



وزارة التعليم العالي والبحث العلمي

الجامعة التقنية الجنوبية

المعهد التقني العمارة

قسم تقنيات الالكترونيات والاتصالات



الحقيبة التدريسية لمادة دوائر التيار الكهربائي المستمر

الصف الاول

Electric Circuits I

تدريسي المادة

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الفصل الدراسي الاول

## جدول مفردات مادة دوائر التيار الكهربائي المستمر

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الهدف من دراسة مادة دوائر التيار الكهربائي المستمر (الهدف العام):

تهدف دراسة مادة دوائر التيار الكهربائي المستمر للصف **الاول** الى:

- ١) أن الطالب قادرا على فهم وتطبيق القوانين الخاص بالدوائر الكهربائية العامة .
- ٢) ان يكون الطالب قادرا على تطبيق النظريات الخاصة بالدوائر الكهربائية .

**الفئة المستهدفة:**

طلبة الصف الاول / قسم التقنيات الالكترونية والاتصالات

التقنيات التربوية المستخدمة:

١. سبورة واقلام
٢. السبورة التفاعلية
٣. عارض البيانات Data Show
٤. جهاز حاسوب محمول Laptop

## الاسبوع الأول

### الهدف التعليمي (الهدف الخاص لكل للمحاضرة):

- ان يتعرف الطالب على وحدات القياس الرئيسية المستخدمة في الدوائر الكهربائية
- ان يتعرف الطالب على مكونات الدوائر الكهربائية
- ان يتعرف الطالب ويكون قادرا على تطبيق قانون اوم
- ان يتعرف الطالب العوامل المؤثرة على قيمة المقاومة
- ان يتعرف الطالب تأثير درجة الحرارة على قيم المقاومة

### الأنشطة المستخدمة:

١. أنشطة تفاعلية صفية
٢. أسئلة عصف ذهني
٣. أنشطة جماعية (إذا تطلب الامر)
٤. واجب بيتي
٥. واجب الكتروني (ويفضل انشاء صفوف الكترونية Classrooms لدمج التعليم الحضوري بالتعليم الالكتروني حسب التوجهات الحديثة للتعليم والتعلم)

### أساليب التقويم:

١. التغذية الراجعة الفورية من قبل التدريسي (التقويم البنائي).
٢. اشراك الطلبة بالتقويم الذاتي (تصحيح أخطائهم بأنفسهم).
٣. التغذية الراجعة النهائية (التقويم الختامي)، ويقصد به حل الأسئلة المعطاة كنشاط صفي في نهاية المحاضرة.

## Electric units system

### Units associated with basic electrical quantities

#### SI units :

*SI unit is an international system of measurements that are used universally in technical and scientific research to avoid the confusion with the units.*

The basic units in the SI system are listed with their symbols, in Table below

| Quantity           | Basic unit | symbol |
|--------------------|------------|--------|
| Length             | meter      | m      |
| Mass               | kilogram   | kg     |
| Time               | second     | s      |
| Electric current   | ampere     | A      |
| Temperature        | kelvin     | K      |
| Luminous intensity | candela    | cd     |

SI units may be made larger or smaller by using prefixes which denote multiplication or division by a particular amount. The six most common multiples, with their meaning, are listed in Table below.

| Prefix |        | Base 10    | Decimal        |
|--------|--------|------------|----------------|
| Name   | Symbol |            |                |
| kilo   | k      | $10^3$     | 1000           |
| mega   | M      | $10^6$     | 1000000        |
| giga   | G      | $10^9$     | 1000000000     |
| tera   | T      | $10^{12}$  | 1000000000000  |
| milli  | m      | $10^{-3}$  | 0.001          |
| micro  | $\mu$  | $10^{-6}$  | 0.000001       |
| nano   | n      | $10^{-9}$  | 0.000000001    |
| pico   | p      | $10^{-12}$ | 0.000000000001 |

## Definition of basic units of voltage, current and resistance

Electric current: is the time rate of change of charge, measured in amperes (A).

Mathematically, the relationship between current  $i$ , charge  $q$ , and time  $t$

$$i = \frac{dq}{dt}$$

$$1 \text{ ampere} = 1 \text{ coulomb/second}$$

Voltage (or potential difference): is the energy required to move a unit charge through an element, measured in volts (V).

$$1 \text{ volt} = 1 \text{ joule/coulomb} = 1 \text{ newton meter/coulomb}$$





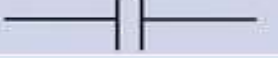



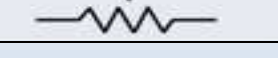

**Resistance** : It may be defined as the property of a substance due to which it opposes (or restricts) the flow of electricity (i.e., electrons) through it.

The unit of electric resistance is the ohm ( $\Omega$ ).

The reciprocal of resistance is called conductance and is measured in siemens (S). Thus conductance, in siemens

$$G = \frac{1}{R}$$

## Electric circuit components

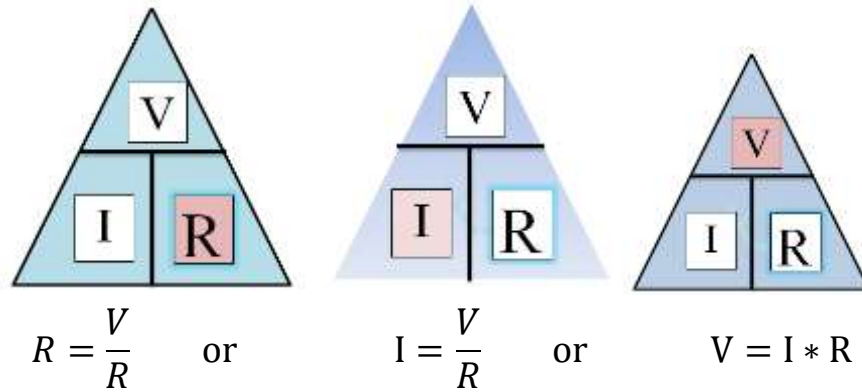
|                   |   |                 |   |
|-------------------|---|-----------------|---|
|                   |   |                 |   |
| Resistance        |  | Battery         |  |
| Inductor          |  | Voltemeter      |  |
| Capacitor         |  | Ammeter         |  |
| Open Switch       |  | Close switch    |  |
| Variable resistor |  | Connecting Wire |  |
|                   |   |                 |   |
|                   |   |                 |   |

## Ohm's law

Ohm's law: The ratio of potential difference (V) between any two points on a conductor to the current (I) flowing between them, is constant, provided the temperature of the conductor does not change.. Thus

$$R = \frac{V}{I}$$

Ohm's law triangle



Power: is the time rate of expending or absorbing energy, measured in watts (W)

Power P in an electrical circuit is given by the product of potential difference V and current I

$$P = V \times I \quad \text{watts}$$

From Ohm's law

$$P = \frac{V^2}{R} \quad \text{watts} \quad , \quad P = I^2 \times R \quad \text{watts}$$

الاسبوع الثاني المحاضرة الثانية

الهدف التعليمي (الهدف الخاص لكل للمحاضرة):

مدة المحاضرة: ٢ ساعة نظري + ٢ ساعة عملي



## Factors effecting on resistance

The resistance of an electrical conductor depends on 4 factors, these

- (a) the length of the conductor (Resistance, R, is directly proportional to length)  $R \propto L$
  - (b) the cross-sectional area of the conductor (Resistance, R, is inversely proportional to cross-sectional area, a, of a conductor)  $R \propto \frac{1}{A}$
  - (c) the type of material
  - (d) the temperature of the material
- resistance

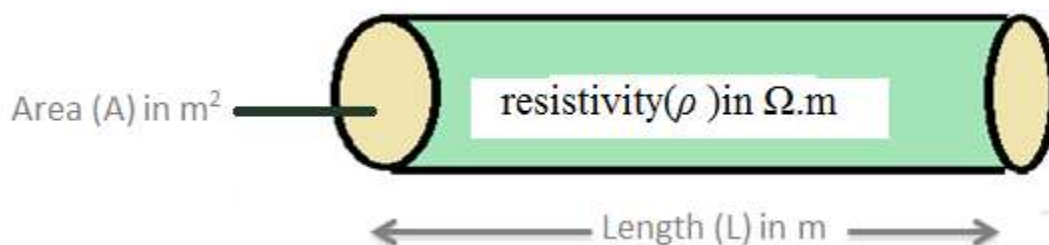
$$R = \frac{\rho L}{A}$$

$\rho$  : resistivity in  $\Omega.m$

L: length in m

A: cross section area in  $m^2$

The value of the resistivity is that resistance of a unit cube of the material measured between opposite faces of the cube



**Example :** Calculate the resistance of a 2 km length of aluminium overhead power cable if the cross-sectional area of the cable is  $100 \text{ mm}^2$ . Take the resistivity of aluminium to be  $0.03 \times 10^{-6} \Omega.m$ .

Solution :

Length = 2 km = 2000 m;

cross-sectional area =  $100 \text{ mm}^2 = 100 \times 10^{-6} \text{ m}^2$

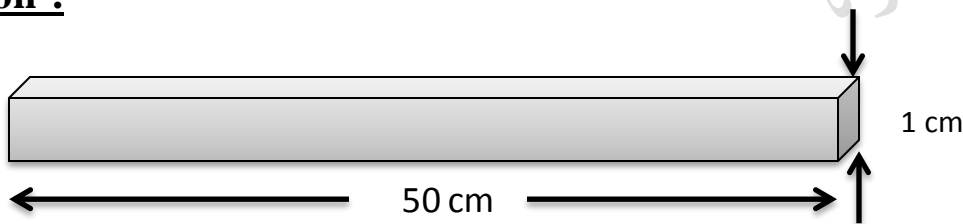
resistivity =  $0.03 \times 10^{-6} \Omega \text{ m}$

$$\text{Resistance } R = \frac{\rho l}{a} = \frac{(0.03 \times 10^{-6} \Omega \text{ m})(2000 \text{ m})}{(100 \times 10^{-6} \text{ m}^2)} = \frac{0.03 \times 2000}{100} \Omega$$
$$= 0.6 \Omega$$

**Example :** A rectangular carbon block has dimensions  $1.0 \text{ cm} \times 1.0 \text{ cm} \times 50 \text{ cm}$ .

(i) What is the resistance measured between the two square ends ? (ii) between two opposing rectangular faces / Resistivity of carbon at  $20^\circ\text{C}$  is  $3.5 \times 10^{-5} \Omega\text{-m}$ .

**Solution :**



$$R = \frac{\rho L}{A}$$

(i)  $A = 1 \times 1 = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$  ;  $l = 0.5 \text{ m}$

$$R = \frac{3.5 \times 10^{-5} \times 0.5}{10^{-4}} = 0.175 \Omega$$

(ii)  $L = 1 \text{ cm}$ ;  $A = 1 \times 50 = 50 \text{ cm}^2 = 5 \times 10^{-3} \text{ m}^2$

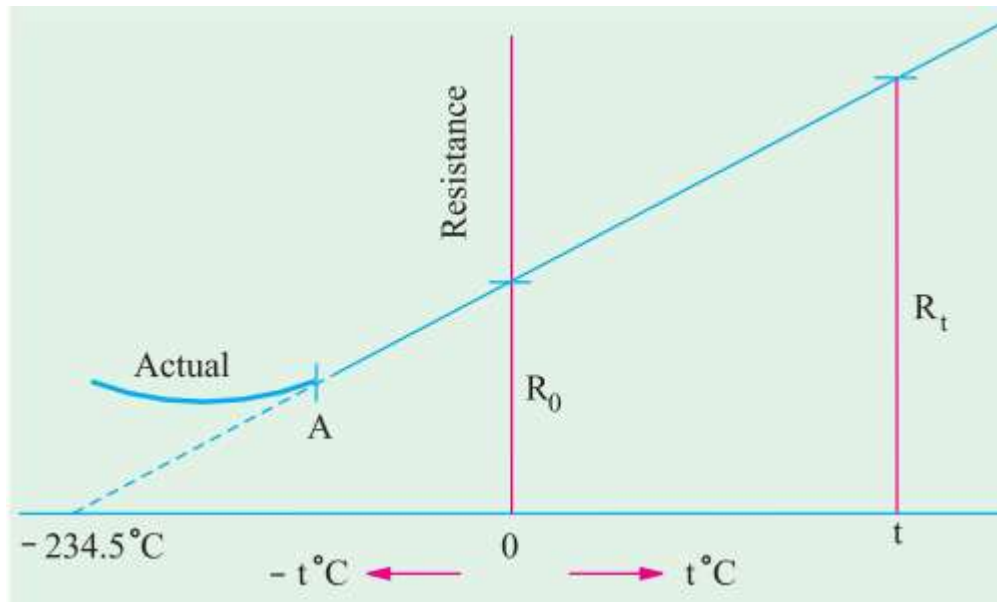
$$R = \frac{3.5 \times 10^{-5} \times 10^{-2}}{5 \times 10^{-3}} = 7 \times 10^{-5} \Omega$$

H.W :A coil consists of 2000 turns of copper wire having a cross-sectional area of  $0.8 \text{ mm}^2$ . The mean length per turn is 80cm and the resistivity of copper is  $0.02 \mu\Omega\text{-m}$ . Find the resistance of the coil.

## Temperature Coefficient of Resistance

The resistance of most good conducting material increases almost linearly with temperature over range of normal operating temperature.

$$R_t = R_0 (1 + \alpha t)$$



$$R_2 = R_1 (1 + \alpha_0 (T_2 - T_1))$$

$$R_T = R_0 (1 + \alpha_0 T)$$

$$\alpha_T = \frac{\alpha_0}{1 + \alpha_0 * T}$$

$R_1$ : resistance at lower temperature  $T_1$

$R_2$ : resistance at higher temperature  $T_2$

$R_T$ : resistance at temperature  $T$

$R_0$ : resistance at zero temperature  $0^\circ\text{C}$

$\alpha_0$ : temperature coefficient at  $0^\circ\text{C}$

$\alpha_T$ : temperature coefficient at  $T^\circ\text{C}$  ( $\Omega/^\circ\text{C}$ )

**Example:** A platinum coil has a resistance is  $3.146 \Omega$  at  $40^\circ\text{C}$  and  $3.717 \Omega$  at  $100^\circ\text{C}$ .

Find

- 1) Temperature coefficient at  $0^\circ\text{C}$ .
- 2) Coil resistance at  $0^\circ\text{C}$ .
- 3) Temperature coefficient at  $40^\circ\text{C}$ .

Solution:

1)

$$R_2 = R_1(1 + \alpha_0(T_2 - T_1))$$

$$3.717 = 3.146(1 + \alpha_0(100 - 40))$$

$$3.717 = 3.146(1 + \alpha_0 * 60)$$

$$3.717 = ((3.146 * 1) + (3.146 \alpha_0 * 60))$$

$$3.717 = 3.146 + 188.76 \alpha_0$$

$$3.717 - 3.146 = 188.76 \alpha_0$$

$$0.571 = 188.76 \alpha_0$$

$$\alpha_0 = \frac{0.571}{188.76} = 0.003 \Omega/^\circ\text{C}$$

(2)

$$R_T = R_0(1 + \alpha_0 T)$$

$$3.146 = R_0(1 + 0.003 * 40)$$

$$3.146 = 1.12 R_0$$

$$R_0 = \frac{3.146}{1.12} = 2.825 \Omega$$

$$\alpha_T = \frac{\alpha_0}{1 + \alpha_0 * T}$$

$$\alpha_{40} = \frac{0.003}{1 + 0.003 * 40} = 0.0026 \Omega/^\circ\text{C}$$

## H.W

**Example:** A copper coil has a resistance of  $(100\Omega)$  at  $(20C^0)$  and the temperature coefficient of this Copper coil  $(0.0043)$  per  $K^0$  at  $(20C^0)$ . Determine the resistance of this coil at  $(100C^0)$ .

الاسبوع الثاني المحاضرة الثانية

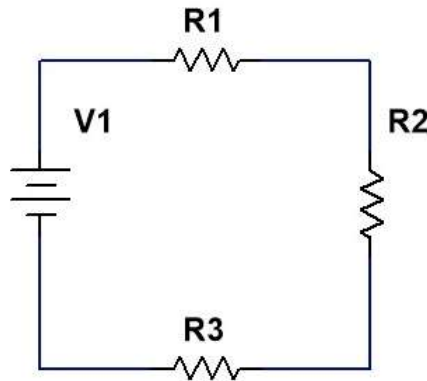
**الهدف التعليمي (الهدف الخاص لكل للمحاضرة):**

- ان يتعرف الطالب ربط المقاومات على التوالي وخصائص ربط التوالي والعلاقات الرياضية
- ان يتعرف الطالب ربط المقاومات على التوازي وخصائص ربط التوالي والعلاقات الرياضية
- ان يتعرف الطالب الربط النجمي والمثلثي والتمييز بينهما.
- ان يتعرف الطالب على قوانين التحويل من الربط النجمي الى الربط المثلثي وبالعكس ويكون قادرا على استخدام القوانين وتطبيقها للتحويل بينهما .

