



وزارة التعليم العالي والبحث العلمي
الجامعة التقنية الجنوبية
المعهد التقني العمارة
قسم تقنيات المساحة



Cadastral Surveying

Stage: 2nd .Surveying/ Second semester

Assistant Lecturer: Athraa abbas kadhim

المفردات النظرية

الاسبوع	تفاصيل المفردات
الأول	حسابات التضليع : أنواع الزوايا والاتجاهات وطرق تصحيحها وحساباتها للمضلع الدائري المغلق والمضلع الرابط وحساب الإحداثيات لأركان المضلع وتصحيحها (بطريقة البوصلة) ، حساب الأطوال والاتجاهات المصححة (الحسابات المعكوسة للأضلاع) .
الثاني	التقاطعات أو القياسات المجهولة في عملية التضليع والتثليث وتشمل : التقاطع الأول (لإيجاد طولين مجهولين) باستخدام طريقتي المثلثات وقوانين التضليع
الثالث	باستخدام طريقتي الهندسة التحليلية ودوران الإحداثيات ، تطبيقات في تقاطعات الطرق وتقسيم الأراضي .
الرابع	التقاطع الثاني . (لإيجاد طول ضلع واتجاه ضلع آخر) باستخدام طريقة المثلثات .
الخامس	باستخدام قوانين التضليع ، الهندسة التحليلية ، تطبيقاتها في تقاطعات الطرق وتقسيم الأراضي.
السادس	التقاطع الثالث . (لإيجاد اتجاهي الضلعين المجهولين) باستخدام طريقة المثلثات .
السابع	باستخدام طريقة الهندسة التحليلية ، تطبيقاتها في تقاطعات الطرق وتقسيم الأراضي .
الثامن	أيجاد القياسات المجهولة (أطوال واتجاهات) في المضلعات الدائرية والرابطة باستخدام التقاطعات المختلفة مع الأمثلة للأنواع الأنف ذكرها.
التاسع	التقاطع الخلفي أو العكسي: لإيجاد موقع نقطة مختارة بالرصد نحو ثلاث نقاط معلومة المواقع الأفقية ولثلاث حالات مختلفة (أو محتملة) .
العاشر	كيفية أعداد جدول بالخطوات المنطقية لإيجاد القياسات المجهولة لمسائل متنوعة باستخدام التقاطعات الثلاثة والحسابات الأمامية والمعكوسة والتقاطع الخلفي .
الحادي عشر	تقسيم الأراضي : تقسيم المضلعات : تقسيم المضلع الى جزئين بواسطة خط ذي نهايتين معلومتين الموقعين . تقسيم المضلع الى جزئين بواسطة خط ذي اتجاه معلوم ويبدأ من نقطة معلومة الموقع (وبعرض معين في حالة طريق أو قناة للري) وحساب مساحات الأجزاء والمواقع الغير محسوبة ، تطبيقات عملية في تقسيم الأراضي لحالات متعددة .
الثاني عشر	تقسيم المضلع الى جزئين متساويين في المساحة بواسطة خط يبدأ من نقطة معلومة الموقع ، تقسيم المضلع الى جزئين متساويين في المساحة بواسطة خط ذي اتجاه معلوم ، تطبيقات عملية في تقسيم الأراضي لحالات متعددة عمليا.
الثالث عشر	مشروع صغير لتقسيم الأراضي الكبيرة باستخدام الحسابات والتقاطعات المختلفة وبموجب مواصفات معينة للمساحات وأبعاد الشوارع وأنصاف أقطارها
الرابع عشر	تكملة حسابات المشروع ورسم المخطط الأفقي له
الخامس عشر	رسم المقطع الطولي له ، وأجراء المناقشات حول النتائج النهائية للتقسيم قطعة الارض

Course Objective

After studying this course the student should be able to

- 1- Known the types of angles and traverses
- 2- Compute the coordinates of points
3. calculate the unknown measurement
- 4.division and subdivision of lands.

