60\left(1-e^{-t}\right) ft/s$ where t is the elapsed time in seconds, Determine the distance traveled in three seconds, and the acceleration at this time.

Solution

$$V = f(t)$$

$$V = 60\left(1-e^{-t}\right)$$

$$V = \frac{ds}{dt} \Rightarrow ds = V\ dt \Rightarrow ds = 60\left(1-e^{-t}\right)dt$$

$$\int_0^s ds = \int_0^t 60\left(1-e^{-t}\right)dt$$

$$S\Big]_0^s = 60\left[t+e^{-t}\right]_0^t \Rightarrow S-0 = 60\left[t+e^{-t}\right]-\left[0+e^{0}\right]$$

$$e^{0} = 1$$

$$S = 60\left[t+e^{-t}-1\right]$$

$$at \quad t = 3$$

$$S = 60\left[3+e^{-3}-1\right]$$

$$S = 122.98 \quad ft$$

$$V = 60\left(1 - e^{-t}\right)$$

$$V = 60 - 60e^{-t}$$

$$a = \frac{dv}{dt} \Rightarrow a = \frac{d}{dt}\left(60 - 60e^{-t}\right)$$

$$a = 0 - 60e^{-t} * (-)$$

$$\mathrm{a} = 60e^{-t}$$

$$at \quad t = 3$$

$$a = 60e^{-3}$$

$$a = 2.99 \quad ft / s^{2}$$

Example (1):- The position of a particle along a straight line is given by $S=(1.5t^{3}-13.5t^{2}+22.5t)$ ft where t is in seconds. Determine the position of the partial when t=6 s and the total distance it travels during the 6-s time interval.

مثال (1): - يُعطى موضع الجسيم على طول خط مستقيم بالمكان t بالثواني. حدد موضع الجزء عندما تكون t = 6 s والمسافة الإجمالية التي يقطعها خلال الفترة الزمنية 6 ثوانٍ.

Solution :

$$S=1.5t^{3}-13.5t^{2}+22.5t$$

$$At \quad t=6$$

$$S=1.5(6)^{3}-13.5(6)^{2}+22.5(6)$$

$$S=-27 \quad ft$$

$$S=1.5t^{3}-13.5t^{2}+22.5t$$

$$At \quad t=0$$

$$S=1.5(0)^{3}-13.5(0)^{2}+22.5(0)$$

$$S=0 \quad ft$$

$$V=\frac{ds}{dt}\Rightarrow V=\frac{d}{dt}(s)\Rightarrow V=\frac{d}{dt}\left(1.5t^{3}-13.5t^{2}+22.5t\right)$$

$$V=4.5t^{2}-27t+22.5$$

$$V=0$$

$$4.5t^{2}-27t+22.5=0$$

$$t=1 \text{ sec}$$

$$t=5 \text{ sec}$$

$$S = 1.5t^3 - 13.5t^2 + 22.5t$$

$$At \quad t = 1$$

$$S = 1.5(1)^3 - 13.5(1)^2 + 22.5(1)$$

$$S = 10.5 \quad ft$$

$$S = 1.5t^3 - 13.5t^2 + 22.5t$$

$$At \quad t = 5$$

$$S = 1.5(5)^3 - 13.5(5)^2 + 22.5(5)$$

$$S = -37.5 \quad ft$$

$$d = 10.5 + 10.5 + 27 + 10.5 + 10.5$$

$$d = 69 \quad ft$$

Example (2):- A particle moves along a straight line such that its position is defined by $S=\left(t^{2}-6t+5\right)m$. Determine the average velocity, the average speed, and the acceleration of the particle when t=6 sec.

مثال (2): - يتحرك جسيم على طول خط مستقيم بحيث يتم تحديد موضعه بواسطة. أوجد السرعة المتوسطة ومتوسط السرعة وتسارع الجسم عندما يكون t = 6 ثوانٍ.

$$S=t^{2}-6t+5 \qquad m$$

$$V=\frac{ds}{dt}=2t-6 \qquad m/s$$

$$a=\frac{dv}{dt}=2 \qquad m/s^{2}$$

$$S=t^{2}-6t+5$$

$$At \quad t=0$$

$$S=(0)^{2}-6(0)+5=5 \;\; m$$

$$S=t^{2}-6t+5$$

$$At \quad t=6$$

$$S=(6)^{2}-6(6)+5$$

$$S=36-36+5=5$$

$$\Delta S=5-5=0 \Rightarrow V_{av}=\frac{\Delta S}{\Delta t}=\frac{0}{6-0} \Rightarrow V_{av}=0$$

$$V=2t-6$$

$$V=0$$

$$2t-6=0$$

$$2t=6 \Rightarrow t=\frac{6}{2} \Rightarrow t=3\sec$$

$$S = t^2 - 6t + 5$$

$$At \quad t = 3$$

$$S = (3)^2 - 6(3) + 5$$

$$S = 9 - 18 + 5$$

$$S = -4 \quad m$$

$$d = 5 + 4 + 4 + 5 = 18 \quad m$$

$$V_{sp} = \frac{d}{\Delta t} = \frac{18}{6 - 0}$$

$$V_{sp} = 3 \; m/s$$

Example (3):- The position of a particle along a straight line path is defined by $S = (t^3 - 6t^2 - 15t + 7)\ ft$ where t is in seconds. Determine the total distance traveled when t=10sec . What are the particle's average velocity, average speed, and the instantaneous velocity and acceleration at this time?

مثال (3): - يتم تحديد موضع الجسيم على طول مسار الخط المستقيم حيث تكون t بالثواني. أوجد المسافة الكلية المقطوعة عندما تكون t = 10sec. ما هي سرعة الجسيم المتوسطة ومتوسط السرعة والسرعة اللحظية والتسارع في هذا الوقت؟

Solution

$$S = t^3 - 6t^2 - 15t + 7$$

$$V = \frac{ds}{dt} = 3t^2 - 12t - 15$$

$$a = \frac{dv}{dt} = 6t - 12$$

$$V = 3t^2 - 12t - 15$$

$$at \quad t = 10$$

$$V = 3(10)^2 - 12(10) - 15$$

$$V = 165 \; ft/\sec$$

$$a = 6t - 12$$

$$at \quad t = 10$$

$$a = 6(10) - 12$$

$$a = 60 - 12$$

$$a = 48 \; ft/s^2$$

$$S = t^3 - 6t^2 - 15t + 7$$

$$at \quad t = 0$$

$$S = (0)^3 - 6(0)^2 - 15(0) + 7$$

$$S = 7 \ ft$$

$$S = t^3 - 6t^2 - 15t + 7$$

$$at \quad t = 10$$

$$S = (10)^3 - 6(10)^2 - 15(t) + 7$$

$$s = 257 \quad ft$$

$$V = 3t^2 - 12t - 15$$

$$V = 0$$

$$3t^2 - 12t - 15 = 0$$

$$t = 5 \qquad t = -1$$

$$S = t^3 - 6t^2 - 15t + 7$$

$$at \quad t = 5$$

$$S = (5)^3 - 6(5)^2 - 15(5) + 7$$

$$S = -93 \ ft$$

$$d = 7 + 93 + 93 + 7 + 250 = 450 \ ft$$

$$V_{av} = \frac{\Delta s}{\Delta t} = \frac{257 - 7}{10 - 0} = \frac{250}{10} = 25 \ m/s$$

$$V_{sp} = \frac{d}{\Delta t} = \frac{450}{10} = 45 \ m/s$$

Simple Strain

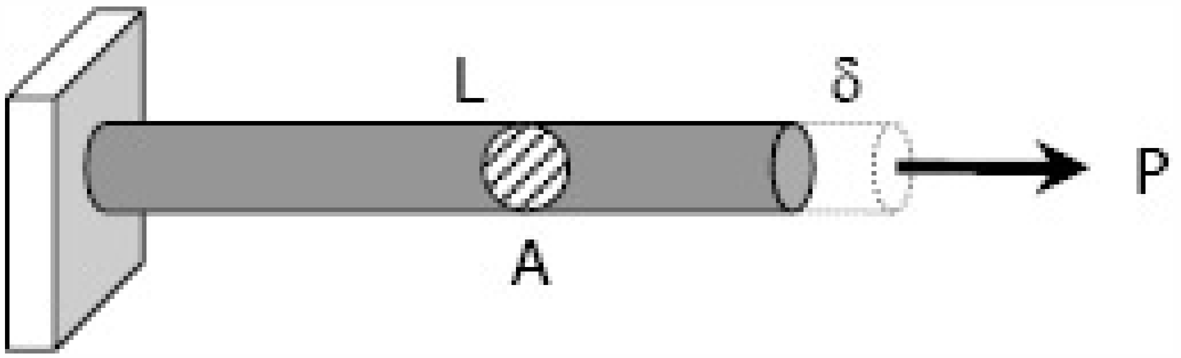
Simple Strain: A strain is the ratio of the change in length caused by the applied force, to the original length:

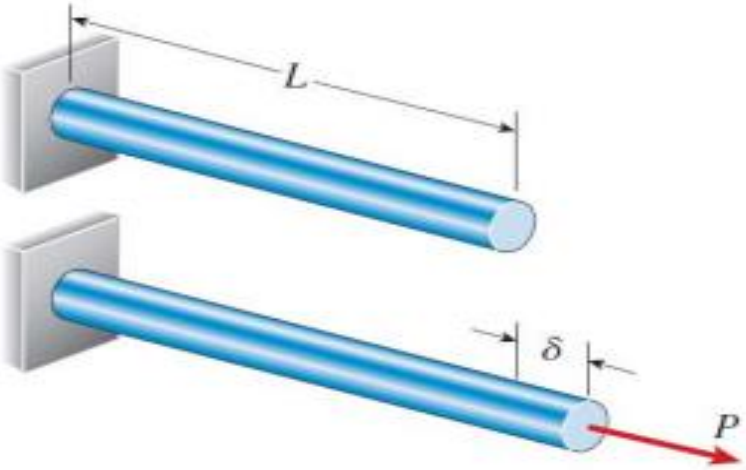
الانفعال البسيط : هو نسبة التغير في الطول بسبب القوة المسلطة إلى الطول الأصلي.

Strain (Є) :
The deformation per unit length .

$$\in = \frac{\delta l}{L}$$

Є	**: Strain**	**(without units)**
δl	**: change in length**	**(mm)**
L	**: original length**	**(mm)**



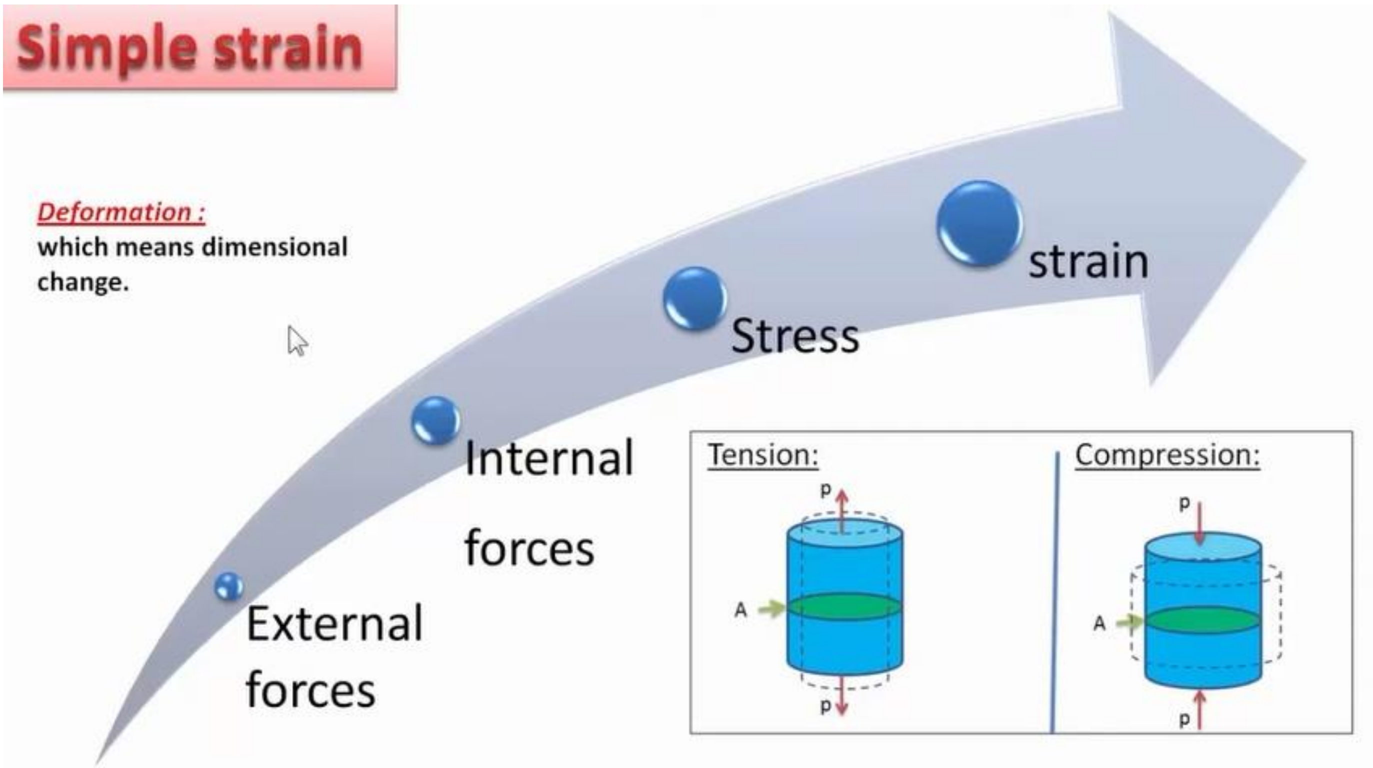


Strain (Є) (ابسيلون): The deformation per unit length:

الانفعال (Є)(ابسيلون):- التشوه لكل وحدة طول

The strain:- means the change in length divided on the original length before deformation.