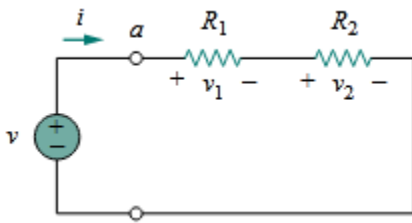
**مدة المحاضرة: ٢ ساعة نظري + ٢ ساعة عملي**

# Series and parallel Circuit

## ◆ Resistance in Series Connection



In fig below The three resistors are in series



In a series circuit

(a) the current  $I$  is the same in all parts of the circuit (  $I_1 = I_2 = I$  )

(b) the sum of the voltages  $V_1$ ,  $V_2$  is equal to the total applied voltage,  $V$ , i.e

$$V = V_1 + V_2 + \dots$$

Applying Ohm's law to each of the resistors, we obtain

$$V_1 = IR_1, V_2 = IR_2,$$

$$\text{then } IR = IR_1 + IR_2$$

Dividing throughout by  $I$  give

$$R = R_1 + R_2 + R_3$$

For  $N$  resistors in series, the equivalent resistor has a value given by

$$R_{eq} = R_1 + R_2 + \dots + R_N = \sum_{n=1}^N R_n$$

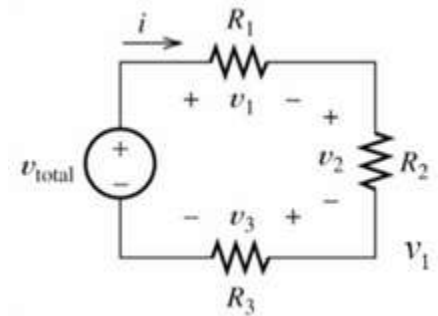
To determine the voltage across each resistor

Voltage divider rule

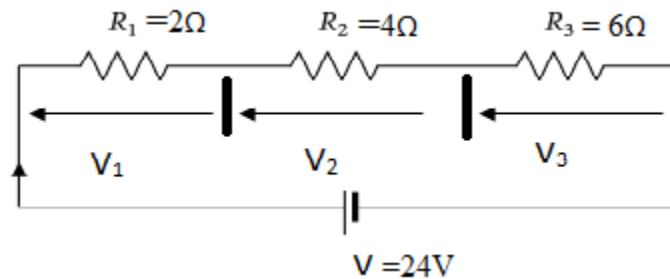
$$v_1 = \frac{R_1}{R_1 + R_2 + R_3} v_{total}, \quad v_2 = \frac{R_2}{R_1 + R_2 + R_3} v_{total}$$

$$v_3 = \frac{R_3}{R_1 + R_2 + R_3} v_{total}$$

$$v_X = \frac{R_X}{R_{eg}} v_{total}$$



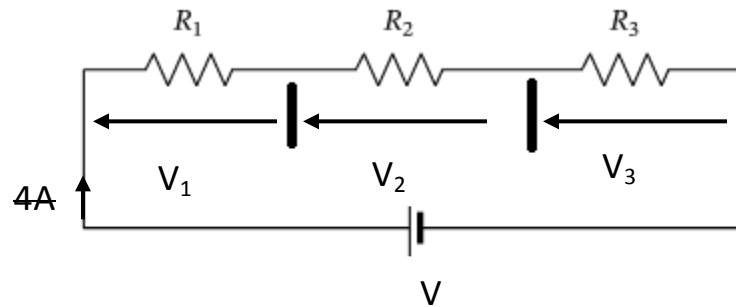
Example: in fig below find  $V_1$ ,  $V_2$  and  $V_3$



$$v_1 = \frac{R_1}{R_1 + R_2 + R_3} v_{total}, \quad v_2 = \frac{R_2}{R_1 + R_2 + R_3} V_t, \quad v_3 = \frac{R_3}{R_1 + R_2 + R_3} V_t$$

$$V_1 = \frac{2 \times 24}{2 + 4 + 6} = 4V, \quad V_2 = \frac{4 \times 24}{2 + 4 + 6} = 8V, \quad V_3 = \frac{6 \times 24}{2 + 4 + 6} = 12V$$

Example2. For the circuit shown in Figure below, determine (a) the battery voltage  $V$ , (b) the total resistance of the circuit, and (c) the values of resistance of resistors  $R_1$ ,  $R_2$  and  $R_3$ , given that the p.d.'s across  $R_1$ ,  $R_2$  and  $R_3$  are 5 V, 2 V and 6 V respectively



**SOL**

$$V_1 = 5V, V_2 = 2V, V_3 = 6V$$

(a) Battery voltage  $V = V_1 + V_2 + V_3$   
 $= 5 + 2 + 6 = 13 \text{ V}$

(b) Total circuit resistance  $R = \frac{V}{I} = \frac{13}{4} = 3.25 \Omega$

(c) Resistance  $R_1 = \frac{V_1}{I} = \frac{5}{4} = 1.25 \Omega$

$$\text{Resistance } R_2 = \frac{V_2}{I} = \frac{2}{4} = 0.5 \Omega$$

$$\text{Resistance } R_3 = \frac{V_3}{I} = \frac{6}{4} = 1.5 \Omega$$

(Check:  $R_1 + R_2 + R_3 = 1.25 + 0.5 + 1.5 = 3.25 \Omega = R$ )

## Resistance in Parallel

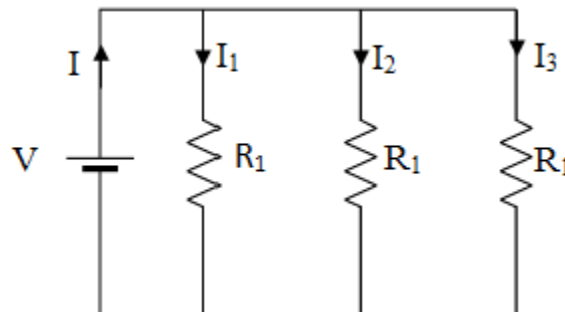




Figure shows three resistors,  $R_1$ ,  $R_2$  and  $R_3$  connected across each other, i.e., in parallel, across a battery source of  $V$  volt

a) the sum of the currents  $I_1$ ,  $I_2$  and  $I_3$  is equal to the total circuit current,  $I$ , i.e.

$$I = I_1 + I_2 + I_3, \text{ and}$$

b) the source p.d.,  $V$  volts, is the same across each of the resistors  $V_1 = V_2 = V_3 = V$

From Ohm's law:

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \text{ and } I = \frac{V}{R}$$

where  $R$  is the total circuit resistance.

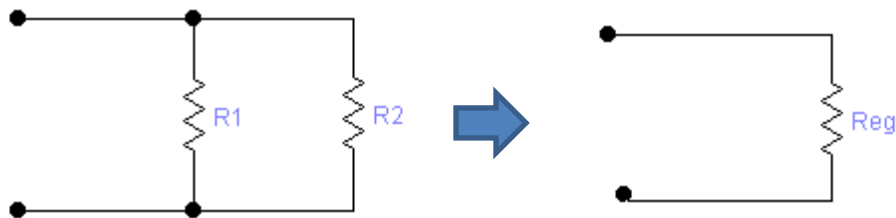
$$\text{Since } I = I_1 + I_2 + I_3$$

$$\text{then, } \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Dividing by  $V$  gives

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

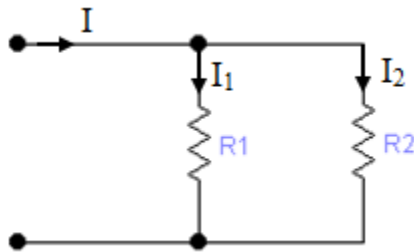
For the special case of two resistors in parallel



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

Hence 
$$R = \frac{R_1 R_2}{R_1 + R_2} \quad \left( \text{i.e. } \frac{\text{product}}{\text{sum}} \right)$$

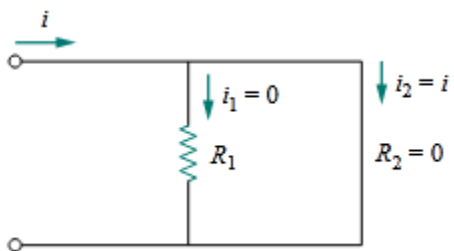
## current divider rule



$$I_1 = \frac{R_2 I}{R_1 + R_2} , \quad I_2 = \frac{R_1 I}{R_1 + R_2}$$

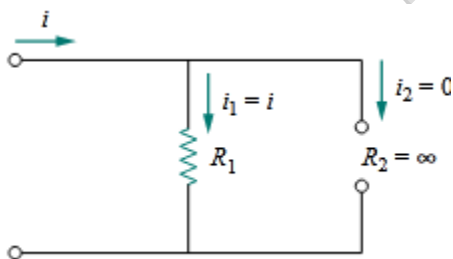
### Cases

A ) Suppose one of the resistors is zero as shown in fig



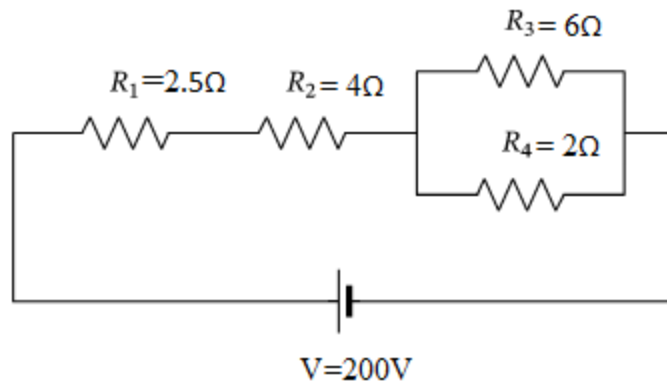
say  $R_2 = 0$ ; that is,  $R_2$  is a short circuit, that the entire current  $i$  bypasses  $R_1$  and flows through the short circuit  $R_2 = 0$ , the path of least resistance

B) Suppose  $R_2 = \infty$ , that is,  $R_2$  is an open circuit, as shown in Fig. below.



The current still flows through the path of least resistance,  $R_1$

Example3: For the series-parallel arrangement shown in Figure below, find (a) the supply current, (b) the current flowing through each resistor and (c) the p.d. across each resistor .



### Solution:

- (a) The equivalent resistance  $R_x$  of  $R_3$  and  $R_4$  in parallel is:

$$R_x = \frac{6 \times 2}{6 + 2} = \frac{12}{8} = 1.5 \Omega$$

The equivalent resistance  $R_T$  of  $R_1$ ,  $R_x$  and  $R_2$  in series is:

$$R_T = 2.5 + 1.5 + 4 = 8 \Omega$$

$$\text{Supply current } I = \frac{V}{R_T} = \frac{200}{8} = 25 \text{ A}$$

- (b) The current flowing through  $R_1$  and  $R_4$  is 25 A

$$\begin{aligned} \text{The current flowing through } R_2 &= \left( \frac{R_3}{R_2 + R_3} \right) I = \left( \frac{2}{6 + 2} \right) 25 \\ &= 6.25 \text{ A} \end{aligned}$$

$$\begin{aligned} \text{The current flowing through } R_3 &= \left( \frac{R_2}{R_2 + R_3} \right) I = \left( \frac{6}{6 + 2} \right) 25 \\ &= 18.75 \text{ A} \end{aligned}$$

(c)

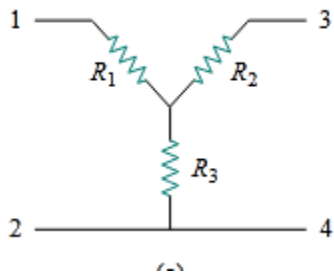
$$\text{p.d. across } R_1, \text{ i.e., } V_1 = IR_1 = (25)(2.5) = 62.5 \text{ V}$$

$$\text{p.d. across } R_x, \text{ i.e., } V_x = IR_x = (25)(1.5) = 37.5 \text{ V}$$

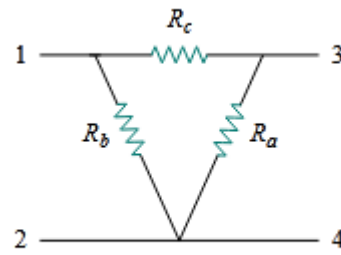
$$\text{p.d. across } R_4, \text{ i.e., } V_4 = IR_4 = (25)(4) = 100 \text{ V}$$

$$\text{Hence the p.d. across } R_2 = \text{p.d. across } R_3 = 37.5 \text{ V}$$

# WYE-DELTA TRANSFORMATIONS



(a) Y

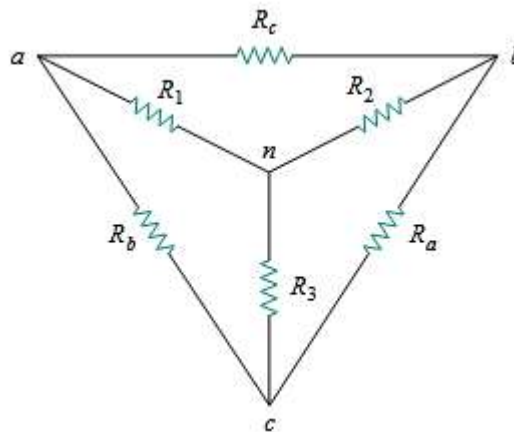


(a)

(b)  $\Delta$

## Delta to Wye Conversion

To transform a  $\Delta$  network to Y



the conversion rule for  $\Delta$  to Y is as follows

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

## Wye to Delta Conversion

the conversion rule for Y to  $\Delta$  is as follows:

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

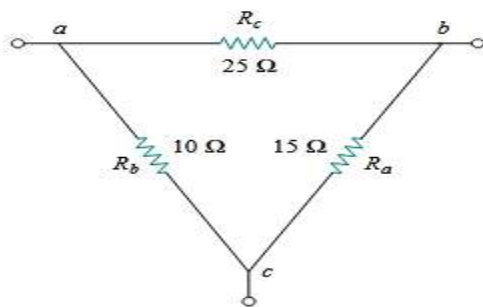
The Y and  $\Delta$  networks are said to be balanced when

$$R_1 = R_2 = R_3 = R_Y, \quad R_a = R_b = R_c = R_\Delta$$

Under these conditions, conversion formulas become

$$R_Y = \frac{R_\Delta}{3} \quad \text{or} \quad R_\Delta = 3R_Y$$

Example: Convert the network in Fig. below to an equivalent Y network



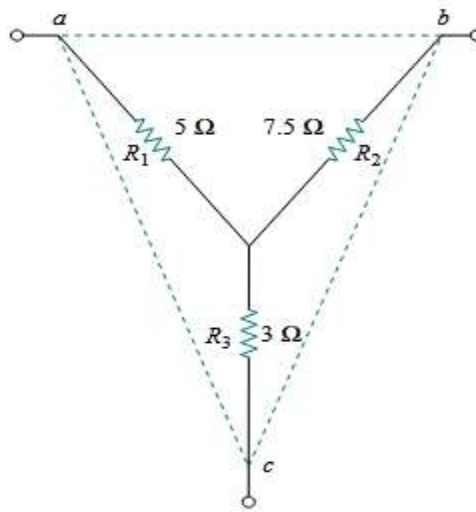
Solution:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{25 \times 10}{25 + 10 + 15} = \frac{250}{50} = 5 \Omega$$

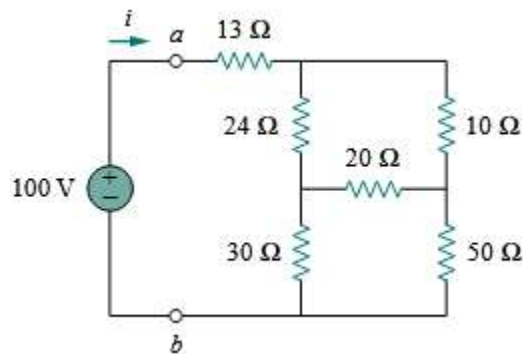
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5 \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3 \Omega$$

The equivalent Y network is shown in Fig below



H.W :For the bridge network in Fig. below, find R ab and i



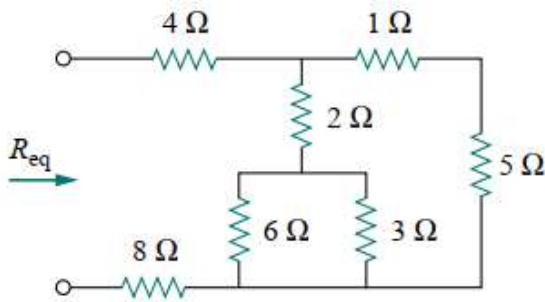
## الهدف التعليمي (الهدف الخاص لكل للمحاضرة):

- ان يكون الطالب على التمييز بين انواع ربط المقاومات (التوالي والتوازي والمركب)
- ان يكون الطالب قادرا على حل المسائل المتعلقة بربط المقاومات وايجاد المقاومة المكافئة والتيارات الكلية و الفرعية وكذلك الفولتيات الكلية او هبوطات الجهد عبر كل مقاومة .

مدة المحاضرة: ٢ ساعة نظري + ٢ ساعة عملي

مدة المحاضرة: ٢ ساعة نظري + ٢ ساعة عملي

Example : Find  $R_{eq}$  for the circuit shown in Fig below.

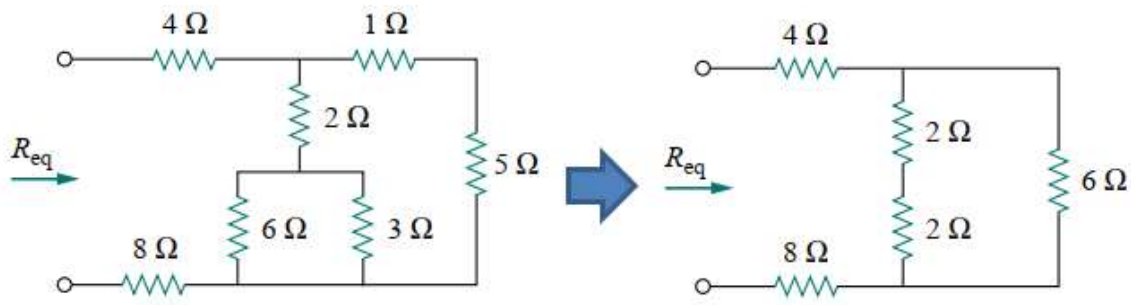


### Solution

$$3\Omega // 6\Omega$$

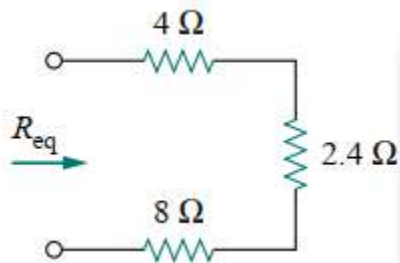
$$\frac{3 \times 6}{3 + 6} = 2\Omega$$

$$1\Omega \text{ and } 5\Omega \text{ in series } (1+5=6\Omega)$$



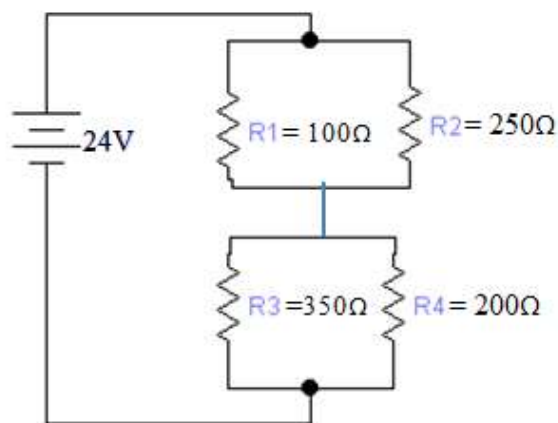
In fig (a) ( $2\Omega$  and  $2\Omega$ ) in series ( $2\Omega+2\Omega=4\Omega$ )

$$4\Omega//6\Omega = 4 \times 6 / 4 + 6 = 2.4\Omega$$



$$R_{eq} = 4 + 2.4 + 8 = 14.4\Omega$$

Example : For the series-parallel arrangement shown in Figure below, find the supply current .