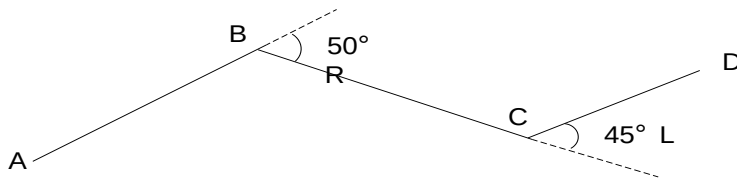
Target Audience: 2nd .Surveying student

Course Description: 2hr Theoretical ,2hr Application

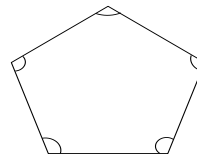
Lecture (1)

Measurement of angles

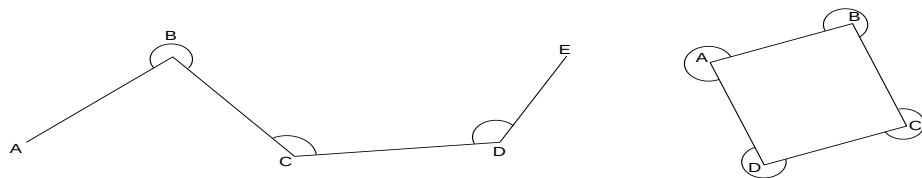
Deflection angle :- The angle between a line and prolongation of the preceding line. It is recorded as right or left.



Interior angles :- In a closed polygon, the angle inside the figure between adjacent lines.



Angles to the Right :- The angle between a line and the preceding line in a clockwise direction.



Observation of Horizontal and Vertical Angles

Angles observed in surveying are classified as either **horizontal** or **vertical**, depending on the plane in which they are measured. Horizontal angles are the basic observations needed for determining bearings and azimuths. Vertical angles are used in trigonometric leveling and for the reduction of distances to horizontal.

Horizontal angle:- is the angle formed in a horizontal plane by two intersecting vertical planes. The theodolite is an instrument used for measuring horizontal and vertical angles.

Two classes of instrument are used to measure horizontal angles. These are :-

- a- Repeating theodolite .
- b- Direction theodolite .

Measuring a horizontal angle :- There are two methods.

1-Direction method :-

- a- Set up the theodolite at station B.
- b- Set the vernier to read 0° or take initial reading.
- c- Set the telescope to bisect station A.
- d- Loosen the upper plate and turn telescope in clockwise direction until the line of sight is set approximately on the right hand sight C .
- e- Tighten the upper clamp bisect point C exactly and take reading .which gives the angle ABC if the initially reading = zero.
- f- Change the face of the instrument and repeat the whole process. The mean of the two readings gives the value of the angle ABC .

Double Sighting:- measuring the angle once with the telescope in the direct position and once with the telescope in the reversed position .

EX./ A direction theodolite is used to measure angle at Q from P to R , the following observation taken .find the angle PQR .

Ins. station	Station bisected	Circle reading Face Left(L)	Horizontal angle	Circle reading Face Right(R)	Horizontal angle
Q	P	$10^\circ 37' 00''$	$34^\circ 38' 40''$	$190^\circ 37' 40''$	$34^\circ 38' 06''$
	R	$45^\circ 15' 40''$		$225^\circ 15' 46''$	

$$\angle PQR = 34^\circ 38' 40'' + 34^\circ 38' 06'' = 34^\circ 38' 23''$$

EX./ Compute the angle SRT from the following observation

Ins. station	Station bisected	Telescope face	Circle reading	Horizontal angle
R	S	L	00° 00' 40"	
	T	L	170° 30' 20"	170° 29' 40"
	T	R	350° 30' 00"	170° 29' 20"

$$\angle SRT = \frac{170^\circ 29' 40'' + 170^\circ 29' 20''}{2} = 170^\circ 29' 30''$$

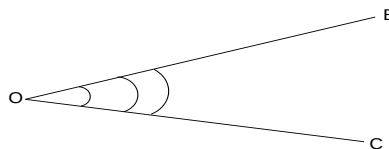
$$F.L = F.R - 180$$

2- Repetition Method:-

To improve upon the accuracy for measuring a single horizontal angle , the method of repetition is used .

For reading angle BOC :-

- 1- The instrument is set up O , make vernier zero or take initial reading .
- 2-The lower clamp is loosened , bisect station B , the adjustment of bisection is done by the lower tangent screw .
- 3- The upper clamp is loosened and the station B bisected any adjustment required in bisection is done by upper clamp , take reading .
- 4-The first station B bisected again by loosening the lower clamp , and by loosening the upper clamp dissect C .
- 5-Repeat the process for example 3 time (No. of repetitions) .
- 6-The final reading dividing by 3 (No. of repetitions) gives the angle .
- 7-The same process is repeated on the other face .
- 8-Mean of two faces is taken as correct horizontal angle .



Examples

Ex. / An angle is measured by repetition . The initial mean reading is $32^{\circ} 12' 20''$. After first repetition the reading is $49^{\circ} 13' 40''$, after sixth repetition the reading is $134^{\circ} 19' 20''$. Compute the value of the angle .