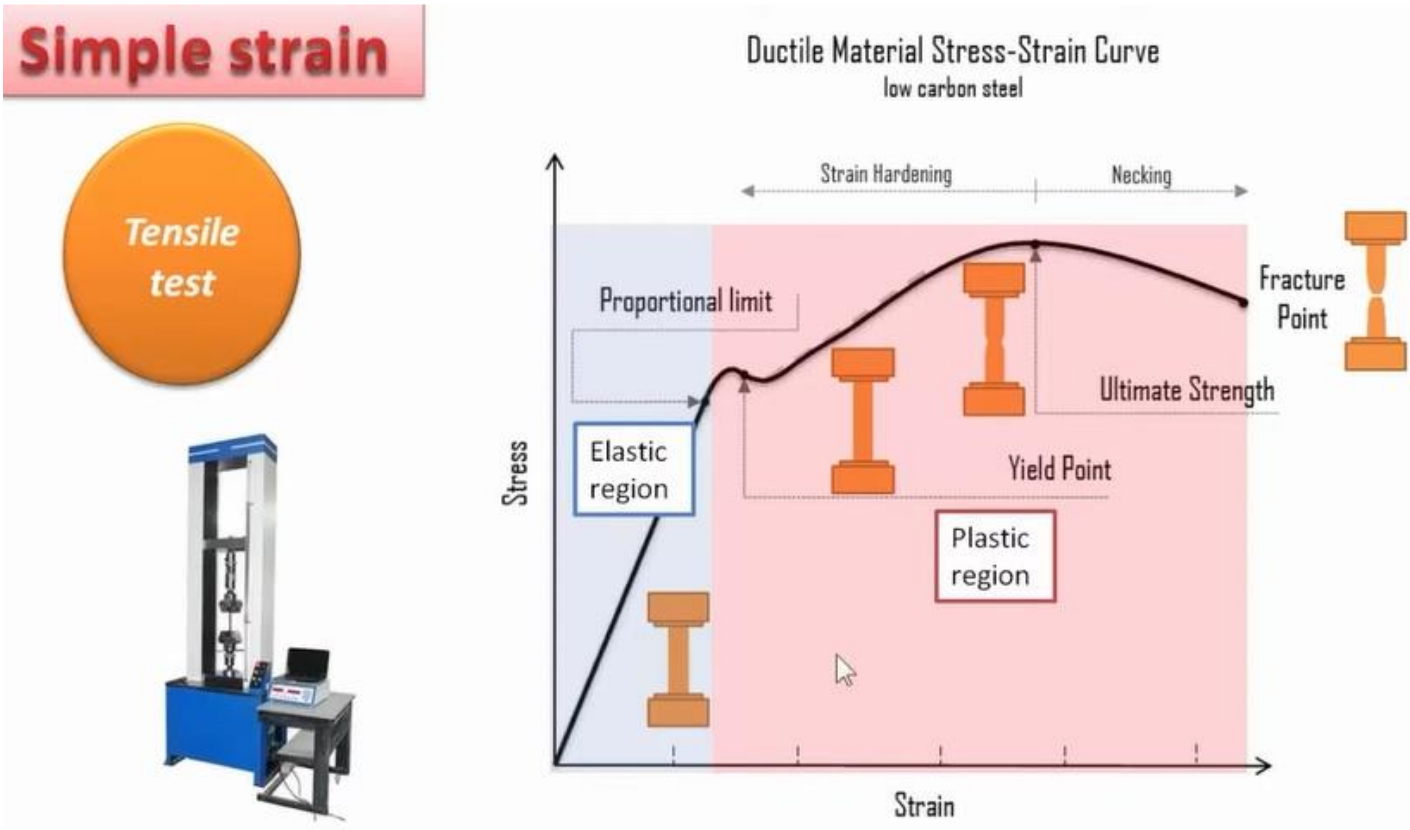
ويقصد بالانفعال :-هو التغير في الطول مقسوماً على الطول الاصلي للعينة قبل التشوه.



Stress-Strain Diagram

Metallic engineering materials are classified as either ductile or brittle materials. A ductile material is one having relatively large tensile strains up to the point of rupture like structural steel and aluminum, whereas brittle materials has a relatively small strain up to the point of rupture like cast iron and concrete.

تصنف المواد الهندسية المعدنية على أنها مواد مطيلة أو هشة. المادة المطيلية هي مادة تحتوي على انفعالات شد كبيرة نسبيًا تصل إلى نقطة التمزق مثل الفولاذ الإنشائي والألمنيوم ، في حين أن المواد الهشة لها انفعال صغير نسبيًا يصل إلى نقطة التمزق مثل الحديد الزهر والخرسانة.



Young's Modulus or Modulus of Elasticity

<u>Hook's law</u> states that when the material is loaded within elastic limit , the stress is proportional to the strain.

<u>قانون هوك:-</u> ينص قانون هوك على انه عندما يتم تحميل المادة ضمن حد مرن ، يكون الضغط متناسبًا طردياً مع الإجهاد.

$$\sigma \propto \varepsilon \Rightarrow \sigma = E\,\varepsilon$$

$$E = \frac{\sigma}{\varepsilon} = \frac{p / A}{\delta L / L}$$

$$E = \frac{p}{\frac{\delta L * A}{L}} \Rightarrow \frac{E * \delta L * A}{L} = p$$

$$P * L = E * \delta L * A$$

$$E = \frac{p * L}{\delta L * A}$$

E : Young Modulus , or modulus of elasticity (N / mm2).

P = Axial compressive force acting on the body.

A = Cross-sectional area of the body,

L = Original length, and.

δ = Decrease in length.

Table 4.1. Values of E for the commonly used engineering materials.

Material	*Modulus of elasticity (E) in GPa i.e. GN/m^2 or kN/mm^2*
Steel and Nickel	200 to 220
Wrought iron	190 to 200
Cast iron	100 to 160
Copper	90 to 110
Brass	80 to 90
Aluminium	60 to 80
Timber	10

Shear Modulus or Modulus of Rigidity

The shear stress is directly proportional to shear strain

$$\tau \propto \varphi \qquad or \qquad G = \frac{\tau}{\varphi}$$

$\varphi = shear\ strain$

$\tau = shear\ stress$

$G = modulus\ of\ rigidity$

Table 4.2. Values of *C* for the commonly used materials.

Material	*Modulus of rigidity (C) in GPa i.e. GN/m^2 or kN/mm^2*
Steel	80 to 100
Wrought iron	80 to 90
Cast iron	40 to 50
Copper	30 to 50
Brass	30 to 50
Timber	10

Example (1) :

A rod 100 cm long and of 2cm X 2cm cross section is subjected to a pull of 1000 kg force . if the modulus of elasticity of the material is $2*10^6$ kg/cm2 , Determine the elongation of the rod ?

شفت طوله 100 سم ومقطع عرضي 2 سم × 2 سم يخضع لسحب بقوة 1000 كجم. إذا كان معامل مرونة المادة كجم / سم 2 ، فقم بتحديد استطالة الشفت؟

Solution :

l =100 cm , A = 2 * 2 = 4 cm^2 , P = 1000 kg , E = 2*10^6

$$\delta l = \frac{P.l}{A.E} = \frac{1000*100}{4*2*10^6} = 0.0125 \quad cm$$

Example (2) :

A load of 5 KN is to be raised with the help of a steel wire . Find the minimum diameter of the steel wire if the stress is not to exceed

100 MN / m^2?

Solution :

P = 5 KN = 5000 N , σ = 100 MN / m^2 = 100 N / mm^2

$$\sigma = \frac{P}{A} \Rightarrow 100 = \frac{5000}{\frac{\pi}{4}(d^2)} \Rightarrow d = 7.98 \approx 8 \text{ mm}$$

4

HOME WORK :

1 – Determine the elongation of the steel bar 1 m long and 1.5 cm^2 cross-sectional area , when subjected to a pull of 1500 kg . Take E = 2*10^6 kg / cm^2 ?

2 – A brass rod 2 cm diameter and 1.5 m long is subjected to an axial pull of 4 tonnes . Find the stress , strain and elongation of the rod , if the modulus of elasticity for the brass is 1.0*10^6 kg / cm^2 ?

3 – A cast iron column has internal diameter of 200 mm , What should be the minimum external diameter so that it may carry a load of 1.6 MN , without the stress exceeding 90 N / mm^2 ?

Example (1) :-

Determine the elongation of the steel bar (1 m) long and $(1.5 cm^2)$ cross-sectional area , when subjected to a Pull of (1500 kg) . Take $(E = 2*10^6) kg / \mathrm{cm}^2$?

Solution:-

$$\delta L = \frac{P*L}{E*A}$$

$$\delta L = \frac{1500*1*100}{2*10^6*1.5} = \frac{150000}{3000000}$$

$$\delta L = 0.05 \text{ cm}$$

Example (2):-

A brass rod (2 cm) diameter and (1.5 m) long is subjected to an axial pull of (4 tonnes) . Find the stress , strain and elongation of the rod , if the modulus of elasticity for the brass is

$1.0*10^{6}\ kg/cm^{2}$?

Solution :-

$$\sigma=\frac{P}{A}=\frac{P}{\frac{\pi}{4}d^{2}}=\frac{4*1000}{\frac{\pi}{4}(2)^{2}}=\frac{4*1000}{\frac{\pi}{4}4}=\frac{4000}{\pi}\Rightarrow\sigma=1273.24\quad kg/cm^{2}$$

$$\sigma=E\varepsilon\Rightarrow\varepsilon=\frac{\sigma}{E}=\frac{1273.24}{1*10^{6}}\Rightarrow\varepsilon=0.00127324$$

$$\varepsilon=\frac{\delta L}{L}\Rightarrow\delta L=\varepsilon*L=0.00127324*1.5*100$$

$$\delta L=0.2\quad cm$$

Example (3):-

A cast iron column has internal diameter of (200 mm) , What should be the minimum external diameter so that it may carry a load of (1.6 MN) , without the stress exceeding $(90 \ N/mm^2)$?

مثال (3):- عمود من الحديد الزهر بقطر داخلي 200 مم ، ما هو الحد الأدنى للقطر الخارجي بحيث يمكن أن يحمل حمولة 1.6 MN ، دون أن يتجاوز الاجهاد 90 نيوتن / مم 2؟

Solution :-

$$\sigma = \frac{P}{A} \Rightarrow P = \sigma * A$$

$$P = \sigma * \frac{\pi}{4}\left(d_o^2 - d_i^2\right)$$

$$1.6*10^6 = 90*\frac{\pi}{4}*\left(d_o^2 - (200)^2\right)$$

$$1600000 = 90*\frac{\pi}{4}*\left(d_o^2 - 40000\right)$$

$$d_o^2 - 40000 = \frac{1600000}{90*\frac{\pi}{4}} \Rightarrow d_o^2 - 40000 = \frac{1600000}{\frac{90*\pi}{4}}$$

$$d_o^2 - 40000 = \frac{1600000*4}{90*\pi} \Rightarrow d_o^2 - 40000 = 22635.4$$

$$d_o^2 = 22635.4 + 40000 \Rightarrow d_o^2 = 62635.4 \Rightarrow d_o = \sqrt{62635.4}$$

$$d_o = 250.2 \ mm$$

Torsional shear stress

Torsional shear stress:- is that occurs by external twist or torque that called torsion stress.

أجهاد قص الالتواء: هو أن يحدث عن طريق الالتواء الخارجي أو عزم الدوران الذي يسمى إجهاد الالتواء.

$$\tau_s = \frac{T * r}{J}$$

Where:

τ_s : *Torsional shear stress*. N/m^2

T : Torque. (N.m)

r : *Radius*. (m)

J : *Polar moment of inertia* (m^4)

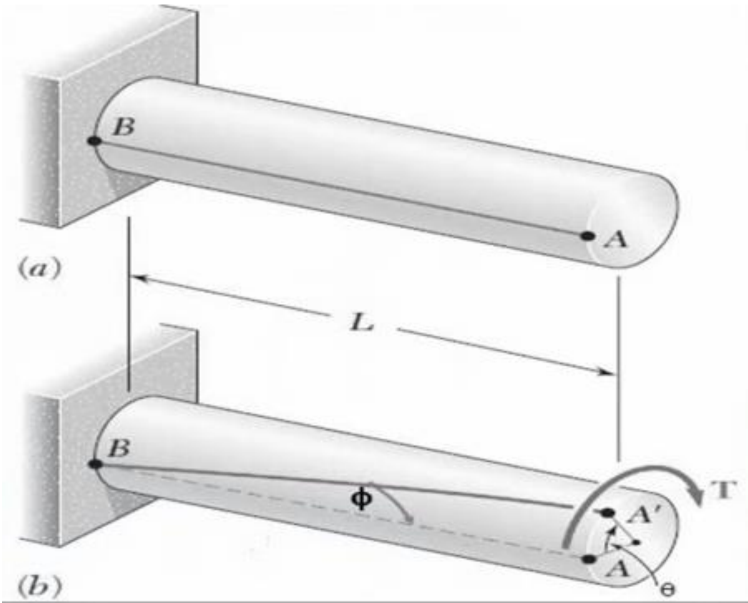
T=F*r

F:force **N**

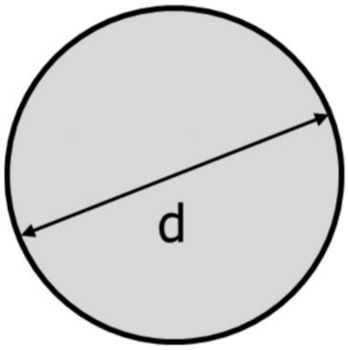
r: force arm m

where :

$$r = \frac{d}{2}$$



a) (**J**) Polar moment of inertia for **(solid shaft)**



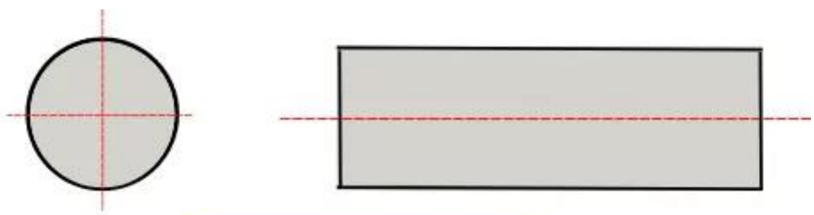
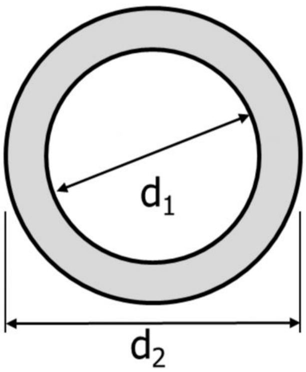


FIG-A : Solid Shaft

$$J = \frac{\pi}{32} d^4$$

b) (**J**) Polar moment of inertia for **(hollow shaft)**



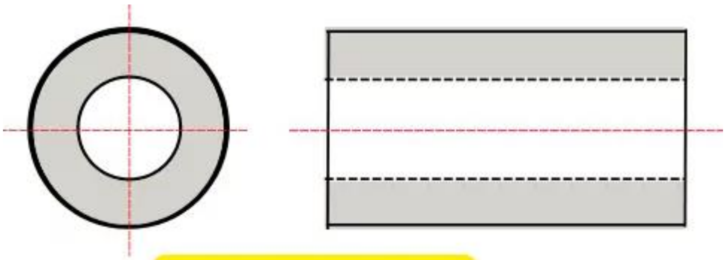


FIG-B : Hollow Shaft

$$J = \frac{\pi}{32}\left(d_o^4 - d_i^4\right)$$

d_o : *external diameter (outer) m*

d_i : internal *diameter* (inter) *m*

Example (1)

Determine the maximum torque that can be applied a hollow circular steel shaft of (100 mm) outer diameter and an (80 mm) inside diameter without exceeding a torsional shear stress use (60 MPa)?