Solution:

$$R1//R2, R3//R4$$

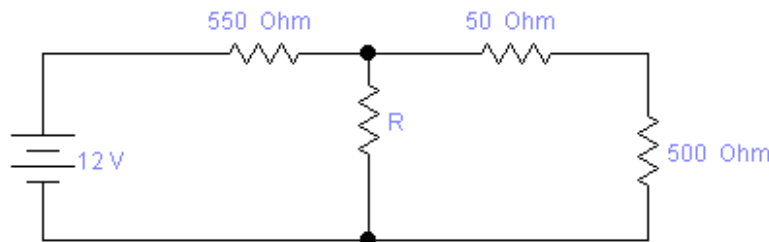


$$R_{eg} = 100\Omega // 250\Omega + 350\Omega // 200\Omega$$

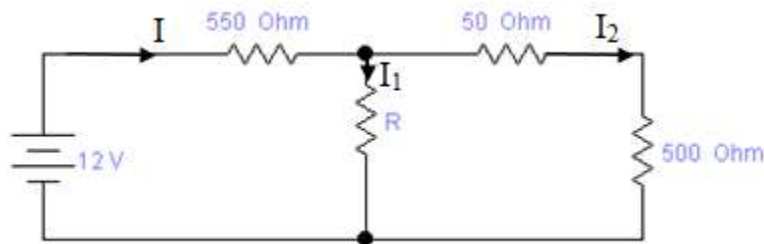
$$R_{eg} = \frac{100 \times 250}{100 + 250} + \frac{350 \times 200}{350 + 200} = 198.701\Omega$$

$$I = \frac{V}{R_{eg}} = \frac{24}{198.701} = 0.121A$$

Example: What is the value of the unknown resistor R in Fig .below if the voltage drop across the 500  $\Omega$  resistor is 2.5 volts ?



Solution:



$$I_2 = \frac{V}{R} = \frac{2.5}{500} = 0.005A$$

$$V_{50\Omega} = 50 \times 0.005 = 0.25V$$

$$\text{Drop voltage across } R = 2.5 + 0.25 = 2.75V$$

$$\text{Drop voltage across } 550\Omega = 12 - 2.75 = 9.25V$$

$$I = 9.25/550 = 0.0168A$$

$$R = \frac{V}{I} = \frac{9.25}{0.0168} = 233\Omega$$

الاسبوع الرابع المحاضرة الرابعة

الهدف التعليمي (الهدف الخاص لكل للمحاضرة):

- ان يتعرف الطالب على قوانين كيرشوف للفولتية والتيار وفهمها.
- ان الطاب قادرا على تطبيق قوانين كيرشوف وحل المسائل الرياضية.

مدة المحاضرة: ٢ ساعة نظري + ٢ ساعة عملي

## KIRCHHOFF'S LAWS

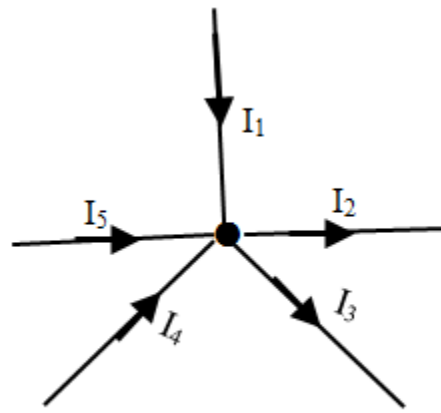
### Kirchhoff's first law

Kirchhoff's Current Law (KCL) : The algebraic sum of the currents present at a junction (node) in a circuit equal zero.

(current flowing towards the junction = current flowing away from the junction)

current flowing towards the junction are positive (+)

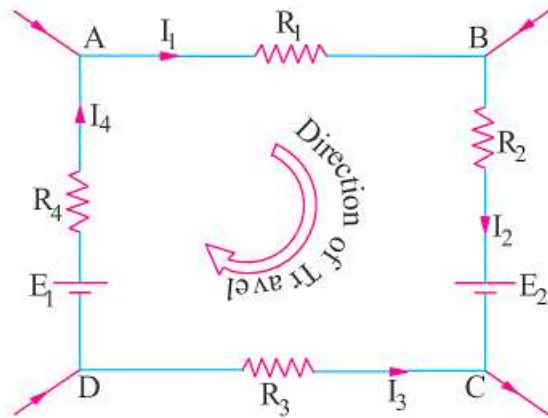
current flowing away from the junction are negative (-)



$$I_1 - I_2 - I_3 + I_4 + I_5 = 0$$

## second law

Kirchhoff's voltage Law (KVL) : The voltage around a loop equals the sum of every voltage drop in the same loop for any closed network and equals zero .



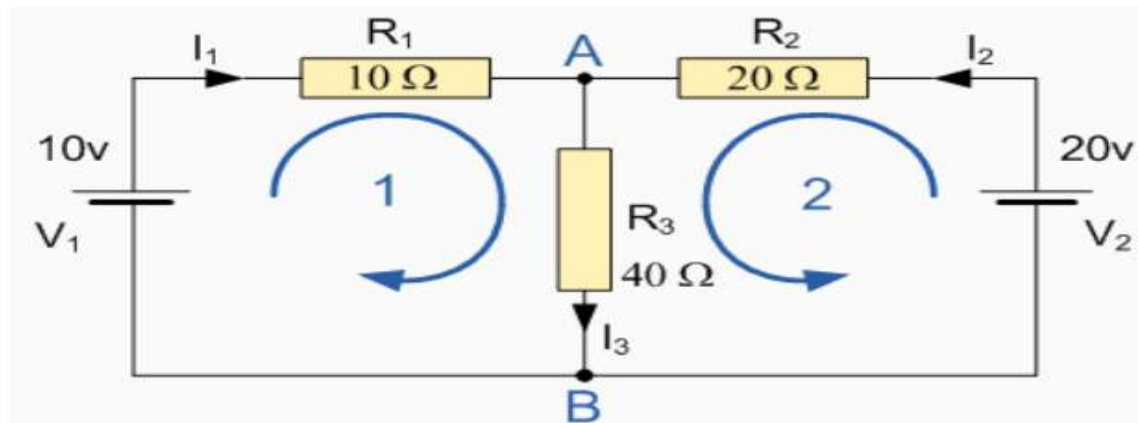
Using Kirchhoff's voltage law, we get

$$-I_1 R_1 - I_2 R_2 - I_3 R_3 - I_4 R_4 - E_2 + E_1 = 0$$

or

$$I_1 R_1 + I_2 R_2 - I_3 R_3 + I_4 R_4 = E_1 - E_2$$

**Example 2:** Use Kirchhoff's laws to determine the currents flowing in each branch of the network shown in Figure below.



### Solution

Using Kirchhoff's first law, at B and A we get,  $I_1 + I_2 = I_3$

$$I_3 = I_1 + I_2$$

Loop1

KVL

$$V_1 = R_1 I_1 + R_3 I_3$$

$$10 = 10I_1 + 40I_3$$

$$10 = 10I_1 + 40(I_1 + I_2)$$

$$10 = 10I_1 + 40I_1 + 40I_2$$

$$10 = 50I_1 + 40I_2$$

$$1 = 5I_1 + 4I_2 \text{ -----1}$$

Loop2

KVL

$$V_2 = R_2 I_2 + R_3 I_3$$

$$20 = 20I_2 + 40I_3$$

$$20 = 20I_2 + 40(I_1 + I_2)$$

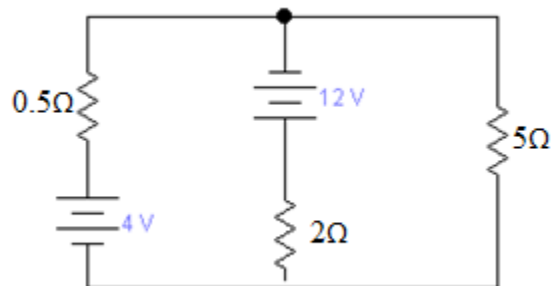
$$20 = 20I_2 + 40I_1 + 40I_2$$

$$20 = 40I_1 + 60I_2$$

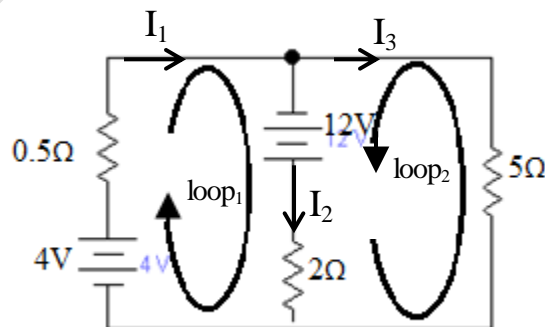
$$1 = 2I_1 + 3I_2 \text{ -----2}$$

$$\begin{aligned}
 1 &= 5I_1 + 4I_2 \quad \text{-----1} & \leftarrow & *3 \\
 1 &= 2I_1 + 3I_3 \quad \text{-----2} & \leftarrow & *4 \\
 3 &= 15I_1 + 12I_2 \quad \text{-----3} \\
 4 &= 8I_1 + 12I_2 \quad \text{-----3} & \text{نطرح معادلة ٤ من} \\
 & & \text{معادلة ٣ فيكون الناتج} \\
 -1 &= 7I_1 + 0 \\
 I_1 &= -0.143\text{A} \\
 \text{نعوض قيمة التيار } I_1 & \text{ في المعادلة رقم (١)} \\
 1 &= 5(-0.143) + 4I_2 \\
 1 &= -0.715 + 4I_2 \\
 1 + 0.715 &= 4I_2 \\
 1.715 &= 4I_2 \\
 I_2 &= 0.429\text{A} \\
 I_3 &= I_1 + I_2 \\
 I_3 &= (-0.143) + 0.429 \\
 I_3 &= 0.286\text{A}
 \end{aligned}$$

**Example:** Determine, using Kirchhoff's laws, each branch current for the network shown in Fig below\_



Solution



KCL

$$I_1 - I_2 - I_3 = 0$$

$$I_3 = I_1 - I_2$$

## KVL

Loop(1)

$$E_1 + E_2 = I_1 R_1 + I_2 R_2$$

$$4 + 12 = 0.5I_1 + 2I_2$$

$$16 = 0.5I_1 + 2I_2 \quad \text{-----(1)}$$

Loop(2)

$$E_2 = I_2 R_2 - I_3 R_3$$

$$12 = 2I_2 - 5(I_1 - I_2)$$

$$12 = -5I_1 + 7I_2 \quad \text{-----(2)}$$

$$16 = 0.5I_1 + 2I_2 \quad *7$$

$$12 = -5I_1 + 7I_2 \quad *2$$

$$112 = 3.5I_1 + 14I_2 \quad \text{-----3}$$

$$24 = -10I_1 + 14I_2 \quad \text{---4}$$

$$88 = 13.5I_1 \quad I_1 = 88/13.5 = 6.518A$$

$$\text{From equation (1)} \quad 16 = 0.5(6.518) + 2I_2 \quad I_2 = \underline{6.37A}$$

$$I_3 = I_1 - I_2$$

$$I_3 = 6.518 - 6.37 = 0.148A$$

الاسبوع الخامس المحاضرة الخامسة

**الهدف التعليمي (الهدف الخاص لكل للمحاضرة):**

- ان يفهم الطالب ويكون قادر على تطبيق طريقة ماكسويل في الدوائر الكهربائية .
- ان يكون الطالب قادرا على تطبيق طريقة حلقات ماكسويل لايجاد التيارات والفولتيات في فروع الدائرة الكهربائية المعقدة.

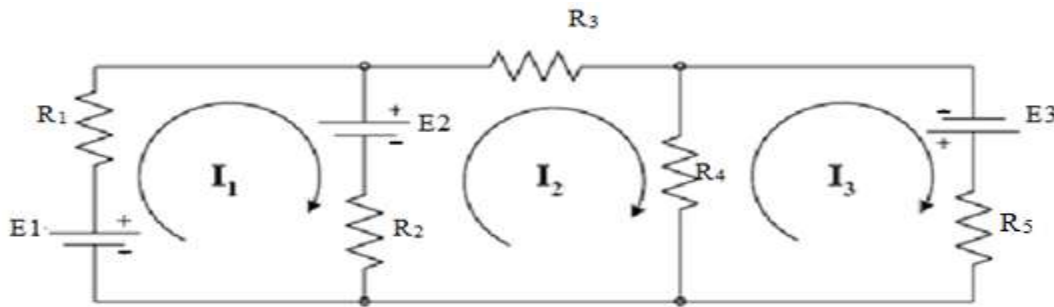
## Maxwell 's Loop Current Methode (Mesh analysis)

Loop : is a closed path (closed connection of branches) in a circuit that starts and ends at the same node without passing through any node more than once.

Mesh : It is a closed loop that does not contain any other loop.

This method can be applied by the following step:

- 1- Suppose that the direction of the currents in each closed loop circulates clockwise.



- 2- Write the loop equations of the circuit as in the following formulas (form each loop one equation).

$$\sum E = \sum I \times R$$

Loop1

$$E_1 - E_2 = I_1 R_1 + R_2 (I_1 - I_2)$$

Loop2

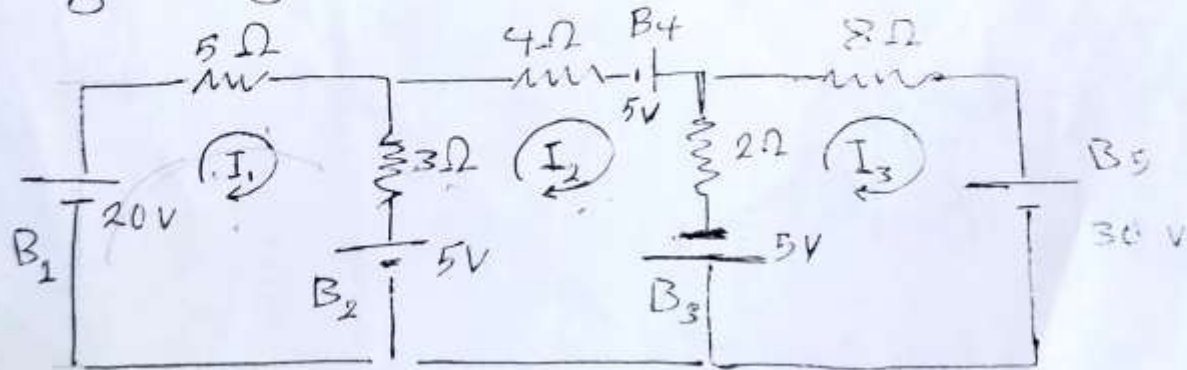
$$E_2 = R_2 (I_2 - I_1) + R_3 I_2 + R_4 (I_2 - I_3)$$

Loop3

$$E_3 = R_4 (I_3 - I_2) + R_5 I_3$$

- 3- Solve the above equation to find out the current value ( $I_1, I_2, I_3, \dots, I_n$ )

Example Determine the current supplied by each battery in the circuit shown in Fig (using maxwell's)



(3)

التقنيات الإلكترونية والاتصالات



For Loop ①

$$20 - 5 = 5I_1 + 3(I_1 - I_2)$$

$$15 = 5I_1 + 3I_1 - 3I_2$$

$$15 = 8I_1 - 3I_2 \quad \text{--- (1)}$$

For loop ②

$$5 + 5 + 5 = 3(I_2 - I_1) + 4I_2 + 2(I_2 - I_3)$$

$$15 = 3I_2 - 3I_1 + 4I_2 + 2I_2 - 2I_3$$

$$15 = -3I_1 + 9I_2 - 2I_3 \quad \text{--- (2)}$$

For Loop ③

$$-5 - 30 = 8I_3 + 2(I_3 - I_2)$$

$$-35 = 8I_3 + 2I_3 - 2I_2$$

$$-35 = -2I_2 + 10I_3 \quad \text{--- (3)}$$

$$15 = 8I_1 - 3I_2 + 0$$

$$15 = -3I_1 + 9I_2 - 2I_3$$

$$-35 = 0 - 2I_2 + 10I_3$$

$$\begin{pmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 15 \\ 15 \\ -35 \end{pmatrix}$$

$$D = \begin{vmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix}$$

$$D = 720 + 0 + 0 - (0 + 32 + 90)$$

$$D = 720 - 122 = 598$$

$$I_1 = \begin{vmatrix} 15 & -3 & 0 & 15 \\ 15 & 9 & -2 & 15 \\ -35 & -2 & 10 & -35 \end{vmatrix}$$

$$D_1 = 1350 - 210 + 0 - (0 + 60 - 450) \\ = 1530$$

$$I_1 = \frac{D_1}{D} = \frac{1530}{598} = 2.558 \text{ A}$$

$$D_2 = \begin{vmatrix} 8 & 15 & 0 & 8 \\ 3 & 15 & -2 & 3 \\ 0 & -35 & 10 & 0 \end{vmatrix}$$

$$D_2 = 1200 + 0 + 0 - (560 - 450) \\ D_2 = 1200 - 110 = 1090$$

$$\therefore I_2 = \frac{D_2}{D} = \frac{1090}{598} = 1.822 \text{ A}$$

$$D_3 = \begin{vmatrix} 8 & -3 & 15 & 8 \\ -3 & 9 & 15 & -3 \\ 0 & -2 & -35 & 0 \end{vmatrix}$$

$$D_3 = -2520 + 0 + 90 - (0 - 240 - 315) \\ = -2520 + 90 + 645 = -1875$$

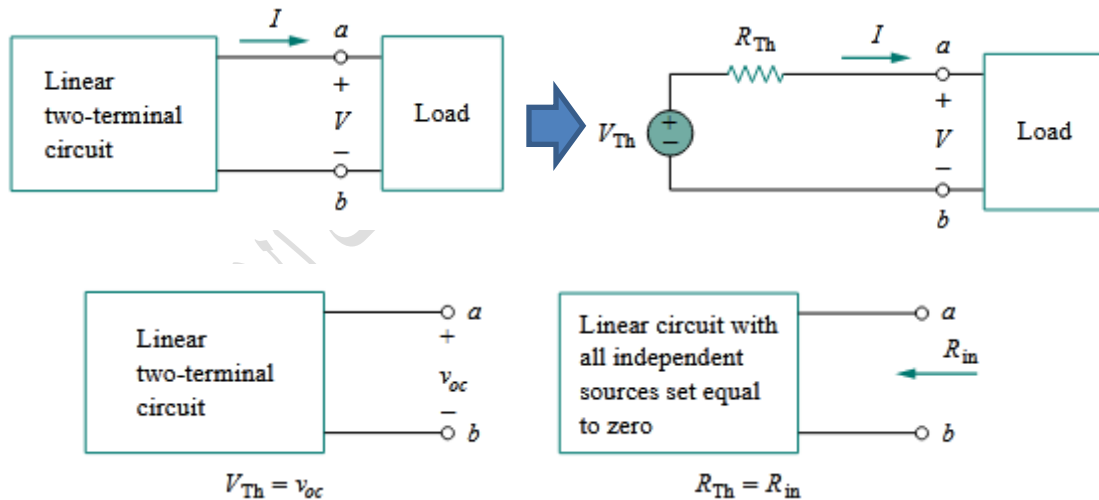
$$\therefore I_3 = \frac{D_3}{D} = \frac{-1875}{598} = -3.135 \text{ A}$$

## الهدف التعليمي (الهدف الخاص لكل للمحاضرة):

- ان يتعرف الطالب على تحليل الدائرة الكهربائية وايجاد مكافئ ثفنن .
- ان يكون الطالب قادرا على تطبيق نظرية ثفنن وايجاد قيم التيارات (الحمل) والفولتيات في مقاومة معينة .