Sol /

The approximate angle $= 49^{\circ} 13' 40'' - 32^{\circ} 12' 20'' = 17^{\circ} 01' 20''$

The angle after repetition $= \frac{\text{last reading} - \text{initial reading}}{\text{No. of repetitions}}$

$$= \frac{134^{\circ} 19' 20'' - 32^{\circ} 12' 20''}{6} = 17^{\circ} 01' 10''$$

Ex. / Compute the value of angle if initial reading $= 136^{\circ} 01' 50''$ and the first reading $= 216^{\circ} 21' 00''$, reading after 6 repetition $= 257^{\circ} 57' 50''$.

(Ans. $= 80^{\circ} 19' 20''$)

Sol. /

The approximate angle $= 216^{\circ} 21' 00'' - 136^{\circ} 01' 50'' = 80^{\circ} 19' 10''$

The approximate reading $= 80^{\circ} 19' 10'' \times 6 = 481^{\circ} 55' 00''$

$\therefore$ Last reading $= 257^{\circ} 57' 50'' + 360^{\circ} = 617^{\circ} 57' 50''$

$$\text{H. angle} = \frac{617^{\circ} 57' 50'' - 136^{\circ} 01' 50''}{6} = 80^{\circ} 19' 20''$$

Ex. / Compute the horizontal angle .

Station Inst.	Station bisected	No. of Rep.	face	Circle reading
F	E	0	L	70° 10' 30"
	G	1	L	170° 30' 30"
	G	6	L	312° 11' 00"
	G	6	R	312° 12' 00"

Sol. /

The approximate angle = $170^\circ 30' 30'' - 70^\circ 10' 30'' = 100^\circ 20' 00''$

The approximate reading = $100^\circ 20' 00'' \times 6 = 602^\circ$

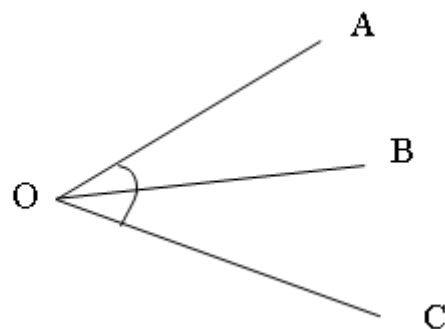
$\therefore$ Last reading = $312^\circ 11' 00'' + 360^\circ = 672^\circ 11' 00''$

$$\text{L.Angle} = \frac{672^\circ 11' 00'' - 70^\circ 10' 30''}{6} = 100^\circ 20' 05''$$

$$\text{R.Angle} = \frac{672^\circ 12' 00'' - 70^\circ 10' 30''}{6} = 100^\circ 20' 15''$$

$$\text{Mean angle} = \frac{F.L + F.R}{2} = 100^\circ 20' 10''$$

H.W. / Compute the angles AOB , BOC from the following observation at O.



Vertical Angle

A vertical angle is an angle measured in a vertical plane from horizontal line upward or downward to give negative or positive value .

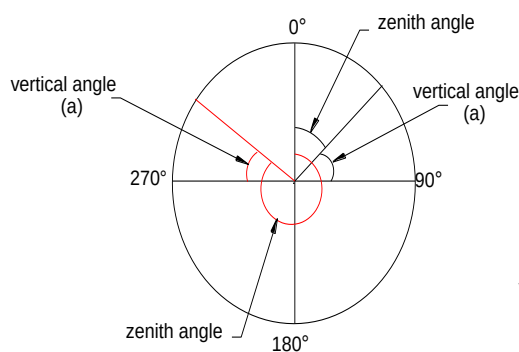
Sometimes referred to elevation or depression .

vertical angle lies between 0° and $\pm 90^\circ$.

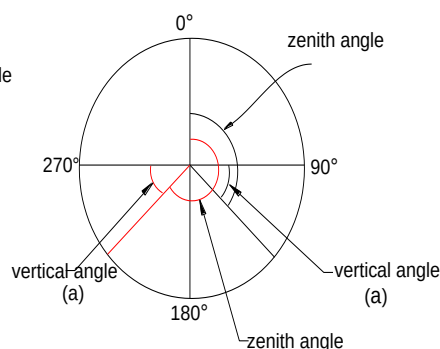
Zenith angle :- is an angle measured in a vertical plane downward from an upward direction vertical line through the instrument .it is thus between 0° and 180°

$$\alpha = 90^\circ - Z(F.L)$$

$$\alpha = Z(F.R) - 270^\circ$$



a bove horizontal



Below horizontal

Ex./ A vertical angle is measured to target on top of hill with telescope in direct position (F.L) , the circle reads ($67^\circ 23' 50''$), with the telescope in the reversed (F.R) position the circle reads ($292^\circ 36' 16''$) compute the vertical angle .