مثال 1: تحديد أقصى عزم يمكن تطبيقه على عمود فولاذي دائري مجوف بقطر خارجي (100 مم) وقطر داخلي (80 مم) دون تجاوز استخدام إجهاد القص الالتوائي (60 ميجا باسكال)؟

Solution

$$\tau_s = \frac{T * r}{J} \Rightarrow \tau_s * J = T * r \Rightarrow T = \frac{\tau_s * J}{r}$$

$$T = \frac{\tau_s * \frac{\pi}{32}\left(d_o^4 - d_i^4\right)}{\frac{d}{2}} \Rightarrow \frac{\tau_s * \pi\left(d_o^4 - d_i^4\right)}{32 * \frac{d}{2}} \Rightarrow \frac{\tau_s * \pi\left(d_o^4 - d_i^4\right)}{16 * d}$$

$$T = \frac{60 * \pi * \left(100^4 - 80^4\right)}{16 * 100} = 6951960 \ N.mm$$

Example (2)

A steel solid shaft (3 m) long that has a diameter of (4 m). is subjected to a torque of (15 N·m) . Determine the maximum shearing stress?

مثال 2: عمود من الصلب بطول (3 م) قطر (4 م). يخضع لعزم دوران (15 نيوتن متر). تحديد أقصى إجهاد القص؟

$$\tau_{\max} = \frac{T * r}{J} = \frac{15*2}{\frac{\pi}{32} * d^4} = \frac{15*2}{\frac{\pi}{32} * (4)^4}$$

$$\tau_{\max} = \text{..........}\ N / m^2$$

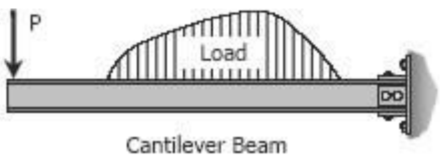
Shear & Moment in Beams

DEFINITION OF A BEAM

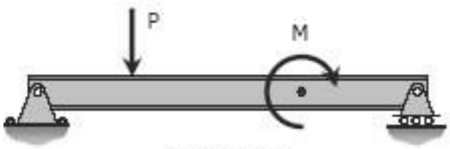
A beam is a bar subject to forces or couples that lie in a plane containing the longitudinal of the bar. According to determinacy, a beam may be determinate or indeterminate.

STATICALLY DETERMINATE BEAMS

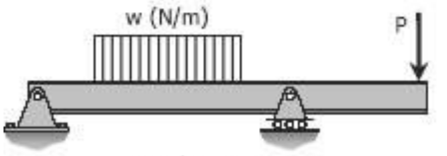
Statically determinate beams are those beams in which the reactions of the supports may be determined by the use of the equations of static equilibrium. The beams shown below are examples of statically determinate beams.



Cantilever Beam



Simple Beam



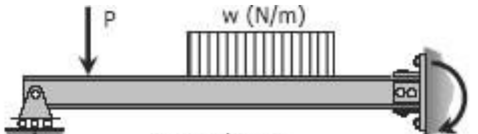
Overhanging Beam

STATICALLY INDETERMINATE BEAMS

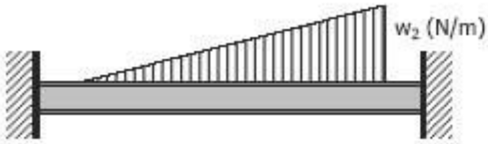
If the number of reactions exerted upon a beam exceeds the number of equations in static equilibrium, the beam is said to be statically indeterminate. In order to solve the reactions of the beam, the static equations must be supplemented by equations based upon the elastic deformations of the beam.

The degree of indeterminacy is taken as the difference between the umber of reactions to the number of equations in static equilibrium that can be applied. In the case of the propped beam shown, there are three reactions R_1, R_2, and M and only two equations ($\sum M = 0$ and sum;$F_v = 0$) can be applied, thus the beam is indeterminate to the first

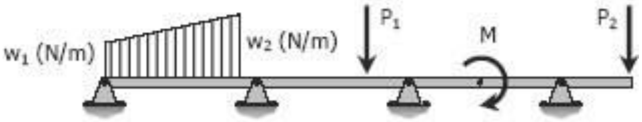
degree (3 – 2 = 1).



Propped Beam



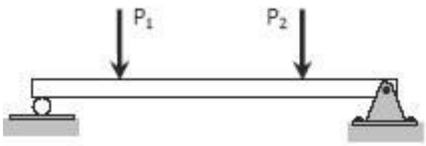
Fixed or Restrained Beam



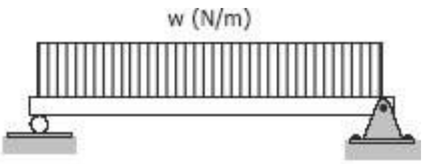
Continuous Beam

TYPES OF LOADING

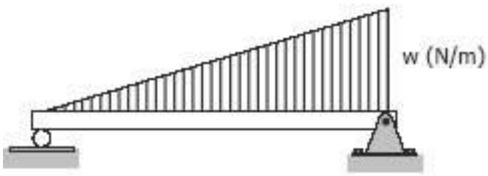
Loads applied to the beam may consist of a concentrated load (load applied at a point), uniform load, uniformly varying load, or an applied couple or moment. These loads are shown in the following figures.



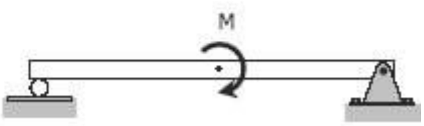
Concentrated Loads



Uniform Load

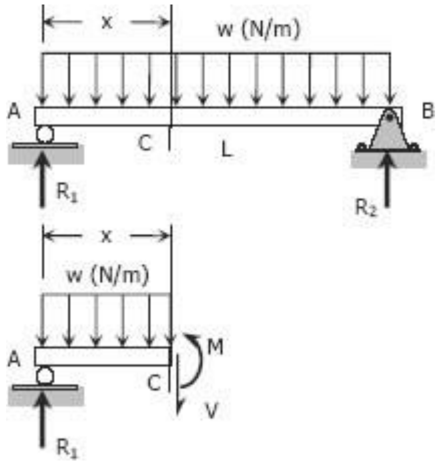


Uniformly Varying Load



Applied Couple

Shear and Moment Diagrams



Consider a simple beam shown of length L that carries a uniform load of w (N/m) throughout its length and is held in equilibrium by reactions R_1 and R_2. Assume that the beam is cut at point C a distance of x from he left support and the portion of the beam to the right of C be removed. The portion removed must then be replaced by vertical shearing force V together with a couple M to hold the left portion of the bar in equilibrium under the action of R_1 and wx. The couple M is called the resisting moment or moment and the force V is called the resisting shear or shear. The sign of V and M are taken to be positive if they have the senses indicated above.

Solved Problems in Shear and Moment Diagrams

INSTRUCTION

Write shear and moment equations for the beams in the following problems. In each problem, let x be the distance measured from left end of the beam. Also, draw shear and moment diagrams, specifying values at all change of loading positions and at points of zero shear. Neglect the mass of the beam in each problem.

Problem 403

Beam loaded as shown in Fig. P-403.

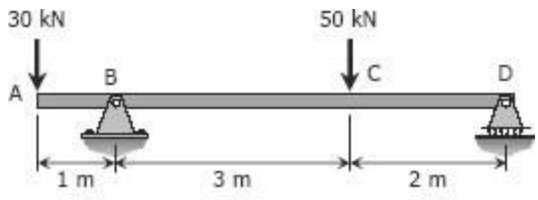


Figure P-403

Solution 403

From the load diagram:

$\sum M_B = 0$

$5R_D + 1(30) = 3(50)$

$R_D = 24$ kN

$\sum M_D = 0$

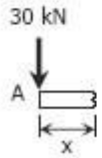
$5R_B = 2(50) + 6(30)$

$R_B = 56$ kN

Segment AB:

$V_{AB} = -30$ kN

$M_{AB} = -30x$ kN·m



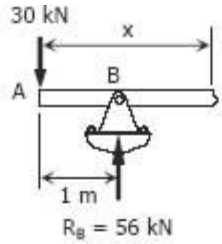
Segment BC:

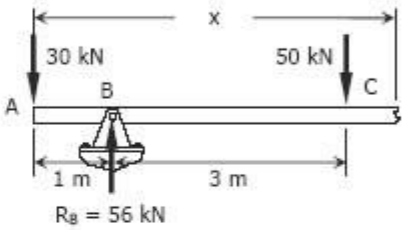
$V_{BC} = -30 + 56$

$= 26$ kN

$M_{BC} = -30x + 56(x - 1)$

$= 26x - 56$ kN·m





Segment CD:

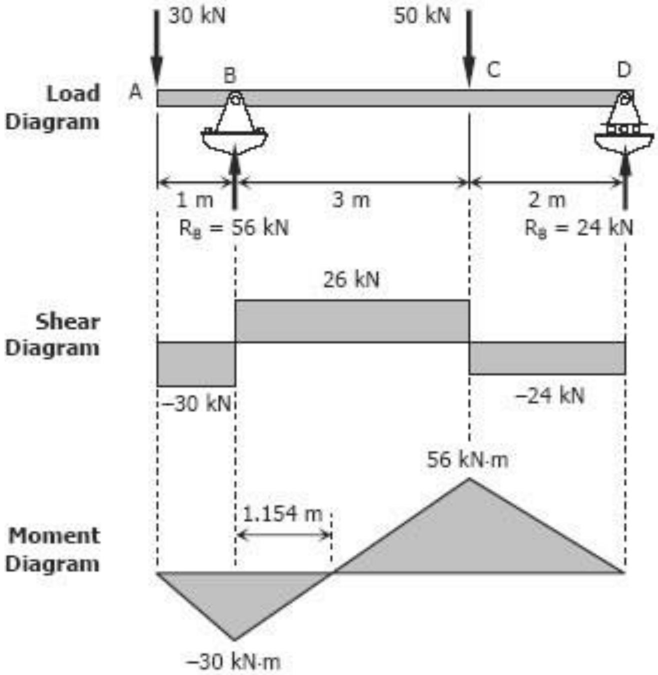
$V_{CD} = -30 + 56 - 50$

$= -24$ kN

$M_{CD} = -30x + 56(x - 1) - 50(x - 4)$

$= -30x + 56x - 56 - 50x + 200$

$= -24x + 144$



To draw the Shear Diagram:

(1) In segment AB, the shear is uniformly distributed over the segment at a magnitude of –30 kN.

(2) In segment BC, the shear is uniformly distributed at a magnitude of 26 kN.

(3) In segment CD, the shear is uniformly distributed at a magnitude of –24 kN.

To draw the Moment Diagram:

(1) The equation $M_{AB} = -30x$ is linear, at x = 0, $M_{AB} = 0$ and at x = 1 m, $M_{AB} = -30$ kN·m.

(2) $M_{BC} = 26x - 56$ is also linear. At x = 1 m, $M_{BC} = -30$ kN·m; at x = 4 m, $M_{BC} = 48$ kN·m. When $M_{BC} = 0$, x = 2.154 m, thus the moment is zero at 1.154 m from B.

(3) $M_{CD} = -24x + 144$ is again linear. At x = 4 m, $M_{CD} = 48$ kN·m; at x = 6 m, $M_{CD} = 0$.

Problem 404

Beam loaded as shown in Fig. P-404.

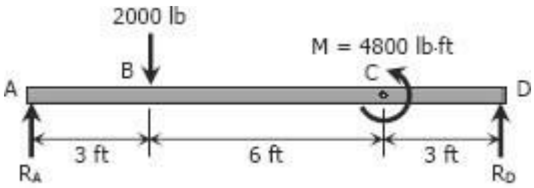


Figure P-404

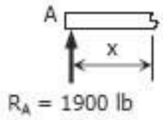
Solution 404

$$\Sigma M_A = 0$$
$$12R_D + 4800 = 3(2000)$$
$$R_D = 100 \text{ lb}$$

$$\Sigma M_D = 0$$
$$12R_A = 9(2000) + 4800$$
$$R_A = 1900 \text{ lb}$$

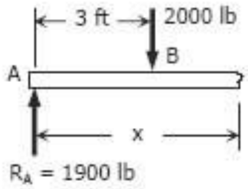
Segment AB:

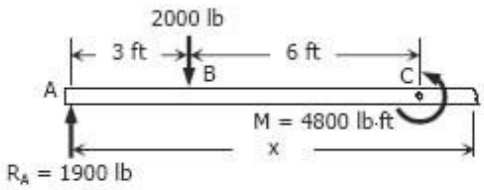
$$V_{AB} = 1900 \text{ lb}$$
$$M_{AB} = 1900x \text{ lb·ft}$$



Segment BC:

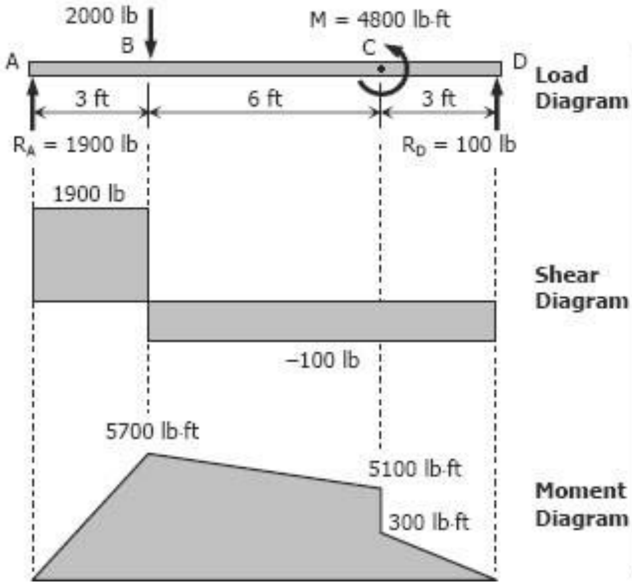
$$V_{BC} = 1900 - 2000$$
$$= -100 \text{ lb}$$
$$M_{BC} = 1900x - 2000(x - 3)$$
$$= 1900x - 2000x + 6000$$
$$= -100x + 6000$$





Segment CD:

$$V_{CD} = 1900 - 2000$$
$$= -100 \text{ lb}$$
$$M_{CD} = 1900x - 2000(x - 3) - 4800$$
$$= 1900x - 2000x + 6000 - 4800$$
$$= -100x + 1200$$



To draw the Shear Diagram:

(1) At segment AB, the shear is uniformly distributed at 1900 lb.
(2) A shear of −100 lb is uniformly distributed over segments BC and CD.