## THEVENIN'S THEOREM

thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source  $V_{Th}$  in series with a resistor  $R_{Th}$ , where  $V_{Th}$  is the open-circuit voltage at the terminals and  $R_{Th}$  is the input or equivalent resistance at the terminals when the independent sources are turned off.



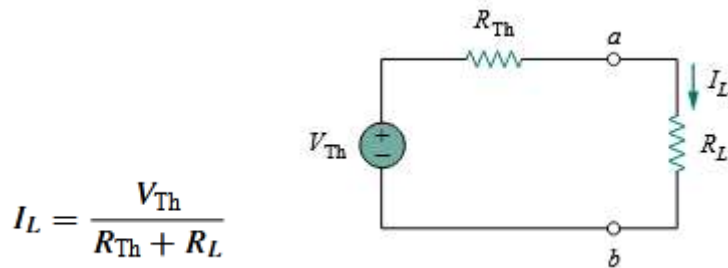
The procedure adopted when using Thevenin's theorem is summarized below. To determine the current in any branch of an active network (i.e. one containing a source of e.m.f.):

- remove the resistance  $R_L$  from that branch
- determine the open-circuit voltage  $V_{oc}$

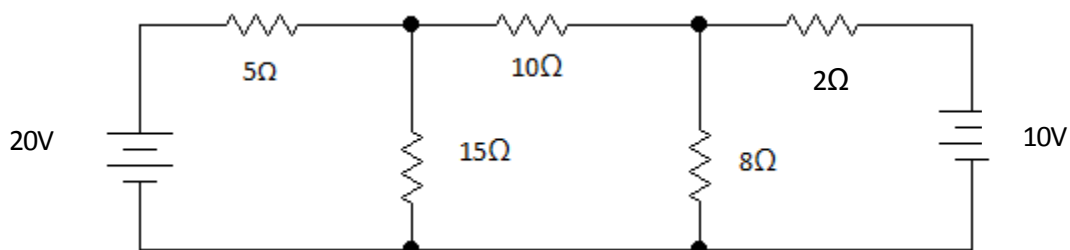
iii) Find the Thevenin's equivalent resistance,  $R_{Th}$  at the terminals when all independent sources are zero (Replacing independent sources)

- If voltage source (short circuit)
- If current source (open circuit)

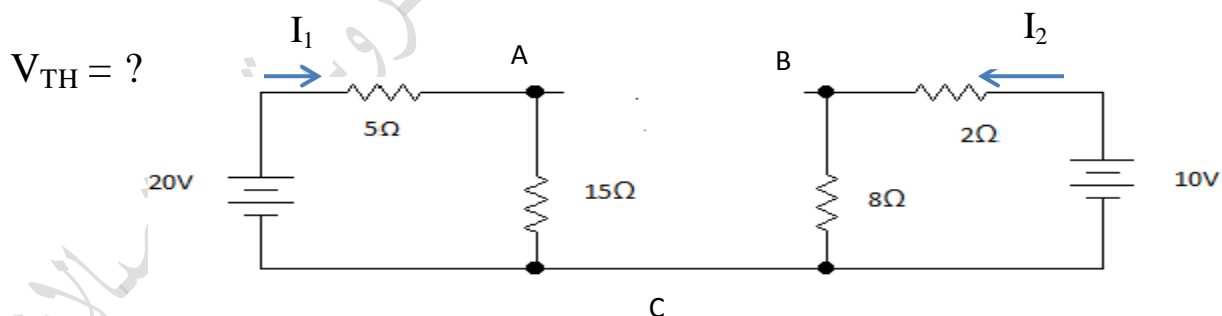
iv) determine the value of the current from the equivalent circuit shown •



Example: For the circuit shown in fig below Calculate the current passes through ( $10\Omega$ ) resistance Using thevenin's theorem



Solution:



$$I_1 = \frac{V}{R}$$

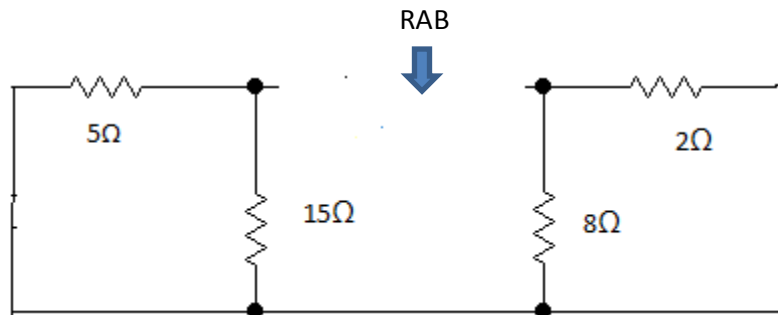
$$I_1 = \frac{20}{5+15} = 1A$$

$$I_2 = \frac{10}{8+2} = 1A$$

$$V_{AC} = 1 \times 15 = 15V$$

$$V_{BC} = 1 \times 8 = 8V$$

$$V_{AB} = 15 - 8 = 7$$



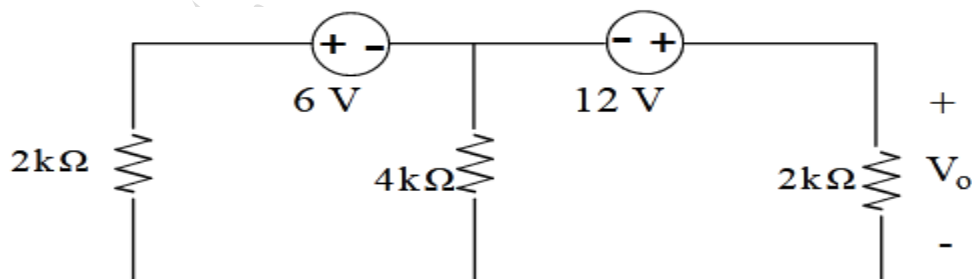
$$R_{AB} = R_{TH}$$

$$R_{AB} = \frac{2 \times 8}{2+8} + \frac{15 \times 5}{15+5} = 3.7 + 1.6 = 5.3\Omega = R_{TH}$$

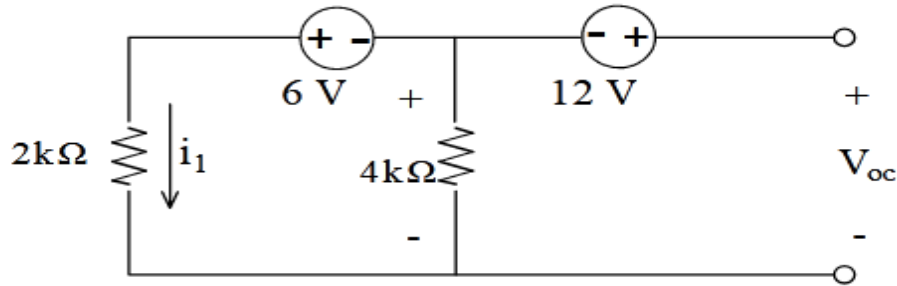
$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

$$I_{10\Omega} = \frac{7}{10 + 5.3} = 0.46A$$

Example: Use Thevenin's Theorms to find  $V_0$



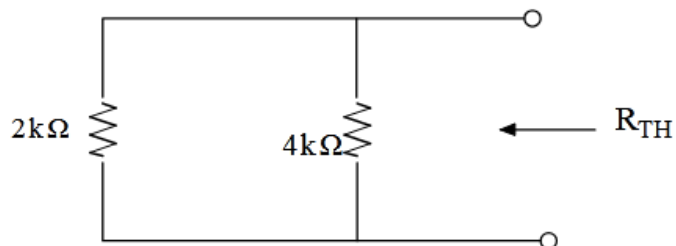
find  $V_{OC}$



$$i_1 = \frac{6 \text{ V}}{2 \text{ k} + 4 \text{ k}} = 1 \text{ m A} \quad \Rightarrow \quad V_{4\text{k}\Omega} = i_1 (4 \text{ k}) = -4 \text{ V}$$

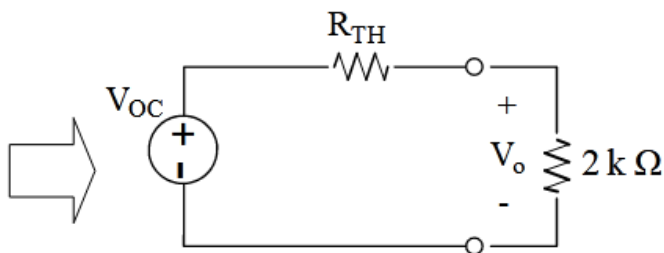
$$V_{oc} = 12 \text{ V} - 4 \text{ V} = 8 \text{ V}$$

Second, find  $R_{TH}$



$$R_{TH} = 2\text{k} // 4\text{k} = 4/3 \text{ k } \Omega$$

Thevenin equivalent circuit is



$$\begin{aligned} V_o &= \frac{2 \text{ k } \Omega}{2 \text{ k} + R_{TH}} V_{oc} \\ &= \frac{2 \text{ k}}{10/3 \text{ k}} (8 \text{ V}) \\ V_o &= 4.8 \text{ V} \end{aligned}$$

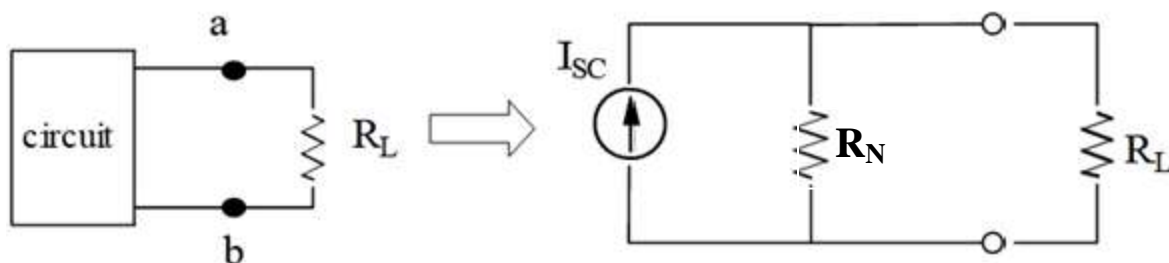
الاسبوع السابع المحاضرة السابعة

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- ان يتعرف الطالب على تحليل الدائرة الكهربائية وايجاد مكافئ نورتن .
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## NORTON'S THEOREM

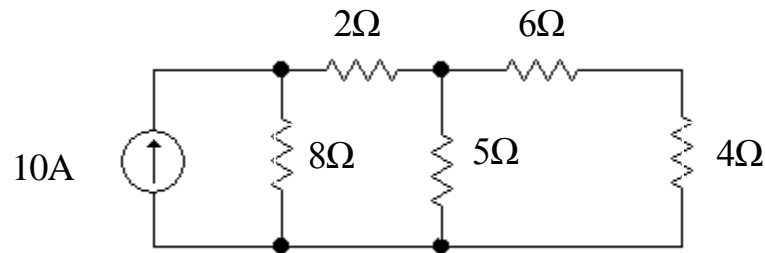
Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source  $I_N$  in parallel with a resistor  $R_N$ , where  $I_N$  is the short-circuit current through the terminals and  $R_N = R_{TH}$  is the input or equivalent resistance at the terminals when the independent sources are turned off



### Procedure of Norton's Theorem

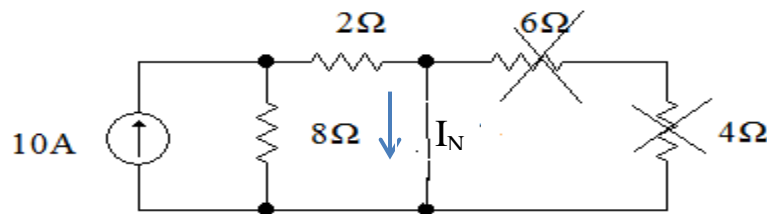
- Find the short circuit current at the terminals,  $I_{sc}$
- Find Thevenin's equivalent resistance,  $R_{TH}$  (as before)
- Reconnect the load to Norton's equivalent circuit

Example: Determine the current in the resistance ( $6\Omega$ ) of the network shown in fig (5) below by using norton's theorem.



### Solution

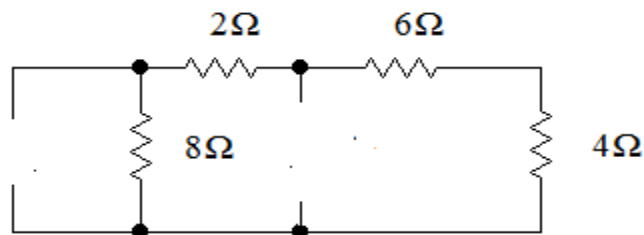
$$I_N = ?$$



$$I_{2\Omega} = I_S = 10 \times \frac{8}{8 + 2} = 8A$$

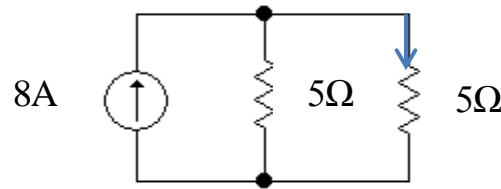
$$I_S = I_N = 8A$$

$$R_N = ?$$



$$R_N = R_{TH} = \frac{(6 + 4) \times (2 + 8)}{(6 + 4) + (2 + 8)} = 5\Omega$$





$$I_{5\Omega} = 8 \times \frac{5}{5 + 5} = 4A$$

الاسبوع التاسع المحاضرة التاسعة

**الهدف التعليمي (الهدف الخاص لكل للمحاضرة):**

- ان يكون الطالب قادرا على تطبيق وتحليل الدائرة باستخدام نظرية التتابق او التراكب.
- ان يكون الطالب قادرا على ايجاد التيار و الفولتية في اي مقاومة باستخدام نظرية التتابق او التراكب.
- ان يكون الطالب قادرا على فهم نظرية نقل اعظم قدرة

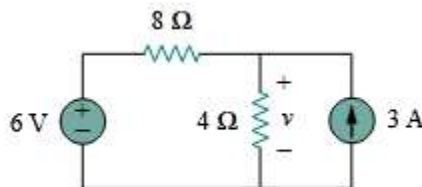
## Superposition theorem

Whenever a linear circuit is excited by more than one independent source, the total response is the algebraic sum of individual responses. The idea is to activate one independent source at a time to get individual response. Then add the individual response to get total response.

Steps to apply the superposition principle

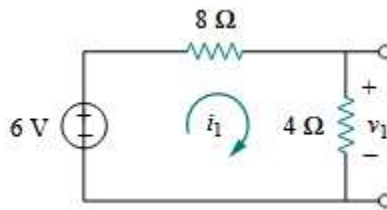
- We consider one independent source at a time while all other independent sources are turned off
- This implies that we replace every voltage source by 0 V (or a short circuit)
- every current source by 0 A (or an open circuit)

Example: Use the superposition theorem to find  $v$  in the circuit in Fig below



Solution:

1- Redraw the original circuit with source (6v) removed set the current source to zero(open circuit)



To obtain  $v_1$ , Applying KVL

$$12i_1 - 6 = 0 \implies i_1 = 0.5 \text{ A}$$

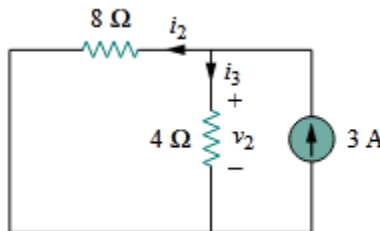
Thus,

$$v_1 = 4i_1 = 2 \text{ V}$$

Or use the voltage division

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

Redraw the original circuit with source (3A), removed set the voltage source to zero(short circuit)



Using current division

$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

obtain  $v_2$

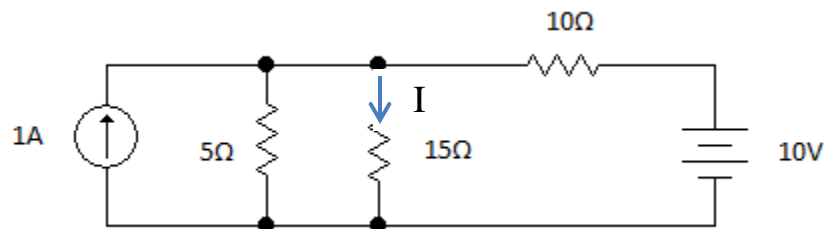
Hence,

$$v_2 = 4i_3 = 8 \text{ V}$$

And we find

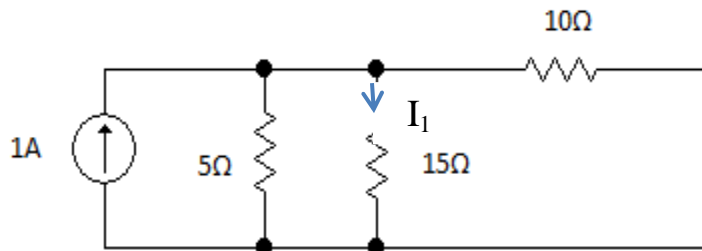
$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

Example: Calculate the voltage across ( $15\Omega$ ) resistance using Superposition Theorem



Solution

removed the voltage source to zero(short circuit)