$$\alpha = 90^\circ - F.L(F.R - 270^\circ)$$

$$\alpha = 90^\circ - 67^\circ 23' 50'' = +22^\circ 36' 10''$$

Sol/ $\alpha = 292^\circ 36' 16'' - 270^\circ = +22^\circ 36' 16''$

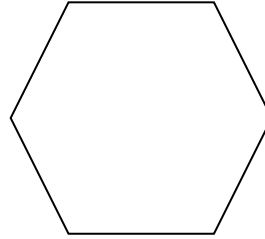
$$\alpha = \frac{22^\circ 36' 10'' + 22^\circ 36' 16''}{2} = 22^\circ 36' 13''$$

Traversing

Traverse :- A polygon consisting of series of straight lines related to one another by known angles .

The traverse may be classified as :-

1- Closed traverse :- if the last point closes on the starting point .



2- Open traverse :- if the last point terminates at any point which except the starting .

Angles of the traverse may be measured by observing :-

1-Interior angles.

2-Deflection angles.

3-Angles to the right.

Balancing a traverse

1- Interior angles الزوايا الداخلية

The sum of the included angles $= (n-2) \times 180$

n = number of the angles .

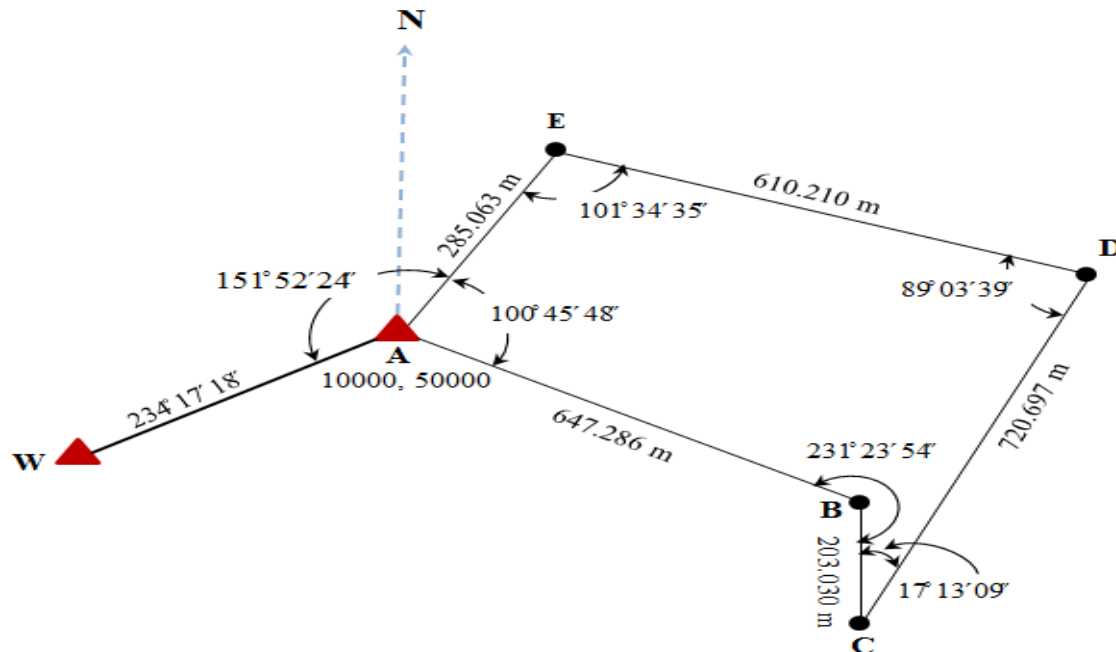
The error should not exceed $= (3-5)a\sqrt{n}$ use $3a\sqrt{n}$ (allowable error) الخطأ المسموح

a = least count of instrument قراءة للجهاز أقل .

(إذا كان الخطأ أقل من الخطأ المسموح به فيوزع على الزوايا, أما إذا كان الخطأ أكثر من المسموح به فيعاد

العمل).