To draw the Moment Diagram:

(1) $M_{AB} = 1900x$ is linear; at $x = 0$, $M_{AB} = 0$; at $x = 3$ ft, $M_{AB} = 5700$ lb·ft.
(2) For segment BC, $M_{BC} = -100x + 6000$ is linear; at $x = 3$ ft, $M_{BC} = 5700$ lb·ft; at $x = 9$ ft, $M_{BC} = 5100$ lb·ft.
(3) $M_{CD} = -100x + 1200$ is again linear; at $x = 9$ ft, $M_{CD} = 300$ lb·ft; at $x = 12$ ft, $M_{CD} = 0$.

Problem 405

Beam loaded as shown in Fig. P-405.

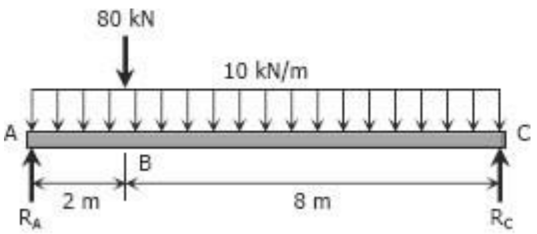
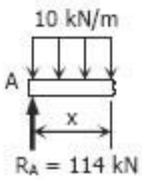


Figure P-405

Solution 405

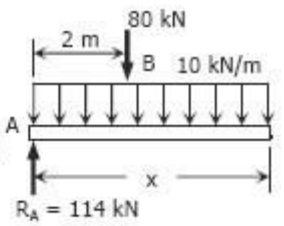
$\Sigma M_A = 0$
$10R_C = 2(80) + 5[10(10)]$
$R_C = 66$ kN

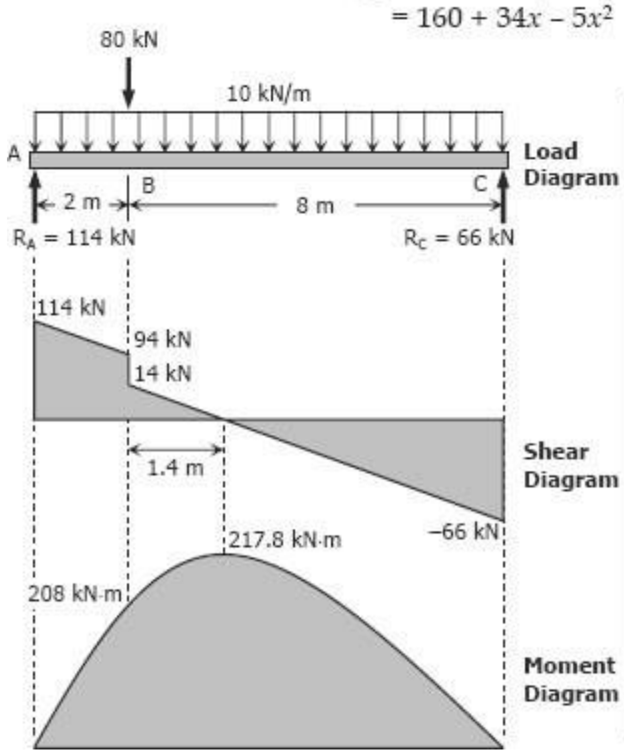
$\Sigma M_C = 0$
$10R_A = 8(80) + 5[10(10)]$
$R_A = 114$ kN



Segment AB:
$V_{AB} = 114 - 10x$ kN
$M_{AB} = 114x - 10x(x/2)$
$= 114x - 5x^2$ kN·m

Segment BC:
$V_{BC} = 114 - 80 - 10x$
$= 34 - 10x$ kN
$M_{BC} = 114x - 80(x - 2) - 10x(x/2)$
$= 160 + 34x - 5x^2$





To draw the Shear Diagram:

(1) For segment AB, $V_{AB} = 114 - 10x$ is linear; at x = 0, V_{AB} = 14 kN; at x = 2 m, V_{AB} = 94 kN.

(2) $V_{BC} = 34 - 10x$ for segment BC is linear; at x = 2 m, V_{BC} = 14 kN; at x = 10 m, V_{BC} = −66 kN. When V_{BC} = 0, x = 3.4 m thus V_{BC} = 0 at 1.4 m from B.

To draw the Moment Diagram:

(1) $M_{AB} = 114x - 5x^2$ is a second degree curve for segment AB; at x = 0, M_{AB} = 0; at x = 2 m, M_{AB} = 208 kN·m.

(2) The moment diagram is also a second degree curve for segment BC given by $M_{BC} = 160 + 34x - 5x^2$; at x = 2 m, M_{BC} = 208 kN·m; at x = 10 m, M_{BC} = 0.

(3) Note that the maximum moment occurs at point of zero shear. Thus, at x = 3.4 m, M_{BC} = 217.8 kN·m.

Problem 406

Beam loaded as shown in Fig. P-406.

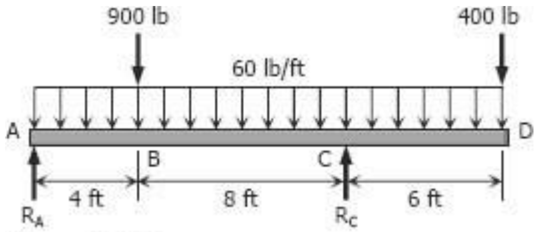


Figure P-406

Solution 406

$\sum M_A = 0$

$12R_C = 4(900) + 18(400) + 9[(60)(18)]$

$R_C = 1710$ lb

$\sum M_C = 0$

$12R_A + 6(400) = 8(900) + 3[60(18)]$

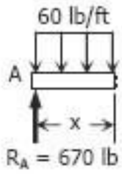
$R_A = 670$ lb

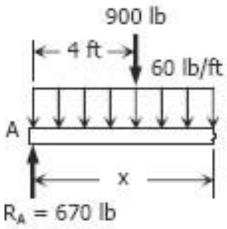
Segment *AB*:

$V_{AB} = 670 - 60x$ lb

$M_{AB} = 670x - 60x(x/2)$

$= 670x - 30x^2$ lb·ft





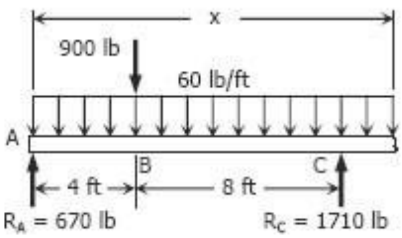
Segment *BC*:

$V_{BC} = 670 - 900 - 60x$

$= -230 - 60x$ lb

$M_{BC} = 670x - 900(x - 4) - 60x(x/2)$

$= 3600 - 230x - 30x^2$ lb·ft



Segment *CD*:

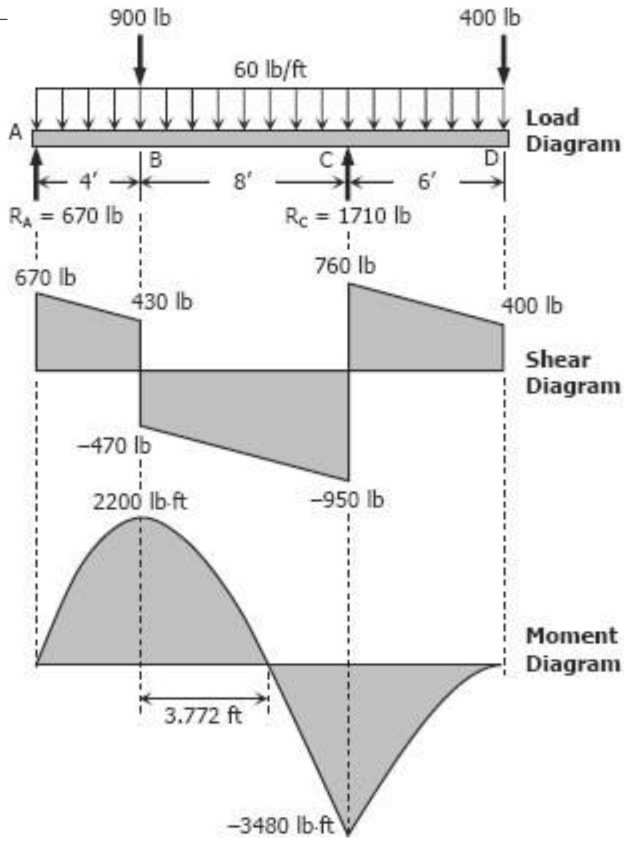
$V_{CD} = 670 + 1710 - 900 - 60x$

$= 1480 - 60x$ lb

$M_{CD} = 670x + 1710(x - 12)$

$- 900(x - 4) - 60x(x/2)$

$= -16920 + 1480x - 30x^2$ lb·ft



To draw the Shear Diagram:

(1) $V_{AB} = 670 - 60x$ for segment AB is linear; at x = 0, $V_{AB} = 670$ lb; at x = 4 ft, $V_{AB} = 430$ lb.

(2) For segment BC, $V_{BC} = -230 - 60x$ is also linear; at x = 4 ft, $V_{BC} = -470$ lb, at x = 12 ft, $V_{BC} = -950$ lb.

(3) $V_{CD} = 1480 - 60x$ for segment CD is again linear; at x = 12, $V_{CD} = 760$ lb; at x = 18 ft, $V_{CD} = 400$ lb.

To draw the Moment Diagram:

(1) $M_{AB} = 670x - 30x^2$ for segment AB is a second degree curve; at x = 0, $M_{AB} = 0$; at x = 4 ft, $M_{AB} = 2200$ lb·ft.

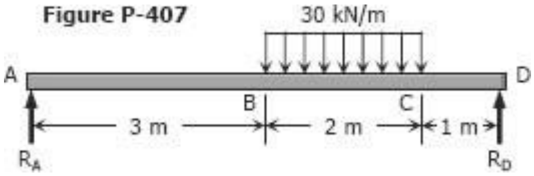
(2) For BC, $M_{BC} = 3600 - 230x - 30x^2$, is a second degree curve; at x = 4 ft, $M_{BC} = 2200$ lb·ft, at x = 12 ft, $M_{BC} = -3480$ lb·ft; When $M_{BC} = 0$, $3600 - 230x - 30x^2 = 0$, x = −15.439 ft and 7.772 ft. Take x = 7.772 ft, thus, the moment is zero at 3.772 ft from B.

(3) For segment CD, $M_{CD} = -16920 + 1480x - 30x^2$ is a second degree curve; at x = 12 ft, $M_{CD} = -3480$ lb·ft; at x = 18 ft, $M_{CD} = 0$.

Problem 407

Beam loaded as shown in Fig. P-407.

Figure P-407



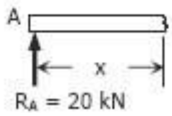
Solution 407

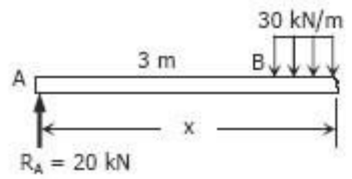
$$\Sigma M_A = 0$$
$$6R_D = 4[2(30)]$$
$$R_D = 40 \text{ kN}$$

$$\Sigma M_D = 0$$
$$6R_A = 2[2(30)]$$
$$R_A = 20 \text{ kN}$$

Segment AB:

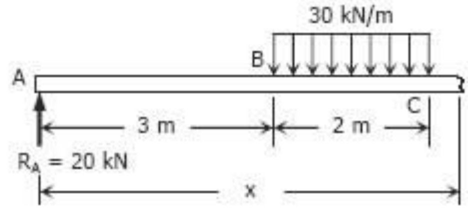
$$V_{AB} = 20 \text{ kN}$$
$$M_{AB} = 20x \text{ kN·m}$$





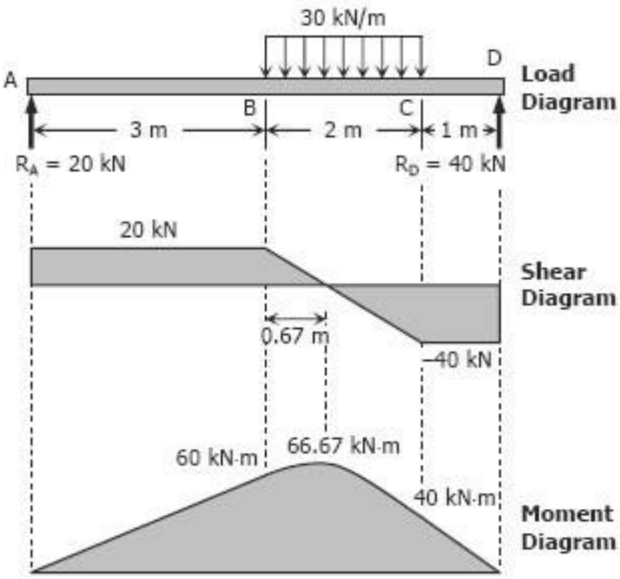
Segment *BC*:

$$V_{BC} = 20 - 30(x - 3)$$
$$= 110 - 30x \text{ kN}$$
$$M_{BC} = 20x - 30(x - 3)(x - 3)/2$$
$$= 20x - 15(x - 3)^2$$



Segment *CD*:

$$V_{CD} = 20 - 30(2)$$
$$= -40 \text{ kN}$$
$$M_{CD} = 20x - 30(2)(x - 4)$$
$$= 20x - 60(x - 4)$$



To draw the Shear Diagram:

1. For segment AB, the shear is uniformly distributed at 20 kN.
2. $V_{BC} = 110 - 30x$ for segment BC; at x = 3 m, V_{BC} = 20 kN; at x = 5 m, V_{BC} = −40 kN. For $V_{BC} = 0$, x = 3.67 m or 0.67 m from B.
3. The shear for segment CD is uniformly distributed at −40 kN.

To draw the Moment Diagram:

1. For AB, $M_{AB} = 20x$; at x = 0, M_{AB} = 0; at x = 3 m, M_{AB} = 60 kN·m.
2. $M_{BC} = 20x - 15(x - 3)^2$ for segment BC is second degree curve; at x = 3 m, M_{BC} = 60 kN·m; at x = 5 m, M_{BC} = 40 kN·m. Note that maximum moment occurred at zero shear; at x = 3.67 m, M_{BC} = 66.67 kN·m.
3. $M_{CD} = 20x - 60(x - 4)$ for segment BC is linear; at x = 5 m, M_{CD} = 40 kN·m; at x = 6 m, M_{CD} = 0.

Problem 408

Beam loaded as shown in Fig. P-408.