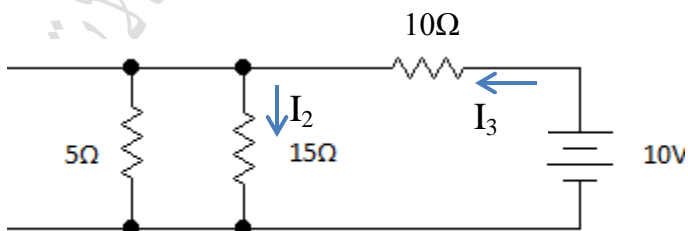


$$\frac{5 \times 10}{5 + 10} = \frac{50}{15} = 3.33\Omega$$

$$I_1 = \frac{3.33}{3.33 + 15} I$$

$$I_1 = \frac{3.33}{3.33 + 15} \times 1 = 0.18A$$

removed set the current source to zero(open circuit)



$$\frac{5 \times 15}{5 + 15} + 10 = 13.75\Omega$$

$$I_3 = \frac{10}{13.75} = 0.73A$$

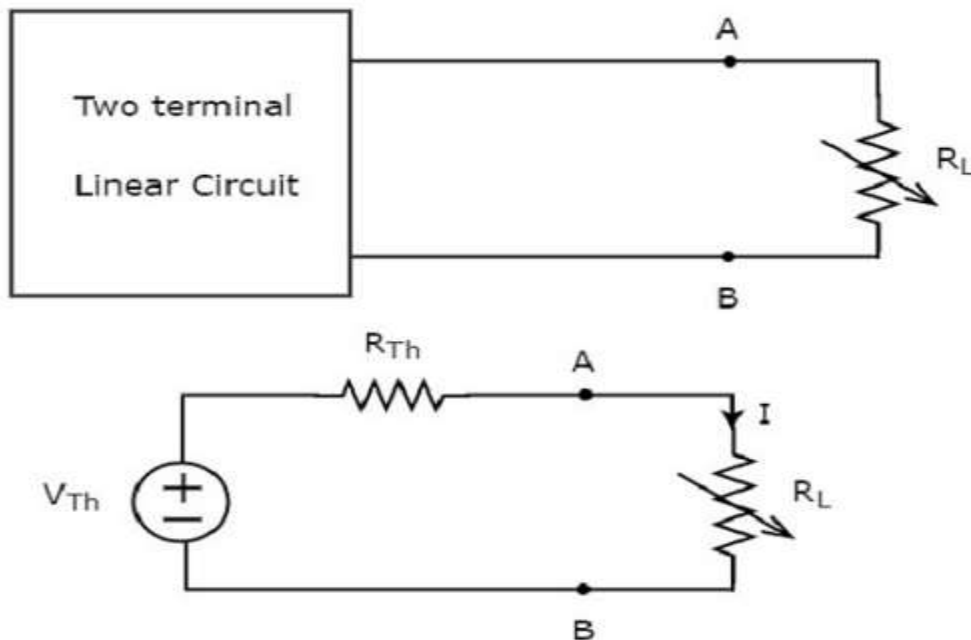
$$I_2 = 0.73 \times \frac{5}{5 + 15} = 0.18A$$

$$I = I_1 + I_2 = 0.18 + 0.18 = 0.36A$$

$$I_{15} = 0.36A$$

### Maximum power transfer theorem

Max power transfer states that the DC voltage source will deliver maximum power to the variable load resistor only when the load resistance is equal to the source resistance.



when the load resistance is equal to the source resistance The amount of power dissipated across the load resistor is

### **Proof of Maximum Power Transfer Theorem**

The power dissipated across the load resistor is

$$P_L = I^2 R_L$$

Substitute  $I = \frac{V_{Th}}{R_{Th} + R_L}$  in the above equation.

$$P_L = \left( \frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$
$$\Rightarrow P_L = V_{Th}^2 \left\{ \frac{R_L}{(R_{Th} + R_L)^2} \right\}$$

For maximum power transfer, differentiate  $P_L$  with respect to  $R_L$  and make it equal to zero

$$\frac{dP_L}{dR_L} = 0$$

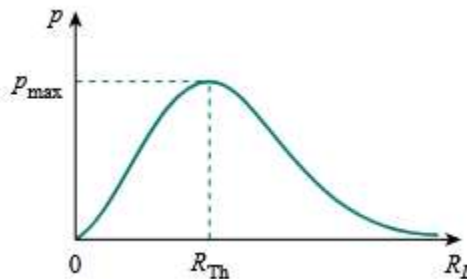
$$\frac{dP_L}{dR_L} = V_{Th}^2 \left\{ \frac{(R_{Th} + R_L)^2 \times 1 - R_L \times 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right\} = 0$$

$$\Rightarrow (R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L) = 0$$

$$\Rightarrow (R_{Th} + R_L)(R_{Th} + R_L - 2R_L) = 0$$

$$\Rightarrow (R_{Th} - R_L) = 0$$

$$R_{Th} = R_L$$



Max power transfer states that the DC voltage source will deliver maximum power to the variable load resistor only when the load resistance is equal to the source resistance.

$$P_{\max} = \frac{V_{Th}^2}{4R_{Th}}$$

## الاسبوع العاشر + الحادي عشر المحاضرة العاشرة + الحادية عشرة

الهدف التعليمي (الهدف الخاص لكل للمحاضرة):

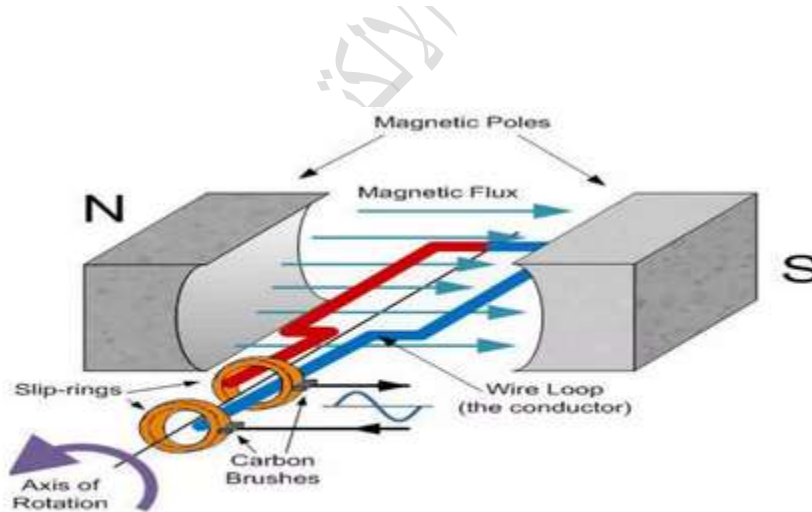
- تعريف الطالب بالكميات المتناوبة والعوامل المؤثرة على القوة الدافعة الكهربائية والتعاريف الاساسية للكميات المتناوبة واشكال الموجات .

مدة المحاضرة: ٤ ساعة نظري

## Alternating Voltage and Current

An alternating voltage may be generated:

1. By rotating a coil at constant angular velocity in a uniform magnetic field.
2. By rotating a magnetic field at a constant angular velocity within a stationary coil.



The value of the voltage generated depends in each case upon:-

- 1-The number of turns in the coil.
- 2-Strength of the field.
- 3-Speed at which the coil or magnetic field rotates.

### Equation of the alternating voltage and current

Consider a rectangular coil having (N) turns and rotating in a uniform magnetic field with an angular velocity of ( $\omega$ ) radian/second as shown in fig below

Let time be measured from the x-axis maximum flux ( $\phi_m$ ) is linked with the coil when its plane coincides with x-axis at the time (t) second this coil rotates through an angle ( $\omega t$ ).

In this deflected position, the component of the flux which is perpendicular to the plane of the coil is

$$\phi = \phi_m \cos \omega t$$

$$\phi N = N \phi_m \cos \omega t$$

$\phi$ : magnetic flux

N: number of turns

According to Faraday's second law of electromagnetic induction, we know that the induced emf in a coil is equal to the rate of change of flux linkage. Therefore,

$$e = \frac{d\phi}{dt}$$

$$e = N \frac{d\phi}{dt}$$

Considering Lenz's law,

$$e = -N \frac{d}{dt} (\phi_m \cos \omega t)$$

$$e = -N \phi_m \omega (-\sin \omega t)$$

$$e = N \phi_m \omega \sin \omega t$$

$$\theta = \omega t$$

$$e = N \phi_m \omega \sin \theta$$

When  $\Theta = 90^\circ$

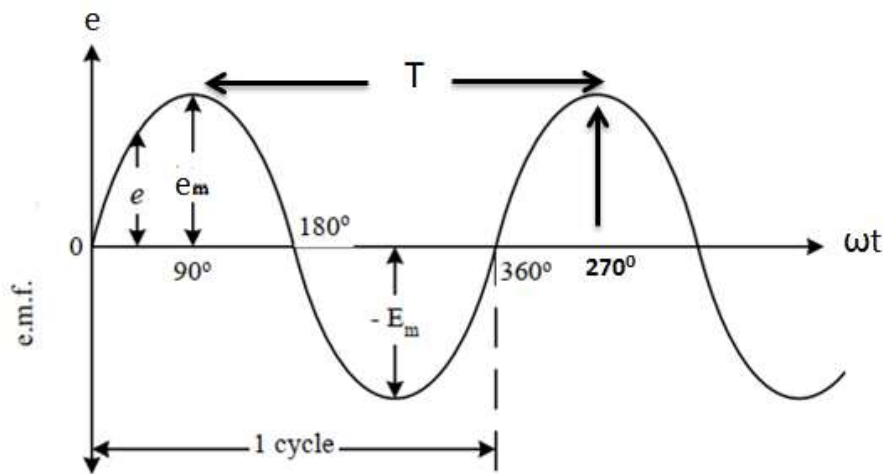
$\sin 90 = 1$

$$e = N\phi m \omega = em$$

$$e = e_m \sin \omega t$$

$$i = \frac{e_m \sin \omega t}{R}$$

$$i = i_m \sin \omega t$$



### Definitions

A few basic terms will be defined in this section which can be applied to any waveform.

- **Periodic waveform**: a waveform that continually repeats itself after the same time interval.
- **Period (T)**: the time interval between successive repetitions of a periodic waveform or the time of one cycle.
- **Frequency (f)**: the number of cycles that occur in 1 second, the unit used for measuring frequency is cycle per second or hertz (Hz).



$$f = \frac{1}{T}$$

$$T = \frac{1}{f}$$

$$\theta = \omega t = \omega \cdot \frac{1}{f}$$

$$\theta = 2\pi \quad \text{one cycle}$$

$$2\pi = \omega \cdot \frac{1}{f} \quad \text{angular velocity } \omega = 2\pi f \text{ rad/second}$$

• **Instantaneous value:** the magnitude of the waveform at any instant of time.

• **Amplitude or Peak value:** the maximum value of a waveform.

**Example2:** An alternating voltage has the equation  $v = 141.4 \sin 377t$ ; what are the values of:

(a) r.m.s. voltage;

(b) frequency;

(c) the instantaneous voltage when  $t = 3 \text{ ms}$ ?

**Sol**

$$V = v_m \sin \omega t$$

$$(a) \quad V_m = 141.4 \text{ V} = \sqrt{2} V$$

$$\text{hence } V = \frac{141.4}{\sqrt{2}} = 100 \text{ V}$$

(b) Also by comparison

$$\omega = 377 \text{ rad/s} = 2\pi f$$

$$\text{hence } f = \frac{377}{2\pi} = 60 \text{ Hz}$$

(c) Finally

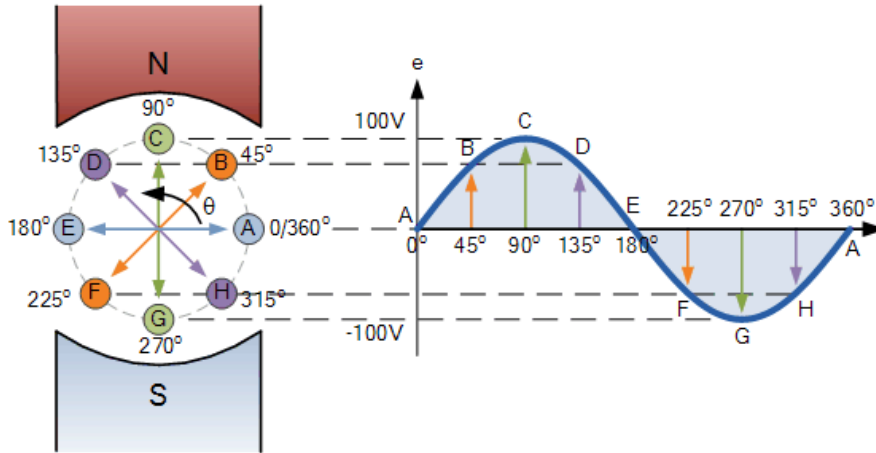
$$v = 141.4 \sin 377t$$

$$\text{When } t = 3 \times 10^{-3} \text{ s}$$

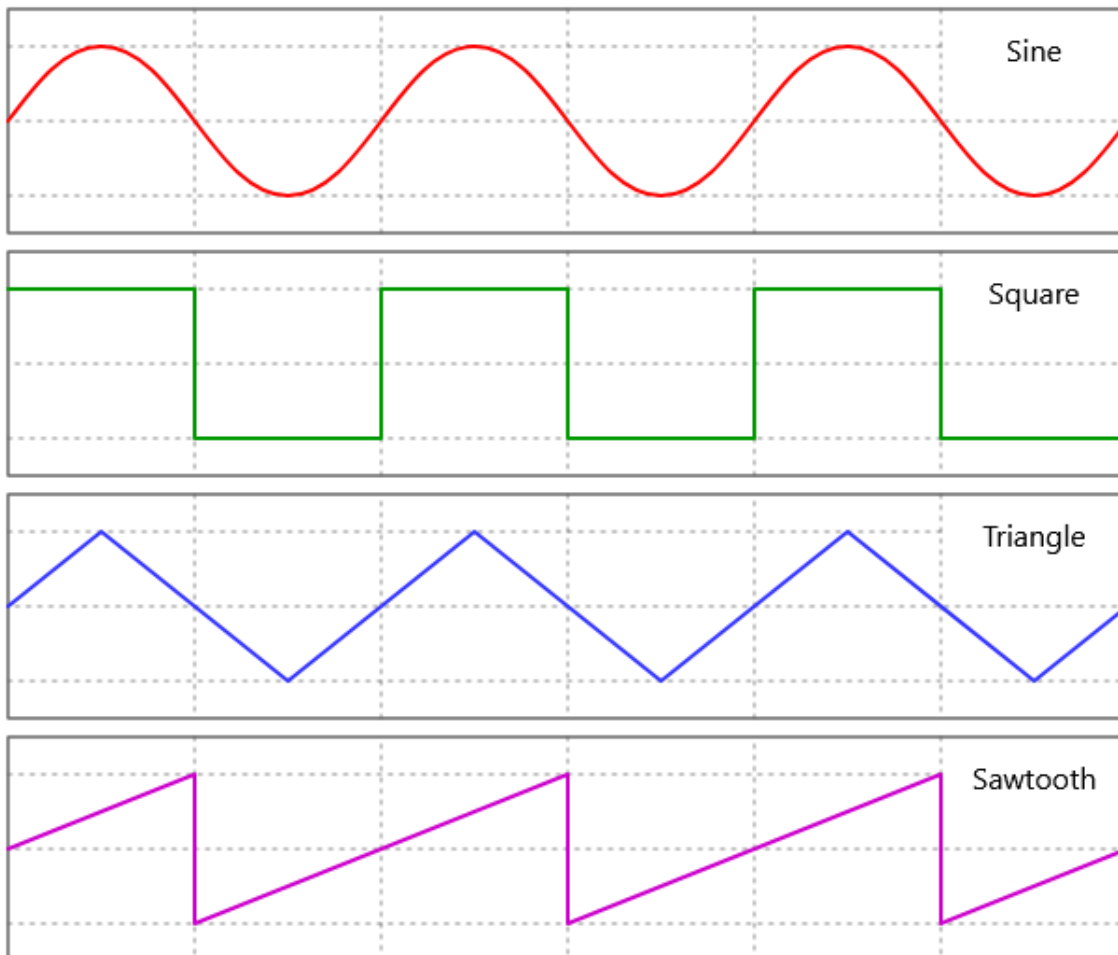
$$v = 141.4 \sin(377 \times 3 \times 10^{-3}) = 141.4 \sin 1.131$$

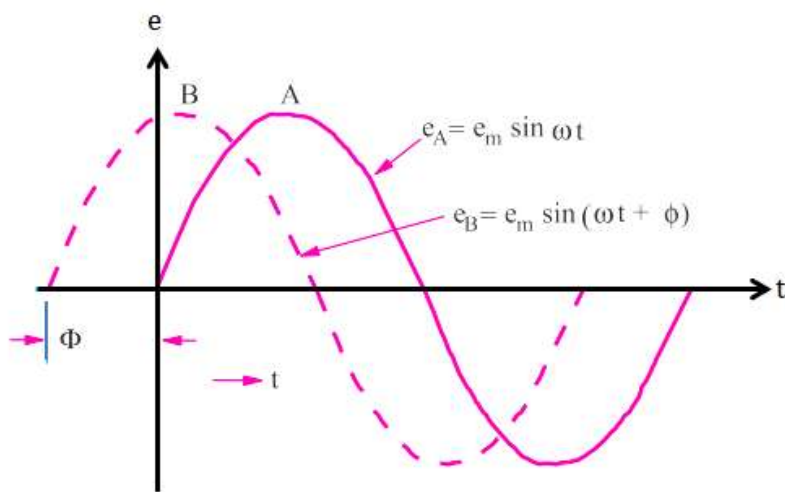
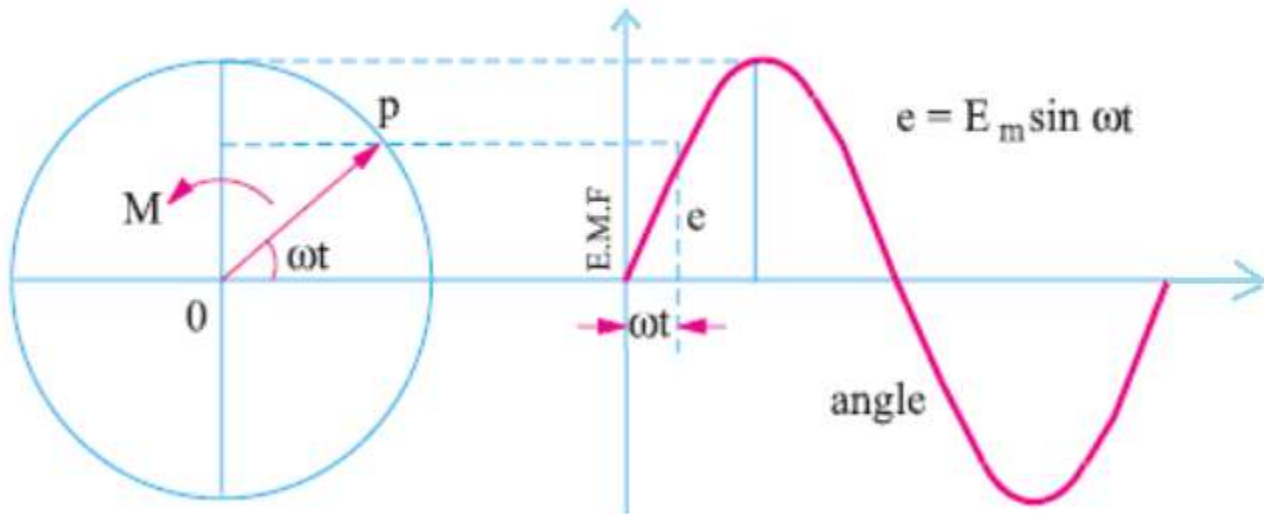
$$= 141.4 \times 0.904 = 127.8 \text{ V}$$

## Representation of an alternating quantity by vector



## Types of Periodic Waveform





$e_B$  leads  $e_A$  by an angle  $\phi$

$e_A = E_m \sin \omega t$  ...reference quantity

$e_B = E_m \sin (\omega t + \phi)$

### Root Mean Square (R.M.S) value

The r.m.s value of an alternating current is given by that steady (d.c.) current which when flowing through a given time produce the same heat (power) as produced by the alternating current when flowing through the same circuit for the same time. It is also known as the effective value of the alternating current.

$$I_{r.m.s} = \sqrt{\frac{1}{T} \int_0^T i^2(\theta) d\theta}$$

## Average Value

The average value  $I_a$  of an alternating current is expressed by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.

In the case of a symmetrical alternating current (*i.e.* one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero.

$$I_{av} = \frac{1}{T} \int_0^T i(\theta) d\theta$$

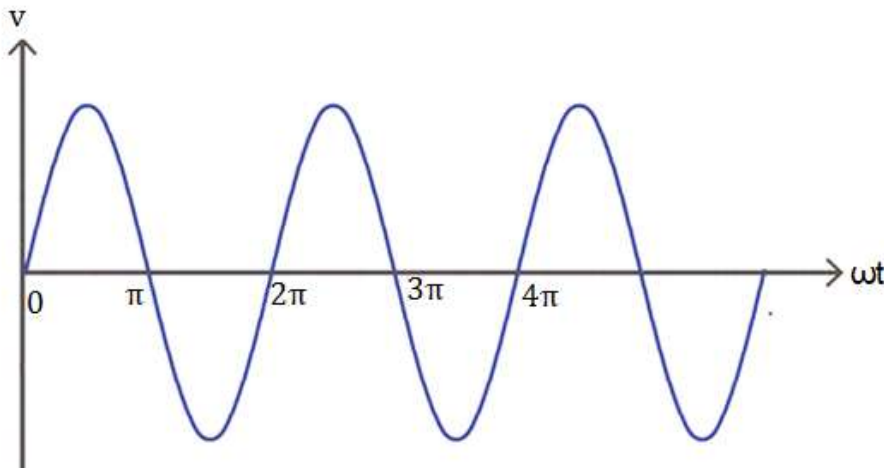
## Form factor

$$K_F = \frac{r.m.s}{average}$$

## Peak or Amplitude Factor

$$K_a = \frac{\text{max value}}{r.m.s \text{ value}}$$

**Example:** Find Root Mean Square (R.M.S) value



$$I_{r.m.s} = \sqrt{\frac{1}{T} \int_0^T i^2(\theta) d\theta}$$

$$T = 2\pi$$

$$I_{r.m.s} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (I_m \sin \omega t)^2 d\omega t}$$

$$\sin^2 \omega t = 1 - \cos 2\omega t$$

$$I_{r.m.s} = \sqrt{\frac{I_m^2}{2\pi} \int_0^{2\pi} \frac{1}{2} (1 - \cos 2\omega t) d\omega t}$$

$$I_{r.m.s} = \sqrt{\frac{I_m^2}{4\pi} \left[ \omega t - \frac{\sin 2\omega t}{2} \right]}$$

$$I_{r.m.s} = \sqrt{\frac{I_m^2}{4\pi} \left[ \omega t - \frac{\sin 2\omega t}{2} \right]_0^{2\pi}}$$

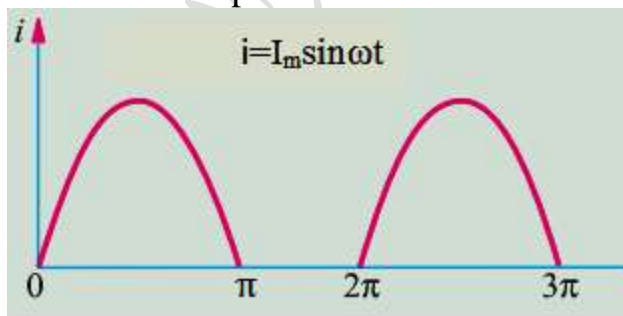
$$I_{r.m.s}^2 = \frac{I_m^2}{4\pi} [2\pi - 0]$$

$$I_{r.m.s}^2 = \frac{I_m^2}{4\pi} [2\pi]$$

$$I_{r.m.s}^2 = \frac{I_m^2}{4\pi} [2\pi]$$

$$I_{r.m.s}^2 = \frac{I_m^2}{2} \quad \therefore I_{r.m.s} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

**Example:** Find Root Mean Square (R.M.S) value, Average Value, Form factor and Amplitude Factor for the wave form in fig below.



**Solution**

$$I_{r.m.s} = \sqrt{\frac{1}{T} \int_0^T i^2(\theta) d\theta}$$

$$T=2\pi$$

$$I_{r.m.s} = \sqrt{\frac{1}{2\pi} \int_0^\pi ((I_m \sin \omega t)^2 d\omega t + \int_\pi^{2\pi} 0 d\omega t)}$$

$$\sin^2 \omega t = 1 - \cos 2\omega t$$

$$I_{r.m.s} = \sqrt{\frac{I_m^2}{2\pi} \int_0^\pi \frac{1}{2} (1 - \cos 2\omega t) d\omega t}$$

$$I_{r.m.s} = \sqrt{\frac{I_m^2}{4\pi} \left[ \omega t - \frac{\sin 2\omega t}{2} \right]}$$

$$I_{r.m.s} = \sqrt{\frac{I_m^2}{4\pi} \left[ \omega t - \frac{\sin 2\omega t}{2} \right]_0^\pi}$$

$$I_{r.m.s}^2 = \frac{I_m^2}{4\pi} [\pi - 0]$$

$$I_{r.m.s}^2 = \frac{I_m^2}{4\pi} [\pi]$$

$$I_{r.m.s}^2 = \frac{I_m^2}{4\pi} [\pi]$$

$$I_{r.m.s}^2 = \frac{I_m^2}{4} \quad \therefore I_{r.m.s} = \frac{I_m}{2}$$

$$I_{av} = \frac{1}{T} \int_0^T i(\theta) d\theta$$

$$I_{av} = \frac{1}{2\pi} \int_0^\pi ((I_m \sin \omega t) d\omega t + \int_\pi^{2\pi} 0 d\omega t)$$

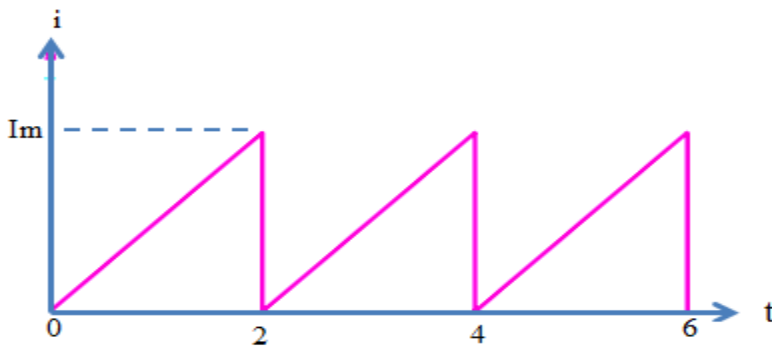
$$I_{av} = \frac{I_m}{2\pi} [-\cos \omega t]_0^\pi$$

$$I_{av} = \frac{I_m}{2\pi} [-(-1 + 1)] = \frac{I_m}{2\pi} (2) \quad \therefore I_{av} = \frac{I_m}{\pi}$$

$$K_F = \frac{I_{r.m.s}}{I_{av}} = \frac{\frac{I_m}{2}}{\frac{I_m}{\pi}} = \frac{\pi}{2}$$

$$K_A = \frac{I_m}{I_{r.m.s}} = \frac{I_m}{\frac{I_m}{2}} = 2$$

**Example:** Find the form-factor of the wave form given in fig below



Solution:

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{I_m - 0}{2 - 0} = \frac{I_m}{2}$$

$$y - y_1 = m(x - x_1) \quad i - 0 = \frac{I_m}{2} (t - 0)$$

$$I_{r.m.s} = \sqrt{\frac{1}{T} \int_0^T i^2(\theta) d\theta} = \sqrt{\frac{1}{2} \int_0^2 i^2 dt} = I_{r.m.s} = \sqrt{\frac{1}{2} \int_0^2 \left(\frac{I_m}{2} t\right)^2 dt}$$

$$I_{r.m.s} = \sqrt{\frac{I_m^2}{8} \int_0^2 (t)^2 dt} = \sqrt{\frac{I_m^2}{8} \left[\frac{t^3}{3}\right]_0^2}$$

$$\sqrt{\frac{I_m^2}{8} \left[\frac{t^3}{3}\right]_0^2}$$

$$I_{r.m.s} = \sqrt{\frac{I_m^2}{8} \left[\frac{2^3}{3}\right]} = \sqrt{\frac{I_m^2}{8} \left[\frac{8}{3}\right]} = \frac{I_m}{\sqrt{3}}$$

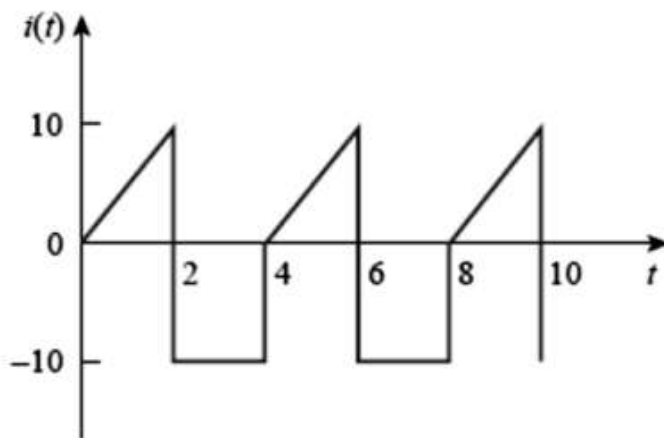
$$I_{av} = \frac{1}{T} \int_0^T i(\theta) d\theta = \frac{1}{2} \int_0^2 i dt = \frac{1}{2} \int_0^2 \frac{I_m}{2} t dt = \frac{I_m}{4} \int_0^2 t dt$$

$$I_{av} = \frac{I_m}{4} \left[ \frac{t^2}{2} \right]_0^2$$

$$I_{av} = \frac{I_m}{2} \quad \therefore K_F = \frac{I_{r.m.s}}{I_{av}} = \frac{\frac{I_m}{\sqrt{3}}}{\frac{I_m}{2}} = \frac{I_m}{\sqrt{3}} \times \frac{2}{I_m} = \frac{2}{\sqrt{3}}$$

## H.W

**1- Determine the rms value of the current waveform in Figure shown below.**



2- prove that Root Mean Square value of the waveform below  $0.707I_m$



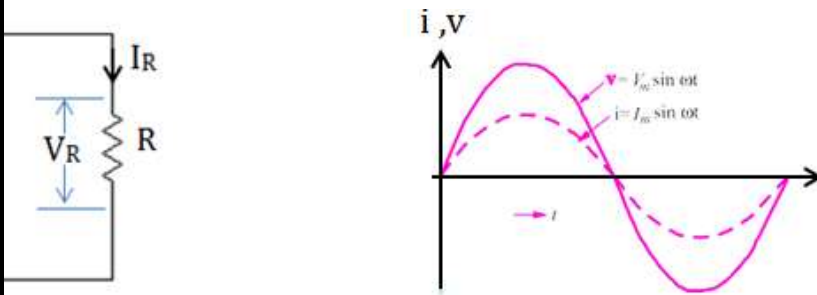
## الاسبوع : الثاني عشر المحاضرة الثانية عشر

الهدف التعليمي (الهدف الخاص لكل للمحاضرة):

- ان يكون الطالب قادرا على تطبيق فهم وتحليل تأثير التيار المتناوب على مقاومة و ملف ومتسعة .
- ان يكون الطالب قادرا على تطبيق فهم وتحليل تأثير التيار المتناوب على دوائر التوالي .

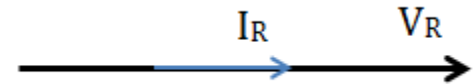
### A.C. Through Resistance, Inductance and Capacitance

#### A.C. Through Pure Resistance



**In Pure resistance:** The current ( $I_R$ ) is in phase with the applied voltage ( $V_R$ ) as shown in the phasor.

$$I_R = \frac{V_R}{R}$$

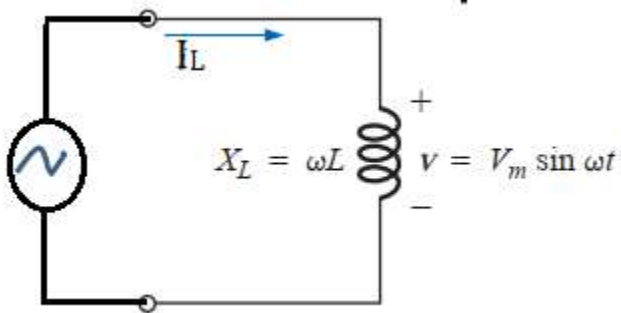


$$Z=R$$

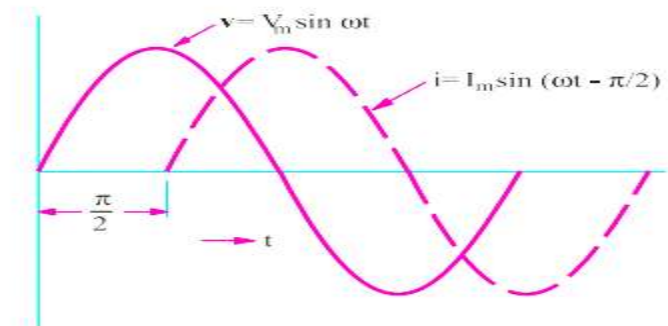
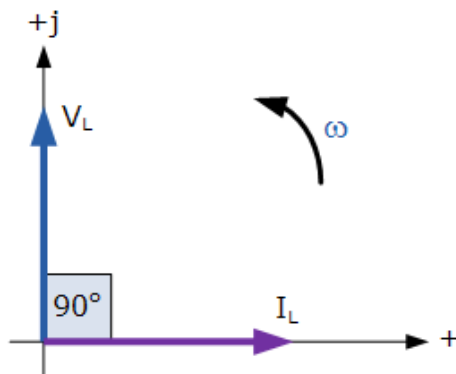
Z: Impedance in ( $\Omega$ )

$$Z = \frac{V}{I}$$

## Pure inductance



In an A.C. circuit containing pure inductance  $L$  only, the current  $I_L$  lags the applied voltage  $V_L$  by  $90^\circ$  as shown in the phasor diagram.



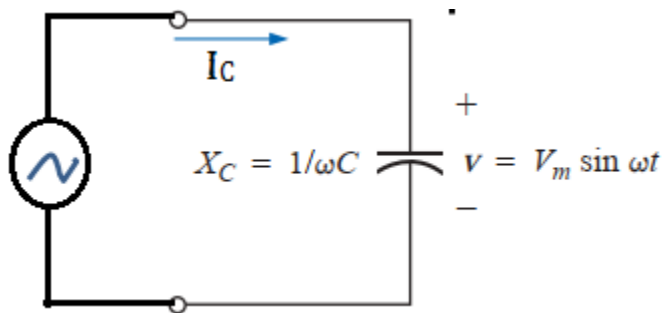
$$X_L = 2\pi fL$$

- Where:
- $X_L$  = Inductive Reactance in Ohms, ( $\Omega$ )
- $\pi$  (pi) = 3.14
- $f$  = Frequency in Hertz, (Hz)
- $L$  = Inductance in Henries, (H)

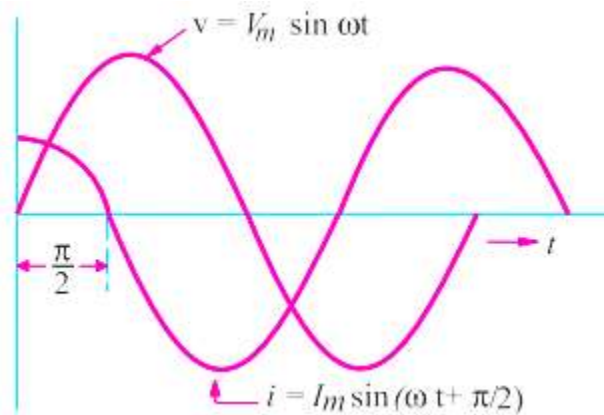
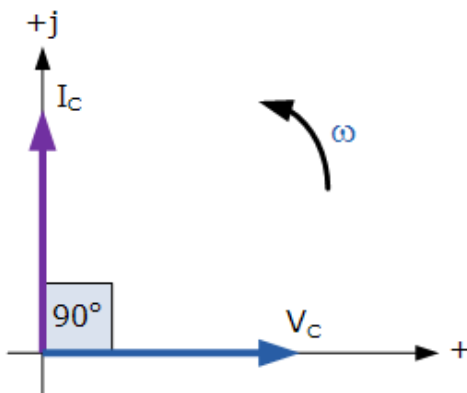
$$X_L = \omega L, Z = X_L$$

$$I_L = \frac{V}{X_L}$$

### Pure capacitance:



In an A.C. circuit containing pure capacitance, the current  $I_C$  leads the applied voltage  $V_C$  by  $90^\circ$ .