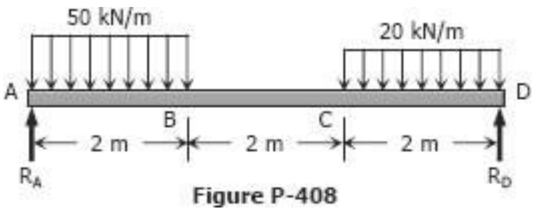


Figure P-408

Solution 408

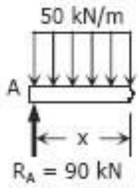
$$\Sigma M_A = 0$$
$$6R_D = 1[2(50)] + 5[2(20)]$$
$$R_D = 50 \text{ kN}$$

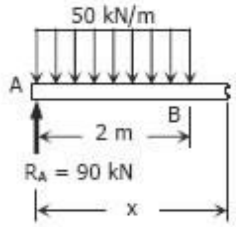
$$\Sigma M_D = 0$$
$$6R_A = 5[2(50)] + 1[2(20)]$$
$$R_A = 90 \text{ kN}$$

Segment AB:

$$V_{AB} = 90 - 50x \text{ kN}$$

$$M_{AB} = 90x - 50x(x/2) = 90x - 25x^2$$

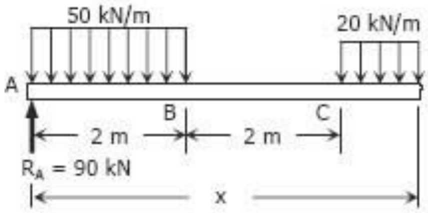




Segment BC:

$$V_{BC} = 90 - 50(2) = -10 \text{ kN}$$

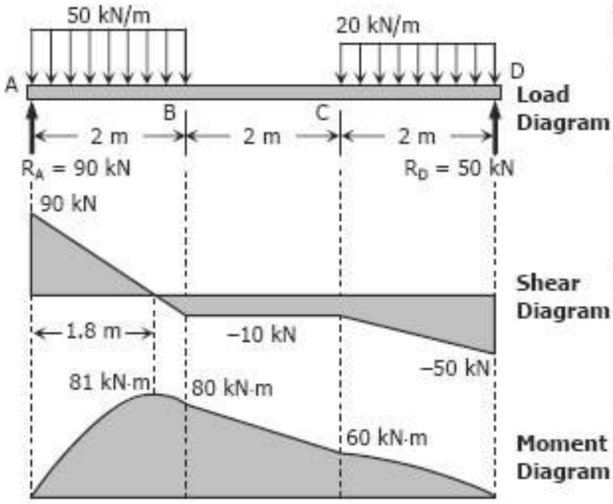
$$M_{BC} = 90x - 2(50)(x - 1) = -10x + 100 \text{ kN·m}$$



Segment CD:

$$V_{CD} = 90 - 2(50) - 20(x - 4) = -20x + 70 \text{ kN}$$

$$M_{CD} = 90x - 2(50)(x - 1) - 20(x - 4)(x - 4)/2 = 90x - 100(x - 1) - 10(x - 4)^2 = -10x^2 + 70x - 60 \text{ kN·m}$$



To draw the Shear Diagram:

(1) $V_{AB} = 90 - 50x$ is linear; at x = 0, V_{BC} = 90 kN; at x = 2 m, V_{BC} = −10 kN. When V_{AB} = 0, x = 1.8 m.

(2) V_{BC} = −10 kN along segment BC.

(3) $V_{CD} = -20x + 70$ is linear; at x = 4 m, V_{CD} = −10 kN; at x = 6 m, V_{CD} = −50 kN.

To draw the Moment Diagram:

(1) $M_{AB} = 90x - 25x^2$ is second degree; at x = 0, M_{AB} = 0; at x = 1.8 m, M_{AB} = 81 kN·m; at x = 2 m, M_{AB} = 80 kN·m.

(2) $M_{BC} = -10x + 100$ is linear; at x = 2 m, M_{BC} = 80 kN·m; at x = 4 m, M_{BC} = 60 kN·m.

(3) $M_{CD} = -10x^2 + 70x - 60$; at x = 4 m, M_{CD} = 60 kN·m; at x = 6 m, M_{CD} = 0.

Problem 409

Cantilever beam loaded as shown in Fig. P-409.

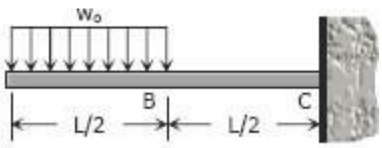


Figure P-409

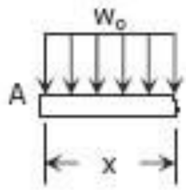
Solution 409

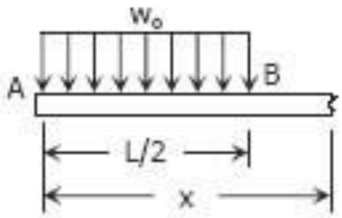
Segment AB:

$V_{AB} = -w_o x$

$M_{AB} = -w_o x(x/2)$

$= -\frac{1}{2} w_o x^2$





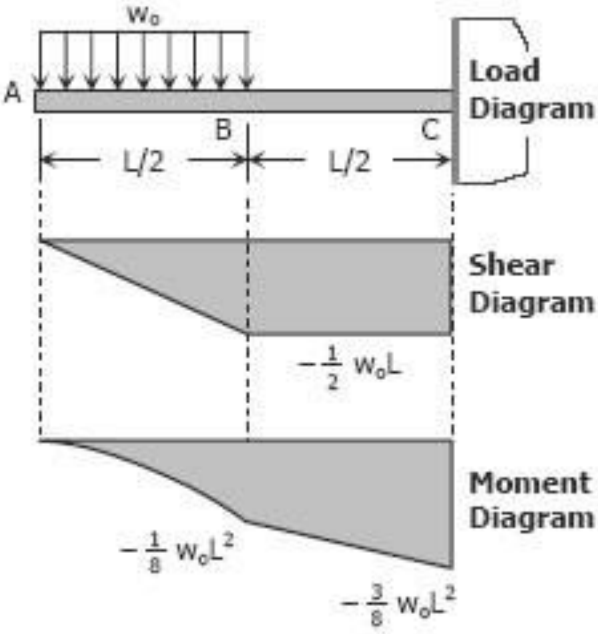
Segment BC:

$V_{BC} = -w_o(L/2)$

$= -\frac{1}{2} w_o L$

$M_{BC} = -w_o(L/2)(x - L/4)$

$= -\frac{1}{2} w_o L x + \frac{1}{8} w_o L^2$



To draw the Shear Diagram:

(1) $V_{AB} = -w_o x$ for segment AB is linear; at x = 0, $V_{AB} = 0$; at x = L/2, $V_{AB} = -\frac{1}{2} w_o L$.

(2) At BC, the shear is uniformly distributed by $-\frac{1}{2} w_o L$.

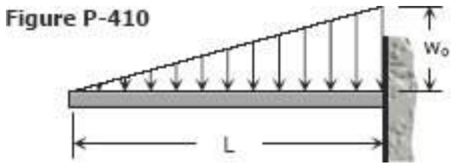
To draw the Moment Diagram:

(1) $M_{AB} = -\frac{1}{2} w_o x^2$ is a second degree curve; at x = 0, $M_{AB} = 0$; at x = L/2, $M_{AB} = -\frac{1}{8} w_o L^2$.

(2) $M_{BC} = -\frac{1}{2} w_o L x + \frac{1}{8} w_o L^2$ is a second degree; at x = L/2, $M_{BC} = -\frac{1}{8} w_o L^2$; at x = L, $M_{BC} = -\frac{3}{8} w_o L^2$.

Problem 410

Cantilever beam carrying the uniformly varying load shown in Fig. P-410.



Solution 410

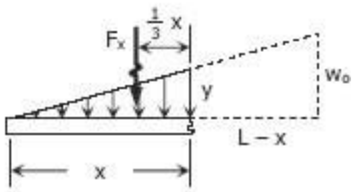
$$\frac{y}{x} = \frac{w_o}{L}$$

$$y = \frac{w_o}{L}x$$

$$F_x = \tfrac{1}{2}xy$$

$$= \frac{1}{2}x\left(\frac{w_o}{L}x\right)$$

$$= \frac{w_o}{2L}x^2$$



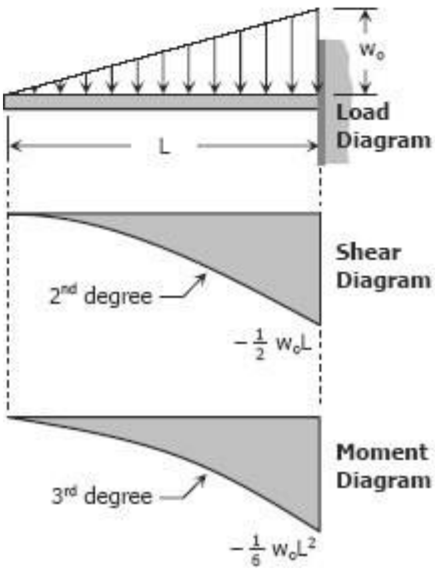
Shear equation:

$$V = -\frac{w_o}{2L}x^2$$

Moment equation:

$$M = -\tfrac{1}{3}xF_x = -\frac{1}{3}x\left(\frac{w_o}{2L}x^2\right)$$

$$= -\frac{w_o}{6L}x^3$$



To draw the Shear Diagram:

$V = -\frac{w_o}{2L}x^2$ is a second degree curve;

at x = 0, V = 0; at x = L, $V = -\frac{1}{2}w_oL$.

To draw the Moment Diagram:

$M = -\frac{w_o}{6L}x^3$ is a third degree curve; at

x = 0, M = 0; at x = L, $M = -\frac{1}{6}w_oL^2$.

Problem 411

Cantilever beam carrying a distributed load with intensity varying from wo at the free end to zero at the wall, as shown in Fig. P-411.

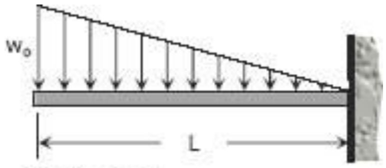
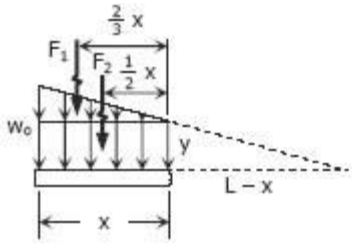


Figure P-411

Solution 411

$$\frac{y}{L-x} = \frac{w_o}{L}$$

$$y = \frac{w_o}{L}(L-x)$$



$$F_1 = \tfrac{1}{2}x(w_o - y)$$

$$= \frac{1}{2}x\left[w_o - \frac{w_o}{L}(L-x)\right]$$

$$= \frac{1}{2}x\left[w_o - w_o + \frac{w_o}{L}x\right]$$

$$= \frac{w_o}{2L}x^2$$

$$F_2 = xy = x\left[\frac{w_o}{L}(L-x)\right]$$

$$= \frac{w_o}{L}(Lx - x^2)$$

Shear equation:

$$V = -F_1 - F_2 = -\frac{w_o}{2L}x^2 - \frac{w_o}{L}(Lx - x^2)$$

$$= -\frac{w_o}{2L}x^2 - w_o x + \frac{w_o}{L}x^2$$

$$= \frac{w_o}{2L}x^2 - w_o x$$

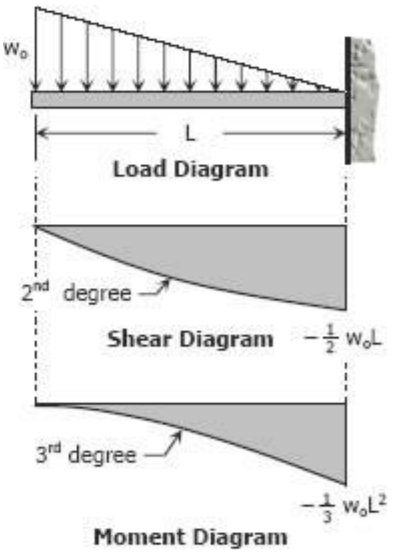
Moment equation:

$$M = -\tfrac{2}{3}xF_1 - \tfrac{1}{2}xF_2$$

$$= -\frac{1}{3}x\left(\frac{w_o}{2L}x^2\right) - \frac{1}{2}x\left[\frac{w_o}{L}(Lx - x^2)\right]$$

$$= -\frac{w_o}{3L}x^3 - \frac{w_o}{2}x^2 + \frac{w_o}{2L}x^3$$

$$= -\frac{w_o}{2}x^2 + \frac{w_o}{6L}x^3$$



To draw the Shear Diagram:

$V = \frac{w_o}{2L}x^2 - w_o x$ is a concave upward second degree curve; at x = 0, V = 0; at x = L, $V = -\frac{1}{2}w_oL$.

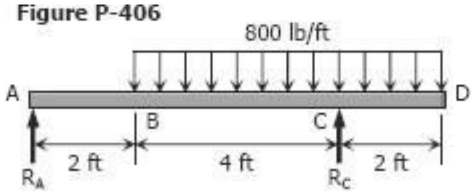
To draw the Moment diagram:

$M = -\frac{w_o}{2}x^2 + \frac{w_o}{6L}x^3$ is in third degree; at x = 0, M = 0; at x = L, $M = -\frac{1}{3}w_oL^2$.

Problem 412

Beam loaded as shown in Fig. P-412.