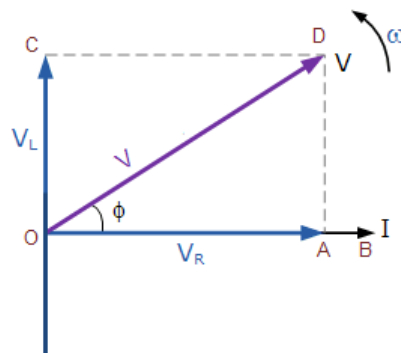
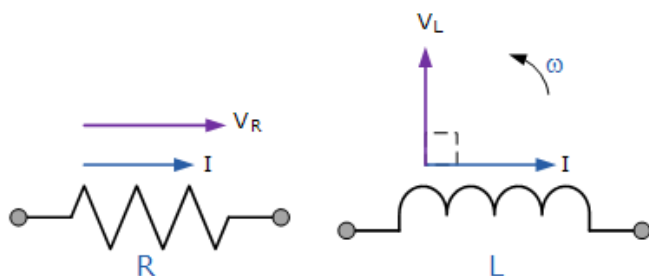
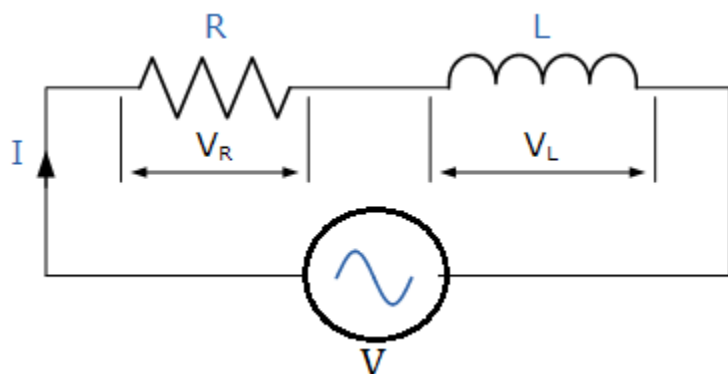
$$I_C = \frac{V}{X_C}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Where:

- $X_C$  is the Capacitive Reactance in Ohms,
- $f$  is the frequency in Hertz
- $C$  is the AC capacitance in Farads,

### Series Resistance-Inductance Circuit



$$V^2 = V_R^2 + V_L^2, \quad V = \sqrt{V_R^2 + V_L^2}$$

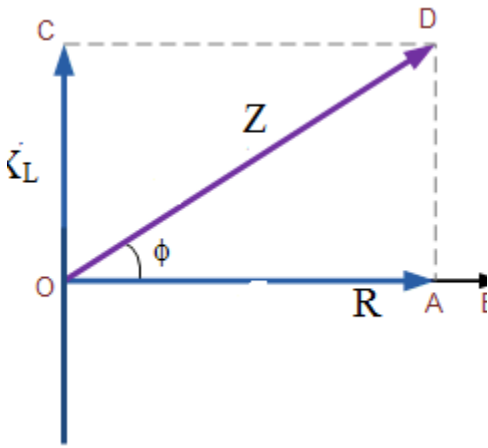
$$V = \sqrt{(I.R)^2 + (I.X_L)^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

## Impedance

$$Z = \frac{V}{I}$$

$$Z = \sqrt{R^2 + X_L^2}$$

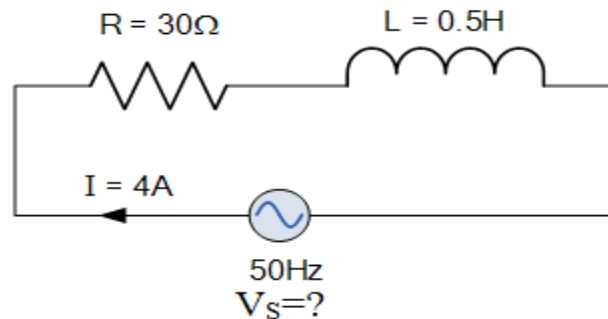


phase angle,  $\theta$  between the voltage and current is calculated as :

$$\cos \theta = \frac{R}{Z} , \quad \tan \theta = \frac{X_L}{R} , \quad \sin \theta = \frac{X_L}{Z}$$

$$\theta = \cos^{-1} \frac{R}{Z} , \quad \theta = \tan^{-1} \frac{X_L}{R} , \quad \theta = \sin^{-1} \frac{X_L}{Z}$$

Example: A coil has a resistance of  $30\Omega$  and an inductance of  $0.5\text{H}$ . If the current flowing through the coil is  $4\text{A}$ . What will be the rms value of the supply voltage if its frequency is  $50\text{Hz}$ .



Solution:

$$V_S = IZ$$

$$X_L = 2\pi fL = 2 \times 3.14 \times 50 \times 0.5 = 157\Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 157^2} = 189.8\Omega$$

$$V_S = 4 \times 157 = 640V$$

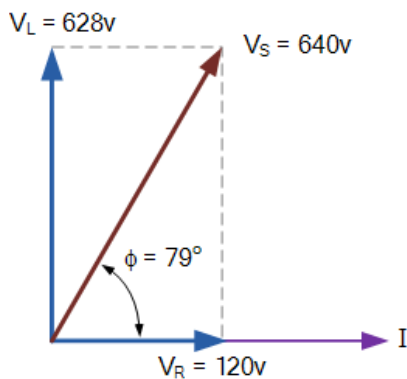
the voltage drops across each component

$$V_R = I.R = 4 \times 30 = 120V$$

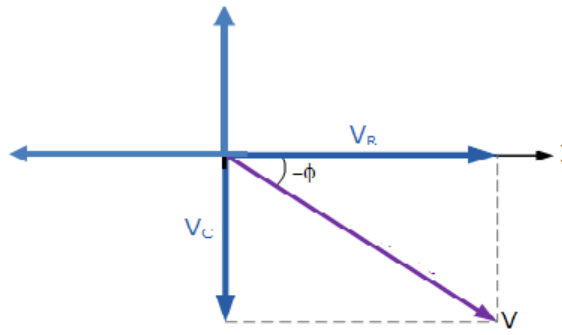
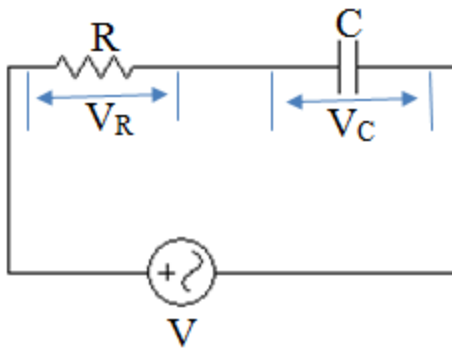
$$V_L = I.X_L = 4 \times 157 = 628V$$

The phase angle between the current and supply voltage

$$\theta = \tan^{-1} \frac{X_L}{R} = \tan^{-1} \frac{157}{30} = 79.2^\circ$$



### Series Resistance-Capacitance Circuit



$$V^2 = V_R^2 + V_C^2, \quad V = \sqrt{V_R^2 + V_C^2}$$

$$V = \sqrt{(I.R)^2 + (I.X_C)^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}}$$

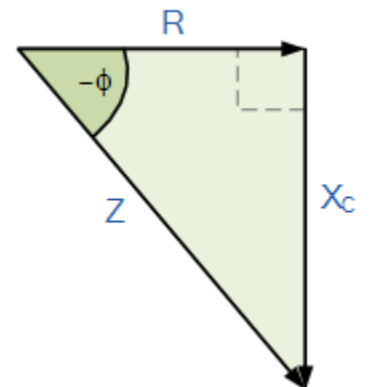
**Impedance**

$$Z = \frac{V}{I}$$

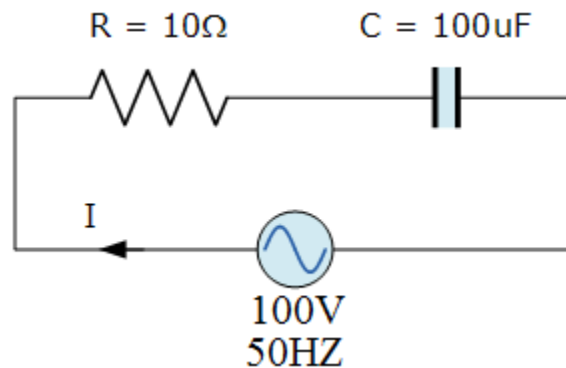
$$Z = \sqrt{R^2 + X_C^2}$$

$$\cos \theta = \frac{R}{Z}, \quad \tan \theta = \frac{X_C}{R}, \quad \sin \theta = \frac{X_C}{Z}$$

$$\theta = \cos^{-1} \frac{R}{Z}, \quad \theta = \tan^{-1} \frac{X_C}{R}, \quad \theta = \sin^{-1} \frac{X_C}{Z}$$



Example: A capacitor which has an internal resistance of  $10\Omega$  and a capacitance value of  $100\mu\text{F}$  is connected to a supply voltage  $100\text{V}$ ,  $50\text{ Hz}$ . Calculate current flowing into the capacitor. Also construct a voltage triangle showing the individual voltage drops.



Solution:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \times 100 \times 10^{-6}} = 31.85\Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{10^2 + 31.85^2} = 33.4\Omega$$

$$I = \frac{V}{Z} = \frac{100}{33.4} = 3\text{A}$$

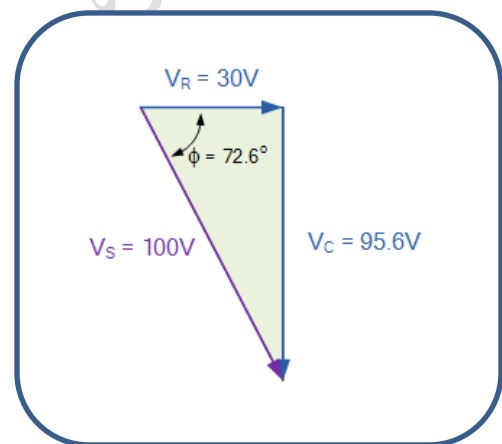
voltage drops around the circuit are calculated as:

$$V_R = I.R = 3 \times 10 = 30\text{V}$$

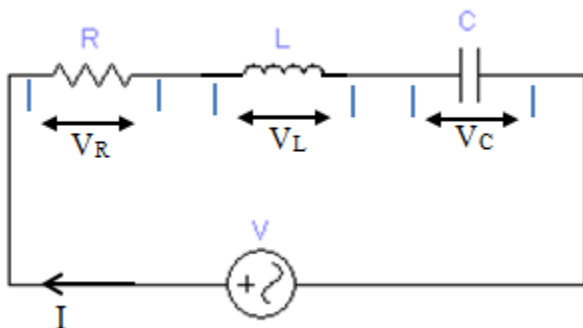
$$V_C = I.X_C = 3 \times 31.85 = 95.6\text{V}$$

The phase angle between the current and voltage

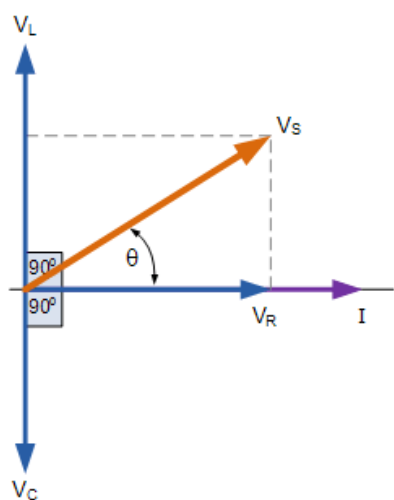
$$\theta = \tan^{-1} \frac{X_C}{R} = \tan^{-1} \frac{31.85}{10} = 72.6^\circ$$



## Series RLC Circuit







$$V^2 = V_R^2 + (V_L - V_C)^2, \quad V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(I.R)^2 + (I.X_L - I.X_C)^2} = I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

### Impedance

$$V = I.Z$$

$$Z = \frac{V}{I}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

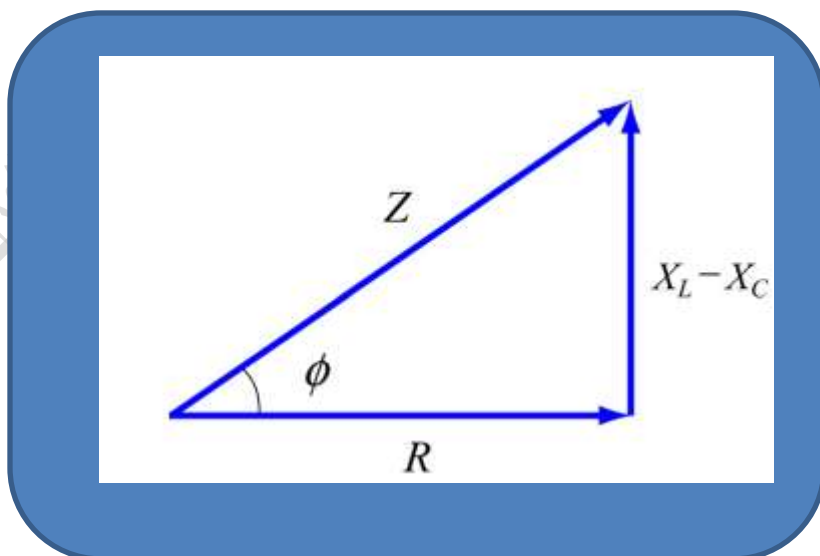
$$Z = \sqrt{R^2 + (X)^2}$$

$$X = X_L - X_C$$

If •  $X_L = X_C$  (resistive circuit)

•  $X_L > X_C$  (inductive circuit)

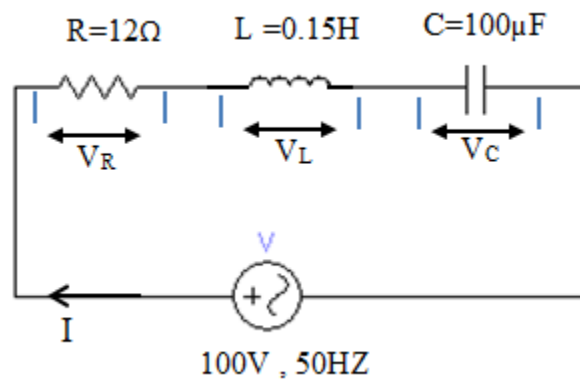
•  $X_L < X_C$  (capacitive circuit)



$$\cos \theta = \frac{R}{Z}, \quad \tan \theta = \frac{X_L - X_C}{R}, \quad \sin \theta = \frac{X_L - X_C}{Z}$$

$$\theta = \cos^{-1} \frac{R}{Z}, \quad \theta = \tan^{-1} \frac{X_L - X_C}{R}, \quad \theta = \sin^{-1} \frac{X_L - X_C}{Z}$$

Example: A series RLC circuit containing a resistance of  $12\Omega$ , an inductance of  $0.15\text{H}$  and a capacitor of  $100\mu\text{F}$  are connected in series across a  $100\text{V}$ ,  $50\text{Hz}$  supply. Calculate the total circuit impedance, the circuits current, power factor and draw the voltage phasor diagram.



Solution:

Inductive Reactance,  $X_L$        $X_L = 2\pi fL = 2 \times 3.14 \times 50 \times 0.15 = 47.13\Omega$

Capacitive Reactance,  $X_C$ .       $X_C = \frac{1}{2 \times \pi \times f \times C} = \frac{1}{2 \times \pi \times 50 \times 100 \times 10^{-6}} = 31.83\Omega$

Impedance,  $Z$ .       $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{12^2 + (47.13 - 31.83)^2} = 19.4\Omega$

Current,  $I$ .       $I = \frac{V}{Z} = \frac{100}{19.4} = 5.14\text{A}$

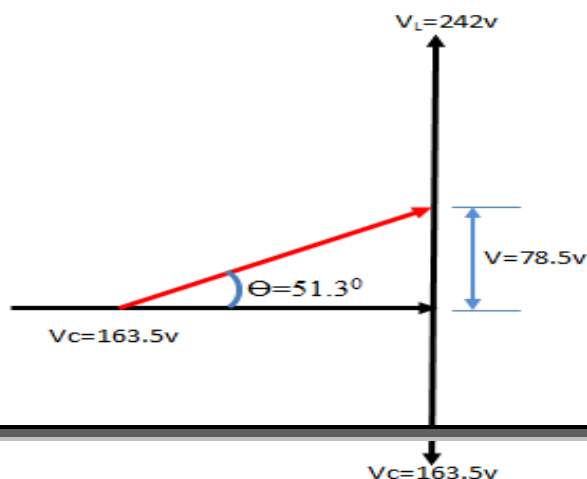
Voltages across  $V_R$ ,  $V_L$ ,  $V_C$ .

$$V_R = I \cdot R = 5.14 \times 12 = 61.7\text{V}, \quad V_L = I \times X_L = 5.14 \times 47.13 = 242.2\text{V},$$

$$V_C = I \times X_C = 163.5\text{V}$$

Phase Angle,  $\theta$

$$\theta = \tan^{-1} \frac{X_L - X_C}{R} = \tan^{-1} \frac{47.13 - 31.83}{12} = 51.8^\circ \text{ lag}$$



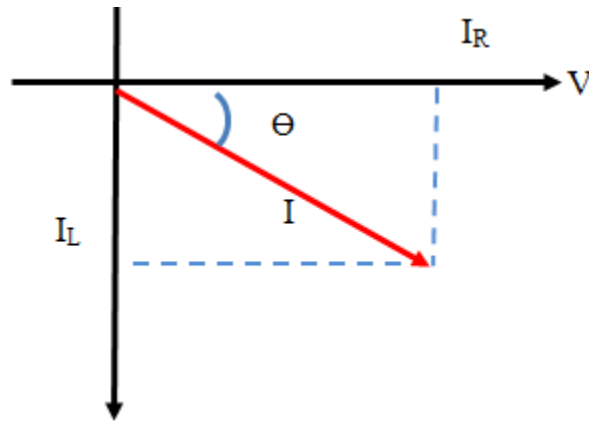
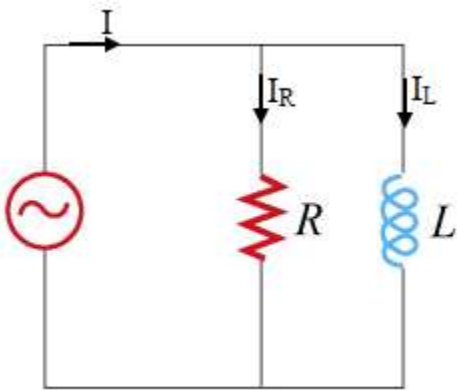
الاسبوع الرابع عشر المحاضرة الرابعة عشر

الهدف التعليمي (الهدف الخاص لكل للمحاضرة):

- ان يكون الطالب قادرا على تطبيق فهم وتحليل تأثير التيار المتناوب على دوائر التوازي .

مدة المحاضرة: ٢ ساعة نظري

## Paraiiel RL Circuit



$$I_R = \frac{V}{R}, \quad I_L = \frac{V}{X_L}$$

$$I^2 = I_R^2 + I_L^2, \quad I = \sqrt{I_R^2 + I_L^2}$$

$$I = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L}\right)^2} = V \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L}\right)^2}$$

$$\frac{V}{I} = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L}\right)^2}}$$

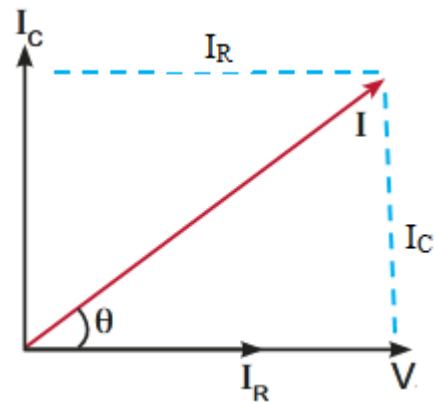
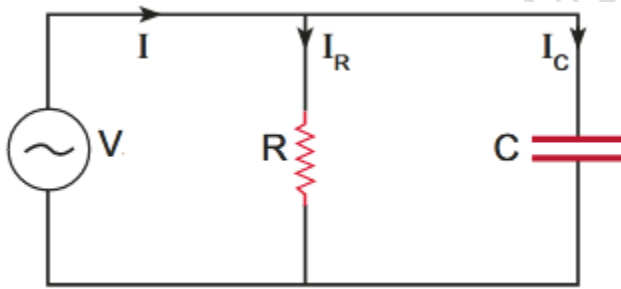
### Impedance

$$Z = \frac{V}{I} = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L}\right)^2}}$$

$$\cos \theta = \frac{I_R}{I} , \quad \tan \theta = \frac{I_L}{I_R} , \quad \sin \theta = \frac{I_L}{I}$$

$$\theta = \cos^{-1} \frac{I_R}{I} , \quad \theta = \tan^{-1} \frac{I_L}{I_R} , \quad \theta = \sin^{-1} \frac{I_L}{I}$$

### Paraiiel RC Circuit



$$I_R = \frac{V}{R} , \quad I_C = \frac{V}{X_C}$$

$$I^2 = I_R^2 + I_C^2 , \quad I = \sqrt{I_R^2 + I_C^2}$$

$$I = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_c}\right)^2} = V \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_c}\right)^2}$$

$$\frac{V}{I} = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_c}\right)^2}}$$

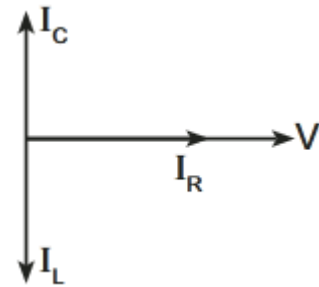
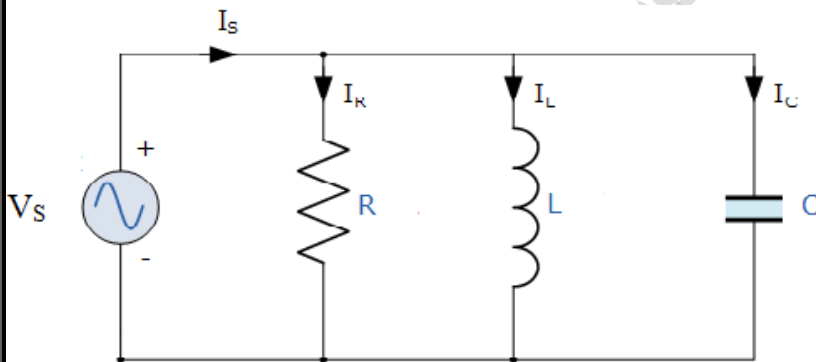
**Impedance**

$$Z = \frac{V}{I} = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_c}\right)^2}}$$

$$\cos \theta = \frac{I_R}{I} \quad , \quad \tan \theta = \frac{I_c}{I_R} \quad , \quad \sin \theta = \frac{I_c}{I}$$

$$\theta = \cos^{-1} \frac{I_R}{I} \quad , \quad \theta = \tan^{-1} \frac{I_c}{I_R} \quad , \quad \theta = \sin^{-1} \frac{I_c}{I}$$

## Paraiiel RC Circuit



$$I_R = \frac{V}{R} \quad , \quad I_L = \frac{V}{X_L} \quad , \quad I_c = \frac{V}{X_c}$$

$$I^2 = I_R^2 + (I_L - I_c)^2 \quad , \quad I = \sqrt{I_R^2 + (I_L - I_c)^2}$$

$$I = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2} = V \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

$$\frac{V}{I} = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

**Impedance**

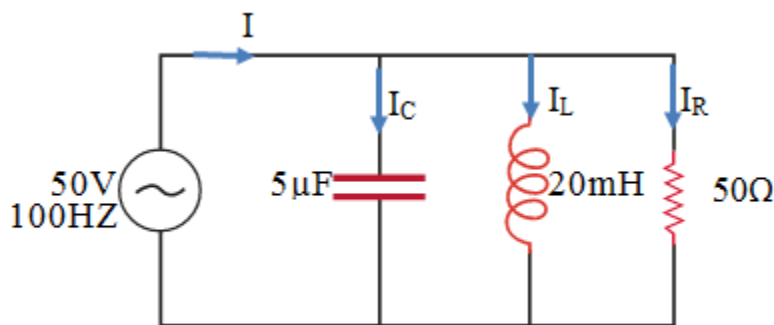
$$Z = \frac{V}{I} = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

**Admittance ( Y ) :** Admittance is the reciprocal of impedance, Z and is given the symbol Y. In AC circuits admittance is defined as the ease at which a circuit composed of resistances and reactances allows current to flow when a voltage is applied taking into account the phase difference between the voltage and the current.  $Y = \frac{1}{Z} \text{ (S)}$

$$Y = \frac{I}{V} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

$$\theta = \cos^{-1} \frac{I_R}{I} \quad , \quad \theta = \tan^{-1} \frac{I_L - I_C}{I_R} \quad , \quad \theta = \sin^{-1} \frac{I_L - I_C}{I}$$

**Example:** A 50Ω resistor, a 20mH coil and a 5μF capacitor are all connected in parallel across a 50V, 100Hz supply. Calculate the total current drawn from the supply, the current for each branch, the total impedance of the circuit and the phase angle.



**Solution:**

Inductive Reactance,  $X_L$

$$X_L = 2\pi fL = 2 \times 3.14 \times 100 \times 20 \times 10^{-3} = 12.6\Omega$$

Capacitive Reactance,  $X_C$ .  $X_C = \frac{1}{2 \times \pi \times F \times C} = \frac{1}{2 \times \pi \times 0.05 \times 10^{-6}} = 318.3 \Omega$

Impedance,  $Z$ .  $Z = \frac{1}{\sqrt{\left(\frac{1}{50}\right)^2 + \left(\frac{1}{12.6} - \frac{1}{318.3}\right)^2}} = 12.7 \Omega$

Current through resistance,  $R$  ( $I_R$ ):  $I_R = \frac{V}{R} = \frac{50}{50} = 1A$

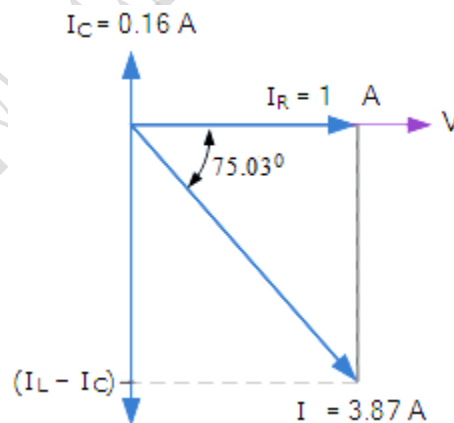
Current through inductor,  $L$  ( $I_L$ ):  $I_L = \frac{V}{X_L} = \frac{50}{12.5} = 3.9A$

Current through capacitor,  $C$  ( $I_C$ ):  $I_C = \frac{V}{X_C} = \frac{50}{318.3} = 0.16A$

Total supply current ( $I$ )

$$I = \sqrt{I_R^2 + (I_L - I_C)^2} = \sqrt{1^2 + (3.9 - 0.16)^2} = 3.87A$$

$$\theta = \tan^{-1} \frac{I_L - I_C}{I_R} = \tan^{-1} \frac{3.9 - 0.16}{1} = 75.03^\circ$$



الاسبوع الخامس عشر المحاضرة الخامسة عشر

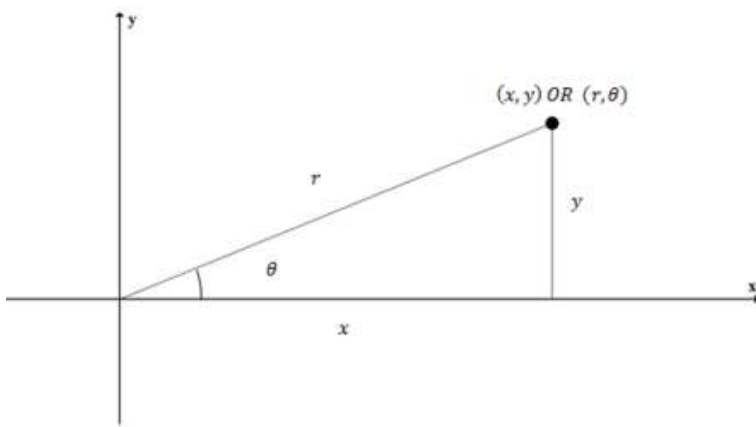
الهدف التعليمي (الهدف الخاص لكل للمحاضرة):

- ان يكون الطالب قادر على فهم  $j$  معامل Rectangular and polar forms والتحويل بينهما والعلاقات الرياضية الخاصة بهم.
- مدة المحاضرة: ٢ ساعة نظري

## j operator

The  $j$  operator in electrical circuit has the same value as the  $i$  operator in mathematics. It is an imaginary number with a numeric value of,  $j = \sqrt{-1}$ . It is used in power systems to reflect the reactive portion of electrical quantities such as impedance, voltage, current and power. For example, with a complex impedance  $Z$

### Rectangular and polar forms



Rectangula form or Complex number :  $A = x + jy$  ,  $j = \sqrt{-1}$

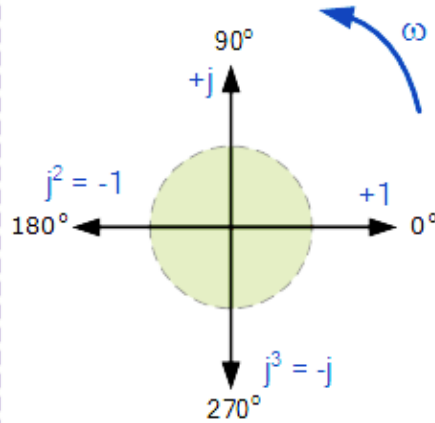


$$90^\circ \text{ rotation: } j^1 = \sqrt{-1} = +j$$

$$180^\circ \text{ rotation: } j^2 = (\sqrt{-1})^2 = -1$$

$$270^\circ \text{ rotation: } j^3 = (\sqrt{-1})^3 = -j$$

$$360^\circ \text{ rotation: } j^4 = (\sqrt{-1})^4 = +1$$

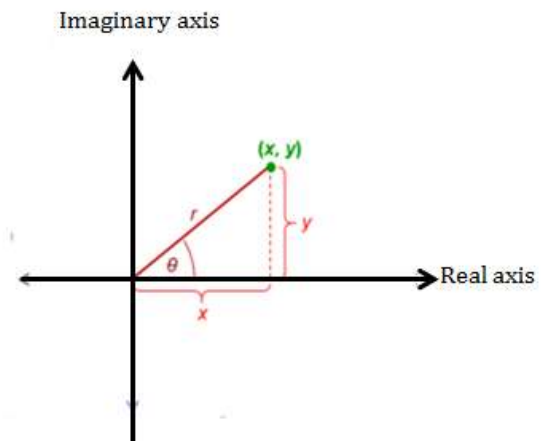


polar forms :  $B = r \angle \theta$

## Convert between Polar and Rectangular Coordinates

### Convert from Polar to Rectangular Coordinate

$$x = r \cos \theta, \quad y = r \sin \theta$$



### Convert from Rectangular to Polar Coordinate

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1} \frac{y}{x}$$

**Example:**

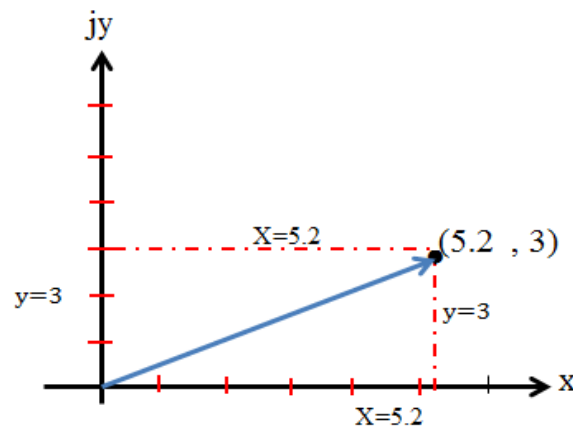
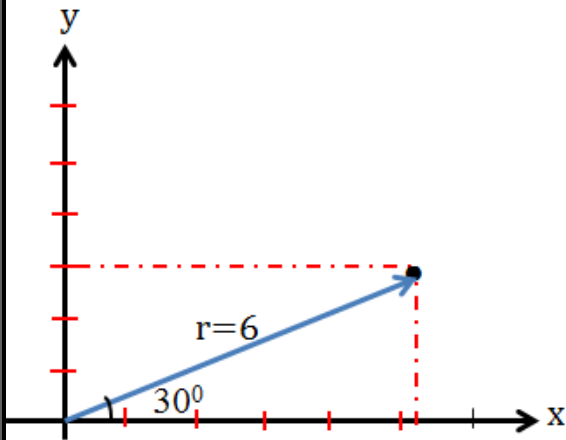
**Example :** Converting Polar Form  $(6 \angle 30^\circ)$  into Rectangular Form.

**Solution:**

$$r \angle \theta \longrightarrow x + jy$$

$$x = r \cos \theta, \quad y = r \sin \theta$$

$$\begin{aligned} 6 \angle 30^\circ &= (6 \cos \theta) + j(6 \sin \theta) \\ &= (6 \cos 30^\circ) + j(6 \sin 30^\circ) \\ &= (6 \times 0.866) + j(6 \times 0.5) \\ &= 5.2 + j3 \end{aligned}$$



**Example :Converting Rectangular (2+j3) Form into Polar Form**

$$x + jy \longrightarrow r \angle \theta$$

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1} \frac{y}{x}$$

$$r = \sqrt{5.2^2 + 3^2} = 6, \quad \theta = \tan^{-1} \frac{3}{5.2} = 30^\circ$$

$$= 6 \angle 30^\circ$$

Addition and subtraction of complex numbers are better performed in *rectangular form*. multiplication and division are better done in *polar form*. Given the complex numbers

$$A_1 = x_1 + jy_1, A_2 = x_2 + jy_2$$

**Addition:**  $A_1 + A_2 = (x_1 + x_2) + j(y_1 + y_2)$

**Subtraction:**  $A_1 - A_2 = (x_1 - x_2) + j(y_1 - y_2)$

**Multiplication :**  $A_1 \times A_2 = r_1 \times r_2 (\angle \theta_1 - \theta_2)$

**Division:**  $\frac{A_1}{A_2} = \frac{r_1}{r_2} (\angle \theta_1 - \theta_2)$

**Complex Conjugate:**  $A^* = x - jy = r \angle -\phi = r e^{-j\phi}$

**Example: Multiplying and Division**  $6 \angle 30^\circ$  and  $8 \angle -45^\circ$ .

$$A_1 \times A_2 = r_1 \times r_2 (\angle \theta_1 - \theta_2)$$

$$A_1 \times A_2 = 6 \times 8 \angle 30^\circ - 45^\circ = 48 \angle -15^\circ$$

$$\frac{A_1}{A_2} = \frac{6}{8} (\angle 30^\circ - (-45^\circ)) = 0.75 \angle 75^\circ$$

**Example: Addition and subtraction** ( $A_1 = 3 + j4$ ,  $A_2 = 5 + j2$ ).

$$A_1 + A_2 = (x_1 + x_2) + j(y_1 + y_2)$$

$$A_1 + A_2 = (3 + 5) + j(4 + 2) = 8 + j6$$

$$A_1 - A_2 = (x_1 - x_2) + j(y_1 - y_2)$$

$$A_1 - A_2 = (3 - 5) + j(4 - 2) = -2 + j2$$

a) pure resistance, then  $R = Z$  and  $Y = \frac{1}{Z} = \frac{1}{R}$

b) pure inductance, then  $Z = jX_L$  and  $Y = \frac{1}{Z} = \frac{1}{jX_L}$

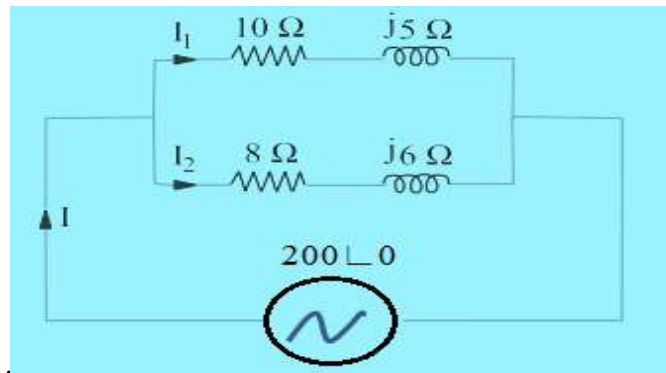
c) pure capacitance, then  $Z = -jX_C$  and  $Y = \frac{1}{Z} = \frac{1}{-jX_C}$

d) resistance and inductance in series, then  $z = R + jX_L$  and  $Y = \frac{1}{Z} = \frac{1}{R + jX_C}$

e) resistance and capacitance in series, then  $z = R - jX_L$  and  $Y = \frac{1}{Z} = \frac{1}{R - jX_C}$

f) resistance and inductance in parallel, then  $\frac{1}{Z} = \frac{1}{R} + j\frac{1}{X_L}$

**Example :** For the circuit shown in fig below Calculate the circuit current.



$$Z_1 = (10 + j5) , \quad Z_2 = (8 + j6)$$

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{Z_1 + Z_2}{Z_2 \cdot Z_1}$$

$$Y = \frac{1}{Z} = \frac{Z_1 + Z_2}{Z_2 \cdot Z_1} = \frac{(10 + j5) + (8 + j6)}{(10 + j5) \cdot (8 + j6)} = \frac{(10 + j5) + (8 + j6)}{(10 + j5) \cdot (8 + j6)}$$

$$= \frac{(10 + 8) + j(5 + 6)}{80 + j60 + j40 - 30} = \frac{(18 + j11)}{(50 + j100)}$$

$$Y = \frac{(18 + j11)(50 - j100)}{(50 + j100)(50 - j100)} = \frac{200 - j1250}{12,500} = 0.16 - j0.1$$

$$I = \frac{V}{Z} = V \cdot Y$$

$$V = 200 \angle 0^\circ = 200 + j0$$

$$I = (200 + j0) \cdot (0.16 - j0.1) = 32 - j20 \text{ A}$$

polar form

$$I = \sqrt{32^2 + 20^2} , \tan^{-1} \frac{-20}{32}$$

$$I = 37.74 \angle -32^\circ \text{ A}$$

THANK YOU