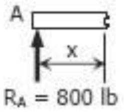
Figure P-406



Solution 412

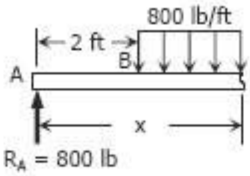
$$\Sigma M_A = 0$$
$$6R_C = 5[6(800)]$$
$$R_C = 4000 \text{ lb}$$

$$\Sigma M_C = 0$$
$$6R_A = 1[6(800)]$$
$$R_A = 800 \text{ lb}$$



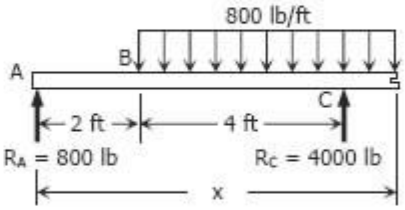
Segment AB:

$$V_{AB} = 800 \text{ lb}$$
$$M_{AB} = 800x$$



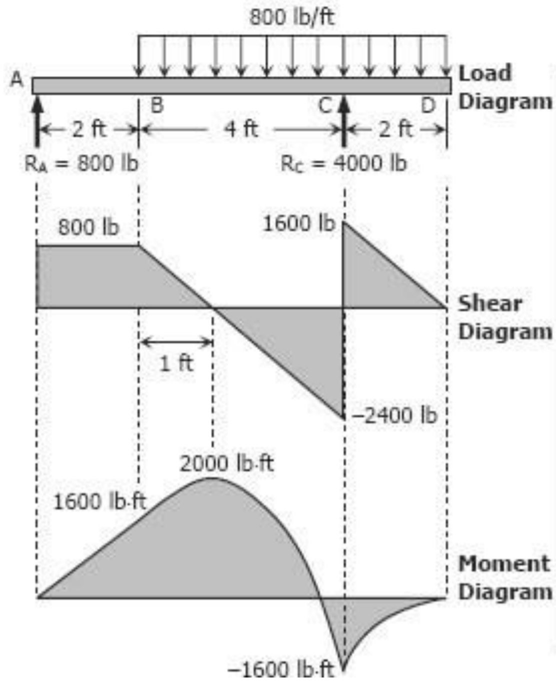
Segment BC:

$$V_{BC} = 800 - 800(x - 2)$$
$$= 2400 - 800x$$
$$M_{BC} = 800x - 800(x - 2)(x - 2)/2$$
$$= 800x - 400(x - 2)^2$$



Segment CD:

$$V_{CD} = 800 + 4000 - 800(x - 2)$$
$$= 4800 - 800x + 1600$$
$$= 6400 - 800x$$
$$M_{CD} = 800x + 4000(x - 6) - 800(x - 2)(x - 2)/2$$
$$= 800x + 4000(x - 6) - 400(x - 2)^2$$



To draw the Shear Diagram:

(1) 800 lb of shear force is uniformly distributed along segment AB.

(2) $V_{BC} = 2400 - 800x$ is linear; at x = 2 ft, V_{BC} = 800 lb; at x = 6 ft, V_{BC} = −2400 lb. When $V_{BC} = 0$, $2400 - 800x = 0$, thus x = 3 ft or V_{BC} = 0 at 1 ft from B.

(3) $V_{CD} = 6400 - 800x$ is also linear; at x = 6 ft, V_{CD} = 1600 lb; at x = 8 ft, V_{BC} = 0.

To draw the Moment Diagram:

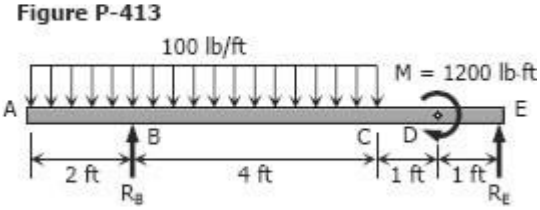
(1) $M_{AB} = 800x$ is linear; at x = 0, M_{AB} = 0; at x = 2 ft, M_{AB} = 1600 lb·ft.

(2) $M_{BC} = 800x - 400(x - 2)^2$ is second degree curve; at x = 2 ft, M_{BC} = 1600 lb·ft; at x = 6 ft, M_{BC} = −1600 lb·ft; at x = 3 ft, M_{BC} = 2000 lb·ft.

(3) $M_{CD} = 800x + 4000(x - 6) - 400(x - 2)^2$ is also a second degree curve; at x = 6 ft, M_{CD} = −1600 lb·ft; at x = 8 ft, M_{CD} = 0.

Problem 413

Beam loaded as shown in Fig. P-413.

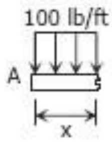


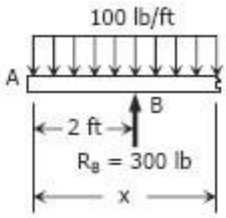
Solution 413

$\sum M_B = 0$
$6R_E = 1200 + 1[6(100)]$
$R_E = 300$ lb

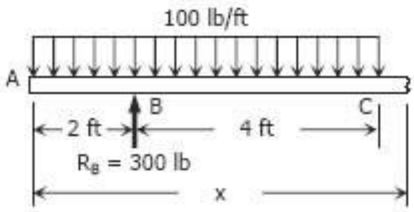
$\sum M_E = 0$
$6R_B + 1200 = 5[6(100)]$
$R_B = 300$ lb

Segment AB:
$V_{AB} = -100x$ lb
$M_{AB} = -100x(x/2)$
$= -50x^2$ lb·ft

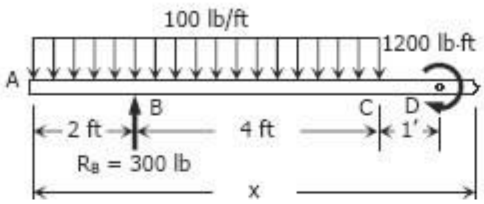




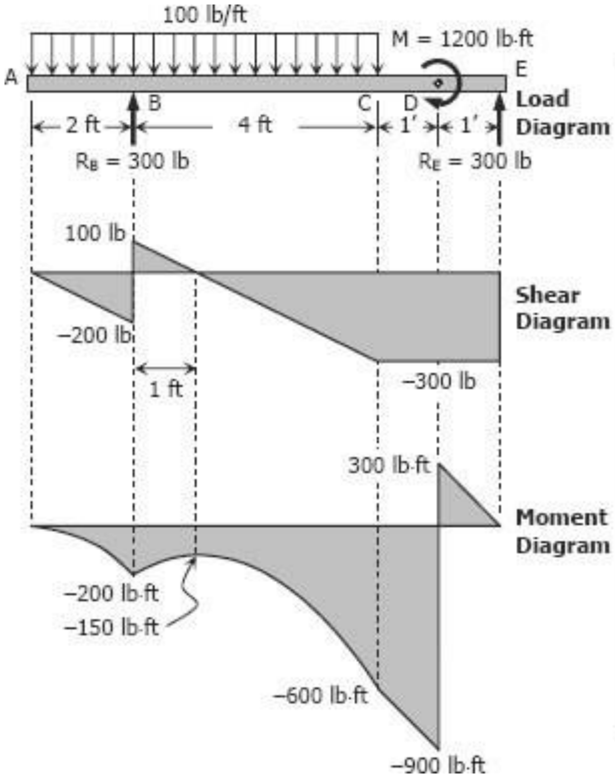
Segment BC:
$V_{BC} = -100x + 300$ lb
$M_{BC} = -100x(x/2) + 300(x - 2)$
$= -50x^2 + 300x - 600$ lb·ft



Segment CD:
$V_{CD} = -100(6) + 300$
$= -300$ lb
$M_{CD} = -100(6)(x - 3) + 300(x - 2)$
$= -600x + 1800 + 300x - 600$
$= -300x + 1200$ lb·ft



Segment DE:
$V_{DE} = -100(6) + 300$
$= -300$ lb
$M_{DE} = -100(6)(x - 3) + 1200 + 300(x - 2)$
$= -600x + 1800 + 1200 + 300x - 600$
$= -300x + 2400$



To draw the Shear Diagram:

(1) $V_{AB} = -100x$ is linear; at x = 0, V_{AB} = 0; at x = 2 ft, V_{AB} = −200 lb.

(2) $V_{BC} = 300 - 100x$ is also linear; at x = 2 ft, V_{BC} = 100 lb; at x = 4 ft, V_{BC} = −300 lb. When V_{BC} = 0, x = 3 ft, or V_{BC} =0 at 1 ft from B.

(3) The shear is uniformly distributed at −300 lb along segments CD and DE.

To draw the Moment Diagram:

(1) $M_{AB} = -50x^2$ is a second degree curve; at x= 0, M_{AB} = 0; at x = ft, M_{AB} = −200 lb·ft.

(2) $M_{BC} = -50x^2 + 300x - 600$ is also second degree; at x = 2 ft; M_{BC} = −200 lb·ft; at x = 6 ft, M_{BC} = −600 lb·ft; at x = 3 ft, M_{BC} = −150 l·ft.

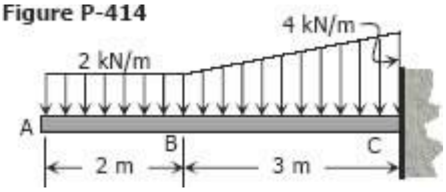
(3) $M_{CD} = -300x + 1200$ is linear; at x = 6 ft, M_{CD} = −600 lb·ft; at x = 7 ft, M_{CD} = −900 lb·ft.

(4) $M_{DE} = -300x + 2400$ is again linear; at x = 7 ft, M_{DE} = 300 lb·ft; at x = 8 ft, M_{DE} = 0.

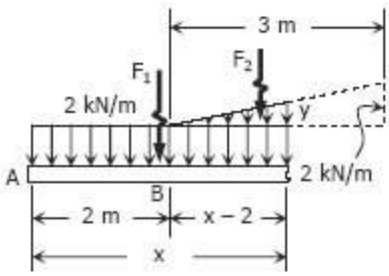
Problem 414

Cantilever beam carrying the load shown in Fig. P-414.

Figure P-414

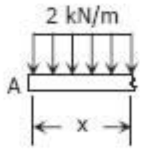


Solution 414



Segment *AB*:

$$V_{AB} = -2x \text{ kN}$$
$$M_{AB} = -2x(x/2)$$
$$= -x^2 \text{ kN·m}$$



Segment *BC*:

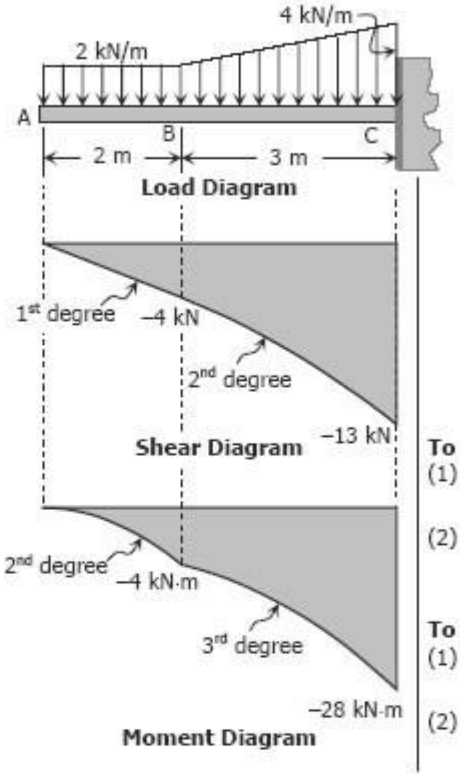
$$\frac{y}{x-2} = \frac{2}{3}$$
$$y = \tfrac{2}{3}(x-2)$$

$F_1 = 2x$

$$F_2 = \tfrac{1}{2}(x-2)y = \tfrac{1}{2}(x-2)[\tfrac{2}{3}(x-2)] = \tfrac{1}{3}(x-2)^2$$

$$V_{BC} = -F_1 - F_2 = -2x - \tfrac{1}{3}(x-2)^2$$

$$M_{BC} = -(x/2)F_1 - \tfrac{1}{3}(x-2)F_2 = -(x/2)(2x) - \tfrac{1}{3}(x-2)[\tfrac{1}{3}(x-2)^2] = -x^2 - \tfrac{1}{9}(x-2)^3$$



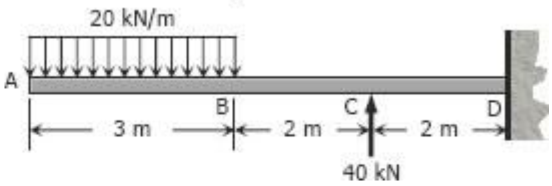
To draw the Shear Diagram:
(1) $V_{AB} = -2x$ is linear; at $x = 0$, $V_{AB} = 0$; at $x = 2$ m, $V_{AB} = -4$ kN.
(2) $V_{BC} = -2x - \frac{1}{3}(x-2)^2$ is a second degree curve; at $x = 2$ m, $V_{BC} = -4$ kN; at $x = 5$ m; $V_{BC} = -13$ kN.

To draw the Moment Diagram:
(1) $M_{AB} = -x^2$ is a second degree curve; at $x = 0$, $M_{AB} = 0$; at $x = 2$ m, $M_{AB} = -4$ kN·m.
(2) $M_{BC} = -x^2 - \frac{1}{9}(x-2)^3$ is a third degree curve; at $x = 2$ m, $M_{BC} = -4$ kN·m; at $x = 5$ m, $M_{BC} = -28$ kN·m.

Problem 415

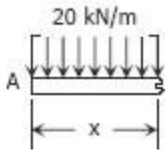
Cantilever beam loaded as shown in Fig. P-415.

Figure P-415



Solution 415

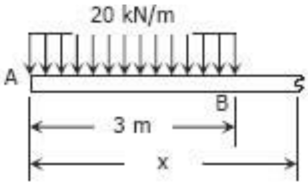
Segment AB:
$V_{AB} = -20x$ kN
$M_{AB} = -20x(x/2)$
$= -10x^2$ kN·m

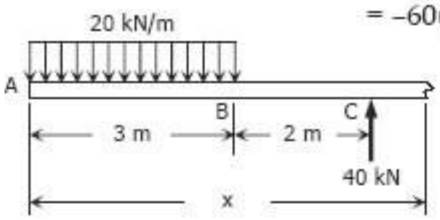


Segment BC:

$V_{BC} = -20(3)$
$= -60$ kN

$M_{BC} = -20(3)(x - 1.5)$
$= -60(x - 1.5)$ kN·m

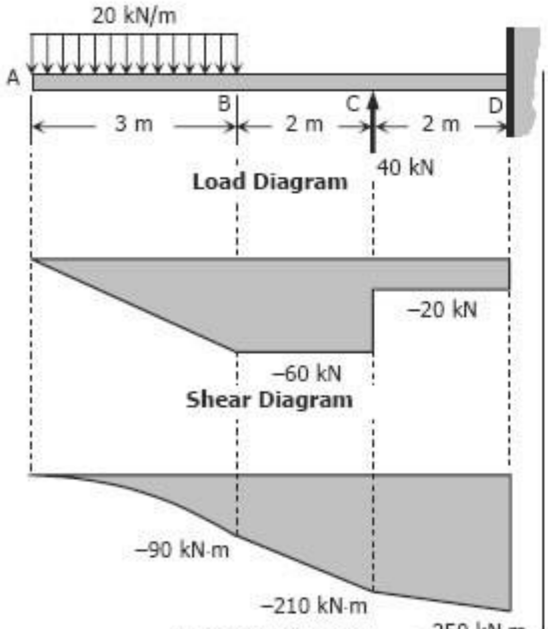




Segment CD:

$V_{CD} = -20(3) + 40$
$= -20$ kN

$M_{CD} = -20(3)(x - 1.5) + 40(x - 5)$
$= -60(x - 1.5) + 40(x - 5)$



To draw the Shear Diagram

(1) $V_{AB} = -20x$ for segment AB is linear; at x = 0, V = 0; at x = 3 m, V = −60 kN.
(2) V_{BC} = −60 kN is uniformly distributed along segment BC.
(3) Shear is uniform along segment CD at −20 kN.

To draw the Moment Diagram

(1) $M_{AB} = -10x^2$ for segment AB is second degree curve; at x = 0, M_{AB} = 0; at x = 3 m, M_{AB} = −90 kN·m.
(2) $M_{BC} = -60(x - 1.5)$ for segment BC is linear; at x = 3 m, M_{BC} = −90 kN·m; at x = 5 m, M_{BC} = −210 kN·m.
(3) $M_{CD} = -60(x - 1.5) + 40(x - 5)$ for segment CD is also linear; at x = 5 m, M_{CD} = −210 kN·m, at x = 7 m, M_{CD} = −250 kN·m.

Problem 416

Beam carrying uniformly varying load shown in Fig. P-416.

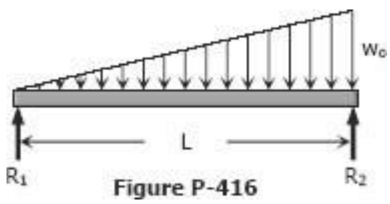
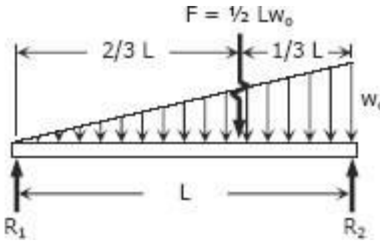


Figure P-416

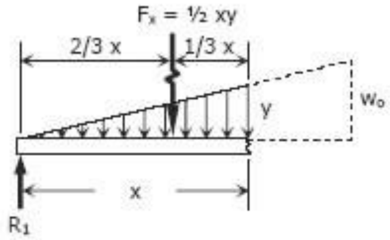
Solution 416

$$\sum M_{R2} = 0$$
$$LR_1 = \tfrac{1}{3} LF$$
$$R_1 = \tfrac{1}{3}(\tfrac{1}{2} Lw_o)$$
$$= \tfrac{1}{6} Lw_o$$



$$\sum M_{R1} = 0$$
$$LR_2 = \tfrac{2}{3} LF$$
$$R_2 = \tfrac{2}{3}(\tfrac{1}{2} Lw_o)$$
$$= \tfrac{1}{3} Lw_o$$

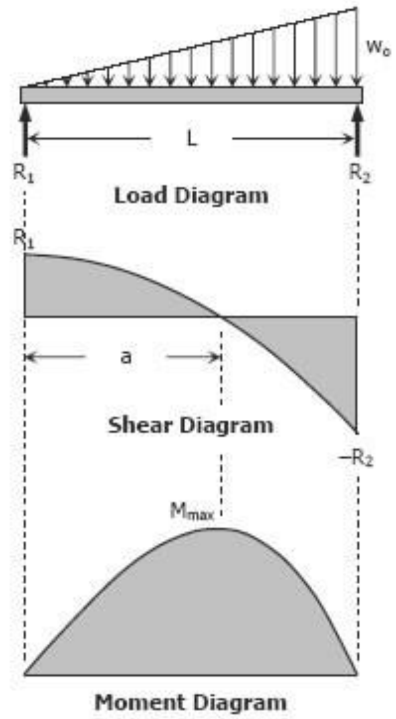
$$\frac{y}{x} = \frac{w_o}{L}$$
$$y = \frac{w_o}{L} x$$



$$F_x = \tfrac{1}{2} xy = \frac{1}{2} x\left(\frac{w_o}{L} x\right)$$
$$= \frac{w_o}{2L} x^2$$

$$V = R_1 - F_x$$
$$= \tfrac{1}{6} Lw_o - \frac{w_o}{2L} x^2$$

$$M = R_1 x - F_x(\tfrac{1}{3} x)$$
$$= \tfrac{1}{6} Lw_o x - \frac{w_o}{2L} x^2 \, (\tfrac{1}{3} x)$$
$$= \tfrac{1}{6} Lw_o x - \frac{w_o}{6L} x^3$$



To draw the Shear Diagram:

$V = 1/6\ Lw_o - w_o x^2/2L$ is a second degree curve; at $x = 0$, $V = 1/6\ Lw_o = R_1$; at $x = L$, $V = -1/3\ Lw_o = -R_2$; If a is the location of zero shear from left end, $0 = 1/6\ Lw_o - w_o x^2/2L$, $x = 0.5774L = a$; to check, use the squared property of parabola:

$a^2/R_1 = L^2/(R_1 + R_2)$
$a^2/(1/6\ Lw_o) = L^2/(1/6\ Lw_o + 1/3\ Lw_o)$
$a^2 = (1/6\ L^3 w_o)/(1/2\ Lw_o) = 1/3\ L^2$
$a = 0.5774L$ a =

To draw the Moment Diagram

$M = 1/6\ Lw_o x - w_o x^3/6L$ is a third degree curve; at $x = 0$, $M = 0$; at $x = L$, $M = 0$; at $x = a = 0.5774L$, $M = M_{max}$

$M_{max} = 1/6\ Lw_o(0.5774L) - w_o(0.5774L)^3/6L$
$M_{max} = 0.0962L^2 w_o - 0.0321L^2 w_o$
$M_{max} = 0.0641L^2 w_o$

Problem 417

Beam carrying the triangular loading shown in Fig. P- 417.

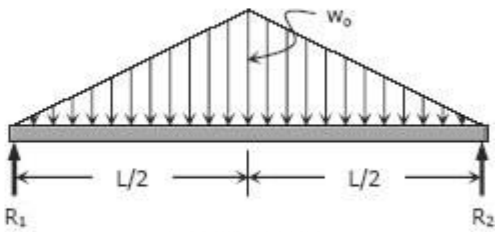
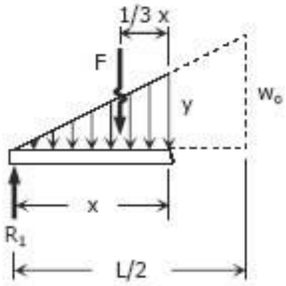


Figure P-417

Solution 417

By symmetry:

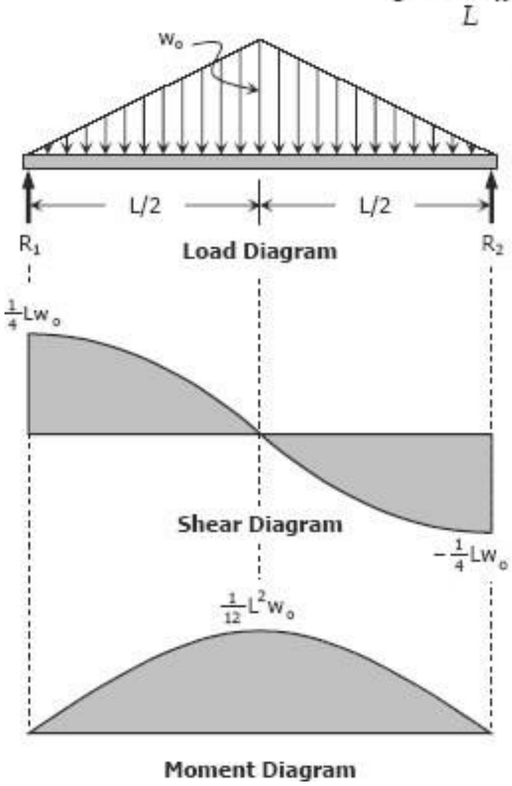
$$R_1 = R_2 = \tfrac{1}{2}(\tfrac{1}{2}Lw_o) = \tfrac{1}{4}Lw_o$$



$$\frac{y}{x} = \frac{w_o}{L/2}; \; y = \frac{2w_o}{L}x$$

$$F = \tfrac{1}{2}xy = \frac{1}{2}x\left(\frac{2w_o}{L}x\right)$$

$$F = \frac{w_o}{L}x^2$$



$$V = R_1 - F$$

$$V = \tfrac{1}{4}Lw_o - \frac{w_o}{L}x^2$$

$$M = R_1x - F(\tfrac{1}{3}x)$$

$$M = \tfrac{1}{4}Lw_ox - \left(\frac{w_o}{L}x^2\right)(\tfrac{1}{3}x)$$

$$M = \tfrac{1}{4}Lw_ox - \frac{w_o}{3L}x^3$$

To draw the Shear Diagram:

$V = Lw_o/4 - w_ox^2/L$ is a second degree curve; at $x = 0$, $V = Lw_o/4$; at $x = L/2$, $V = 0$. The other half of the diagram can be drawn by the concept of symmetry.

To draw the Moment Diagram

$M = Lw_ox/4 - w_ox^3/3L$ is a third degree curve; at $x = 0$, $M = 0$; at $x = L/2$, $M = L^2w_o/12$. The other half of the diagram can be drawn by the concept of symmetry.

Problem 418

Cantilever beam loaded as shown in Fig. P-418.

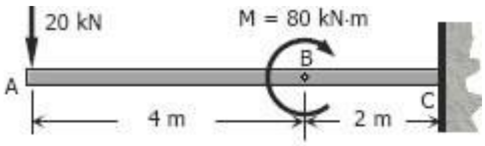
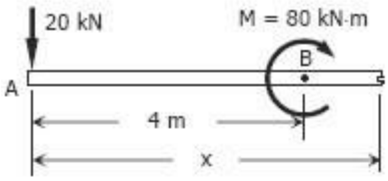


Figure P-418

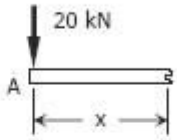
Solution 418



Segment AB:

$V_{AB} = -20$ kN

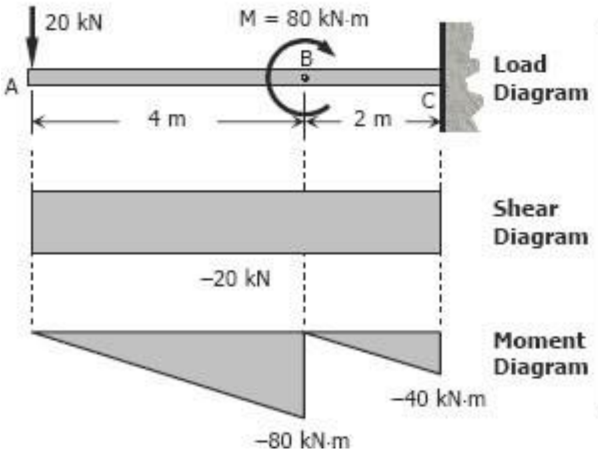
$M_{AB} = -20x$ kN·m



Segment BC:

$V_{AB} = -20$ kN

$M_{AB} = -20x + 80$ kN·m



To draw the Shear Diagram:

V_{AB} and V_{BC} are equal and constant at −20 kN.

To draw the Moment Diagram:

(1) $M_{AB} = -20x$ is linear; when x = 0, $M_{AB} = 0$; when x = 4 m, $M_{AB} = -80$ kN·m.

(2) $M_{BC} = -20x + 80$ is also linear; when x = 4 m, $M_{BC} = 0$; when x = 6 m, $M_{BC} = -60$ kN·m

Problem 419

Beam loaded as shown in Fig. P-419.

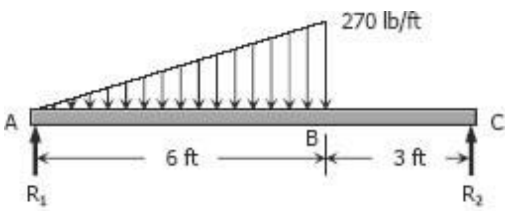
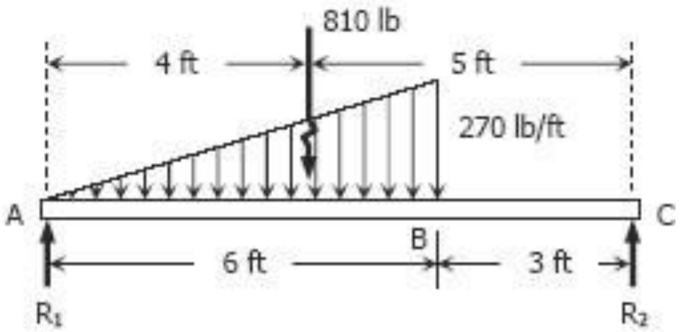


Figure P-419

Solution 419



$[\,\Sigma M_C = 0\,]$ $\quad 9R_1 = 5(810)$

$R_1 = 450$ lb

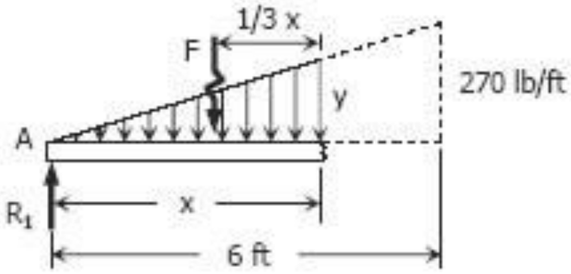
$[\,\Sigma M_A = 0\,]$ $\quad 9R_2 = 4(810)$

$R_2 = 360$ lb

Segment AB:

$$\frac{y}{x} = \frac{270}{6}$$

$$y = 45x$$

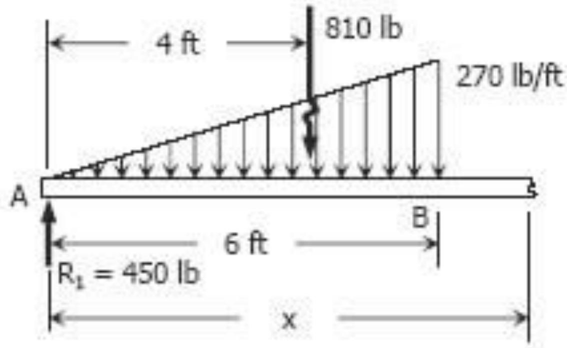


$$F = \tfrac{1}{2}xy = \tfrac{1}{2}x(45x)$$

$$F = 22.5x^2$$

$$V_{AB} = R_1 - F$$
$$= 450 - 22.5x^2 \text{ lb}$$

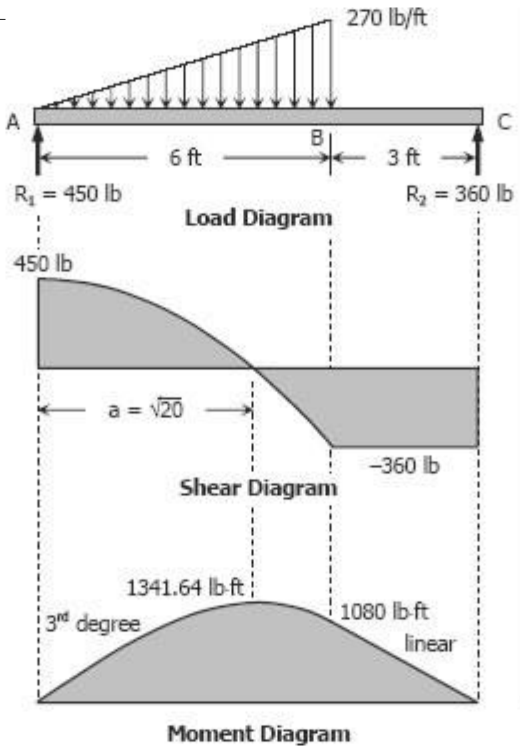
$$M_{AB} = R_1 x - F(\tfrac{1}{3}x)$$
$$= 450x - 22.5x^2(\tfrac{1}{3}x)$$
$$= 450x - 7.5x^3 \text{ lb·ft}$$



Segment BC:

$$V_{BC} = 450 - 810$$
$$= -360 \text{ lb}$$

$$M_{BC} = 450x - 810(x - 4)$$
$$= 450x - 810x + 3240$$
$$= 3240 - 360x \text{ lb·ft}$$



To draw the Shear Diagram:

(1) $V_{AB} = 450 - 22.5x^2$ is a second degree curve; at x = 0, V_{AB} = 450 lb; at x = 6 ft, V_{AB} = −360 lb.

(2) At x = a, V_{AB} = 0,

$$450 - 22.5x^2 = 0$$
$$22.5x^2 = 450$$
$$x^2 = 20$$
$$x = \sqrt{20}$$

To check, use the squared property of parabola.

$$a^2/450 = 6^2/(450 + 360)$$
$$a^2 = 20$$
$$a = \sqrt{20}$$

(3) V_{BC} = −360 lb is constant.

To draw the Moment Diagram:

(1) $M_{AB} = 450x - 7.5x^3$ for segment AB is third degree curve; at x = 0, M_{AB} = 0; at x = $\sqrt{20}$, M_{AB} = 1341.64 lb·ft; at x = 6 ft, M_{AB} = 1080 lb·ft.

(2) $M_{BC} = 3240 - 360x$ for segment BC is linear; at x = 6 ft, M_{BC} = 1080 lb·ft; at x = 9 ft, M_{BC} = 0.

Problem 420

A total distributed load of 30 kips supported by a uniformly distributed reaction as shown in Fig. P-420.

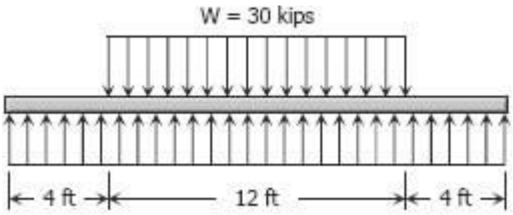
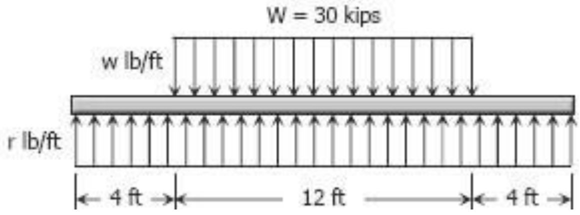


Figure P-420

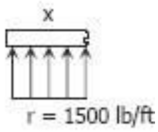
Solution 420



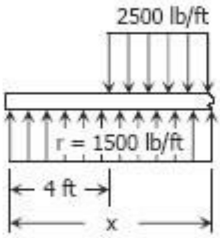
$w = 30(1000)/12$
$w = 2500 \text{ lb/ft}$

$\Sigma F_V = 0$
$R = W$
$20r = 30(1000)$
$r = 1500 \text{ lb/ft}$

First segment (from 0 to 4 ft from left):

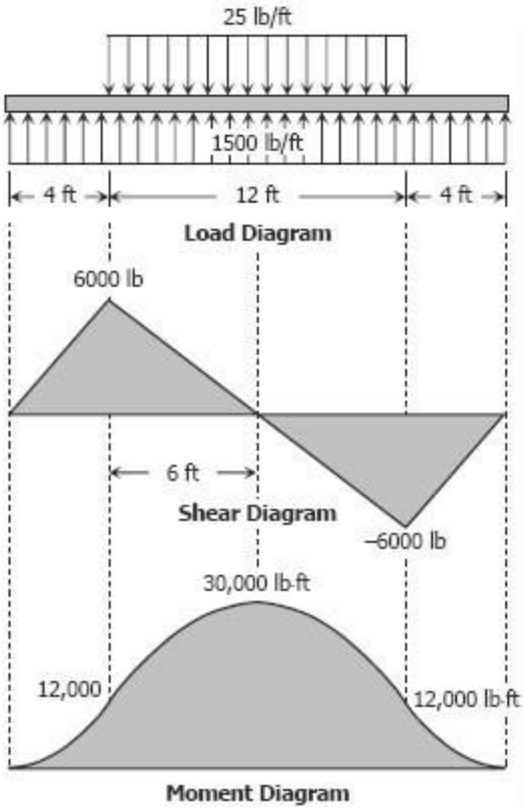


$V_1 = 1500x$
$M_1 = 1500x(x/2)$
$= 750x^2$



Second segment (from 4 ft to mid-span):

$V_2 = 1500x - 2500(x - 4)$
$= 10000 - 1000x$
$M_2 = 1500x(x/2) - 2500(x - 4)(x - 4)/2$
$= 750x^2 - 1250(x - 4)^2$



To draw the Shear Diagram:

(1) For the first segment, $V_1 = 1500x$ is linear; at x = 0, $V_1 = 0$; at x = 4 ft, $V_1 = 6000$ lb.

(2) For the second segment, $V_2 = 10000 - 1000x$ is also linear; at x = 4 ft, $V_1 = 6000$ lb; at mid-span, x = 10 ft, $V_1 = 0$.

(3) For the next half of the beam, the shear diagram can be accomplished by the concept of symmetry.

To draw the Moment Diagram:

(1) For the first segment, $M_1 = 750x^2$ is a second degree curve, an open upward parabola; at x = 0, $M_1 = 0$; at x = 4 ft, $M_1 = 12000$ lb-ft.

(2) For the second segment, $M_2 = 750x^2 - 1250(x - 4)^2$ is a second degree curve, an downward parabola; at x = 4 ft, $M_2 = 12000$ lb-ft; at mid-span, x = 10 ft, $M_2 = 30000$ lb-ft.

(2) The next half of the diagram, from x = 10 ft to x = 20 ft, can be drawn by using the concept of symmetry.

Problem 421

Write the shear and moment equations as functions of the angle θ for the built-in arch shown in Fig. P-421.

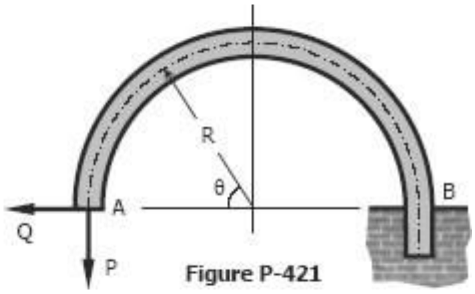
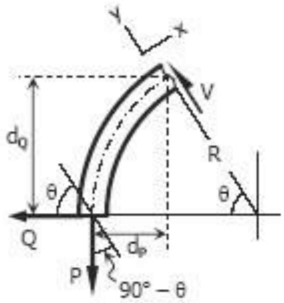


Figure P-421

Solution 421



For θ that is less than 90°

Components of Q and P:

$Q_x = Q \sin\theta$

$Q_y = Q \cos\theta$

$P_x = P \sin(90° - \theta)$

$= P(\sin 90° \cos\theta - \cancel{\cos 90° \sin\theta})$

$= P \cos\theta$

$P_y = P \cos(90° - \theta)$

$= P(\cancel{\cos 90° \cos\theta} + \sin 90° \sin\theta)$

$= P \sin\theta$

Shear:

$V = \Sigma F_y$

$V = Q_y - P_y$

$\mathbf{V = Q \cos\theta - P \sin\theta}$

Moment arms:

$d_Q = R \sin\theta$

$d_P = R - R\cos\theta$

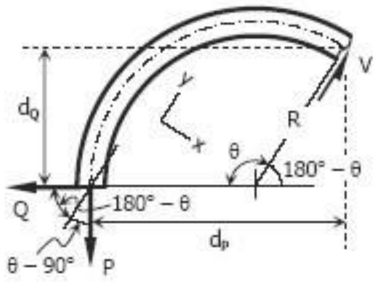
$= R(1 - \cos\theta)$

Moment:

$M = \Sigma M_{counterclockwise} - \Sigma M_{clockwise}$

$M = Q(d_Q) - P(d_P)$

$\mathbf{M = QR \sin\theta - PR(1 - \cos\theta)}$



For θ that is greater than 90°

Components of Q and P:

$Q_x = Q \sin(180° - \theta)$
$= Q(\sin 180° \cos\theta - \cos 180° \sin\theta)$
$= Q\cos\theta$

$Q_y = Q\cos(180° - \theta)$
$= Q(\cos 180° \cos\theta + \sin 180° \sin\theta)$
$= -Q\sin\theta$

$P_x = P\sin(\theta - 90°)$
$= P(\sin\theta\cos 90° - \cos\theta \sin 90°)$
$= -P\cos\theta$

$P_y = P\cos(\theta - 90°)$
$= P(\cos\theta\cos 90° + \sin\theta\sin 90°)$
$= P\sin\theta$

Shear:

$V = \Sigma F_y$
$V = -Q_y - P_y$
$V = -(-Q\sin\theta) - P\sin\theta$
$\mathbf{V = Q\sin\theta - P\sin\theta}$

Moment arms:

$d_Q = R\sin(180° - \theta)$
$= R(\sin 180° \cos\theta - \cos 180° \sin\theta)$
$= R\sin\theta$

$d_P = R + R\cos(180° - \theta)$
$= R + R(\cos 180° \cos\theta + \sin 180° \sin\theta)$
$= R - R\cos\theta$
$= R(1 - \cos\theta)$

Moment:

$M = \Sigma M_{counterclockwise} - \Sigma M_{clockwise}$
$M = Q(d_Q) - P(d_P)$
$\mathbf{M = QR\sin\theta - PR(1 - \cos\theta)}$

Problem 422

Write the shear and moment equations for the semicircular arch as shown in Fig. P-422 if (a) the load P is vertical as shown, and (b) the load is applied horizontally to the left at the top of the arch.

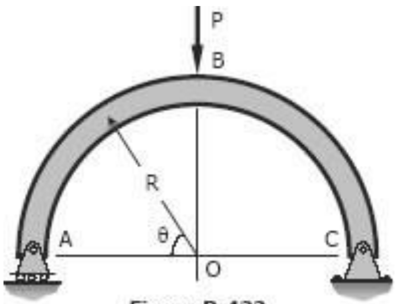
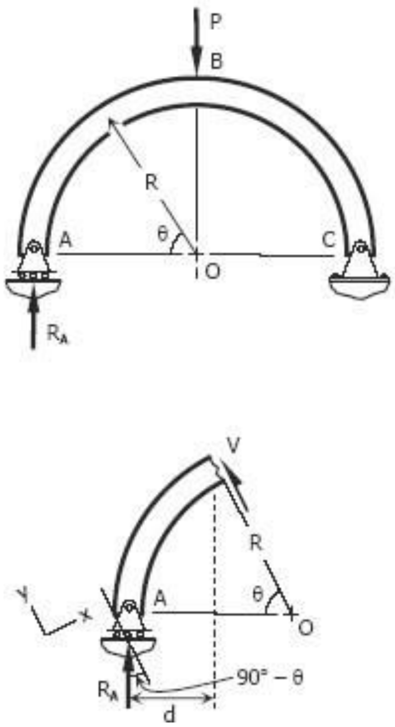


Figure P-422

Solution 422



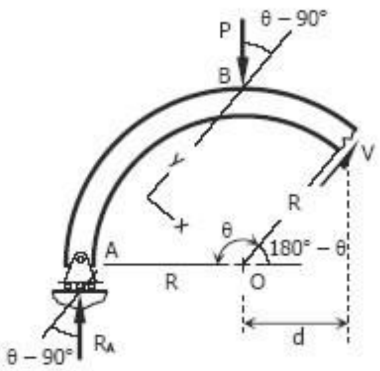
$\Sigma M_C = 0$
$2R(R_A) = RP$
$R_A = \frac{1}{2}P$

For θ that is less than 90°

Shear:
$V_{AB} = R_A \cos(90° - \theta)$
$V_{AB} = \frac{1}{2}P(\cos 90° \cos\theta + \sin 90° \sin\theta)$
$V_{AB} = \frac{1}{2}P\sin\theta$

Moment arm:
$d = R - R\cos\theta$
$d = R(1 - \cos\theta)$

Moment:
$M_{AB} = R_a(d)$
$M_{AB} = \frac{1}{2}PR(1 - \cos\theta)$



For θ that is greater than 90°

Components of P and R_A:

$P_x = P \sin(\theta - 90°)$
$\quad = P(\sin\theta \cos 90° - \cos\theta \sin 90°)$
$\quad = -P \cos\theta$

$P_y = P \cos(\theta - 90°)$
$\quad = P(\cos\theta \cos 90° + \sin\theta \sin 90°)$
$\quad = P \sin\theta$

$R_{Ax} = R_A \sin(\theta - 90°)$
$\quad = \frac{1}{2}P(\sin\theta \cos 90° - \cos\theta \sin 90°)$
$\quad = -\frac{1}{2}P \cos\theta$

$R_{Ay} = R_A \cos(\theta - 90°)$
$\quad = \frac{1}{2}P(\cos\theta \cos 90° + \sin\theta \sin 90°)$
$\quad = \frac{1}{2}P \sin\theta$

Shear:

$V_{BC} = \Sigma F_y$
$V_{BC} = R_{Ay} - P_y$
$V_{BC} = \frac{1}{2}P \sin\theta - P \sin\theta$
$V_{BC} = -\frac{1}{2}\boldsymbol{P} \sin\theta$

Moment arm:

$d = R \cos(180° - \theta)$
$d = R(\cos 180° \cos\theta + \sin 180° \sin\theta)$
$d = -R \cos\theta$

Moment:

$M_{BC} = \Sigma M_{counterclockwise} - \Sigma M_{clockwise}$
$M_{BC} = R_A(R + d) - Pd$
$M_{BC} = \frac{1}{2}P(R - R\cos\theta) - P(-R\cos\theta)$
$M_{BC} = \frac{1}{2}PR - \frac{1}{2}PR\cos\theta + PR\cos\theta$
$M_{BC} = \frac{1}{2}PR + \frac{1}{2}PR\cos\theta$
$M_{BC} = \frac{1}{2}\boldsymbol{PR}(1 + \cos\theta)$