EX: Compute the coordinates points for the following traverse if,
 Az.AW=234° 17' 18", XA=10000, YA=5000



Solution:

$$\Sigma \text{ Theoretical Interior Angles} = (n - 2) * 180^\circ$$

$$\Sigma \text{ Theoretical Interior Angles} = (5 - 2) * 180^\circ = 540^\circ 00' 00''$$

$$\Sigma \text{ measured angles} = 540^\circ 01' 05''$$

$$e = 540^\circ 00' 00'' - 540^\circ 01' 05'' = 00^\circ 01' 05''$$

$$c = \pm k \cdot \sqrt{n}$$

$$c = \pm 00^\circ 00' 30'' \sqrt{5}$$

$$c = \pm 00^\circ 01' 07''$$

صواب حتى اثنين مئتي نظرياً.

$$\text{Correction / angle} = \frac{\text{T.C. for Interior angles}}{n}$$

$$\text{Correction / angle} = \frac{00^\circ 01' 05''}{5} = 00^\circ 00' 13''$$

Point	Int. angle	Corr.	Corrected Int. angle
A	100° 45' 48"	-13"	100° 45' 35"
B	231° 23' 54"	-13"	231° 23' 41"
C	17° 13' 09"	-13"	17° 12' 56"
D	89° 03' 39"	-13"	89° 03' 26"
E	101° 34' 35"	-13"	101° 34' 22"
Σ	540° 01' 05"	-1' 05"	540° 00' 00"

Directions

The directions of the lines:- are fixed by measuring the angle between the lines and fixed line of reference .

This angle is called ((bearing)) of the line .

The reference line is called ((Meridian)) and it may be one of the following :-

a- True Meridian:- The line connected the earth's poles it is fixed .

الاتجاه الحقيقي :- هو الخط المار بالقطبين الجغرافيين الشمالي والجنوبي للكرة الأرضية ويسمى أحيانا بالهجير الجغرافي وهو ثابت لا يتغير ويعين بواسطة الارصاد الفلكية .

b- Magnetic Meridian:- is the direction which is indicated by freely suspended magnetic needle at specific time .It varies from time to time .

c- Assumed Meridian:- Any line of survey may be assumed to be meridian .

d- Grid Meridian:- A line through one point of survey has been adopted as areference meridian .

e- Azimuths, and Bearings

Determining the locations of points and orientations of lines frequently depends on the observation of angles and directions. In surveying, directions are given by **azimuths** and **bearings**

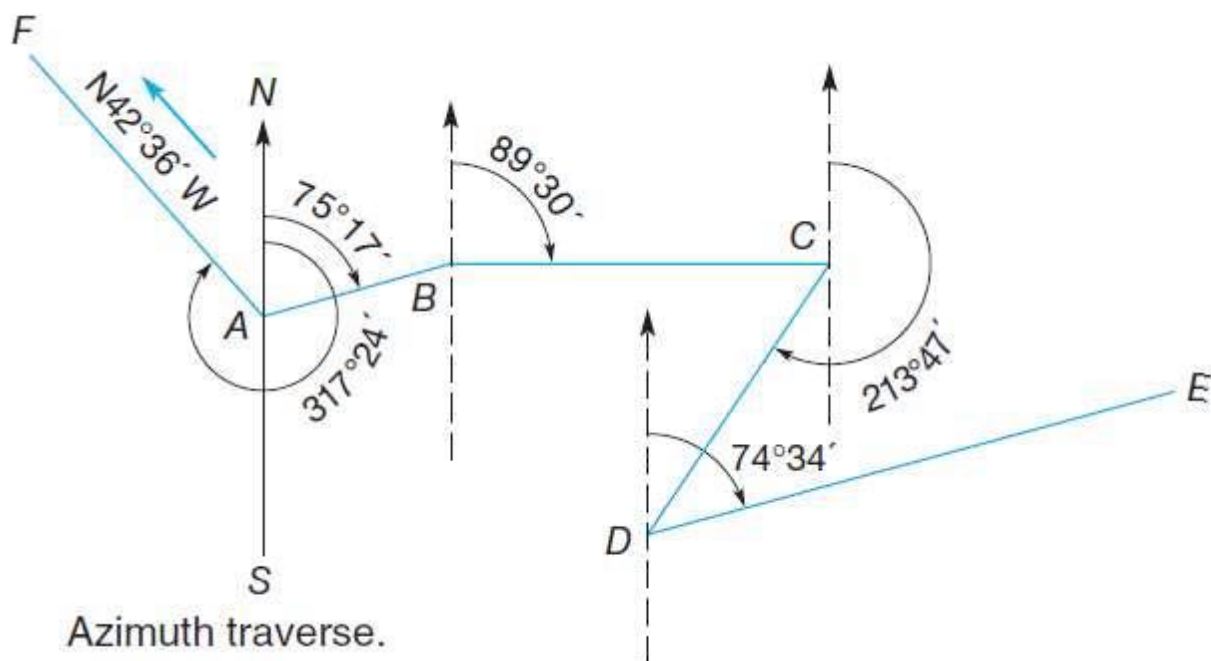
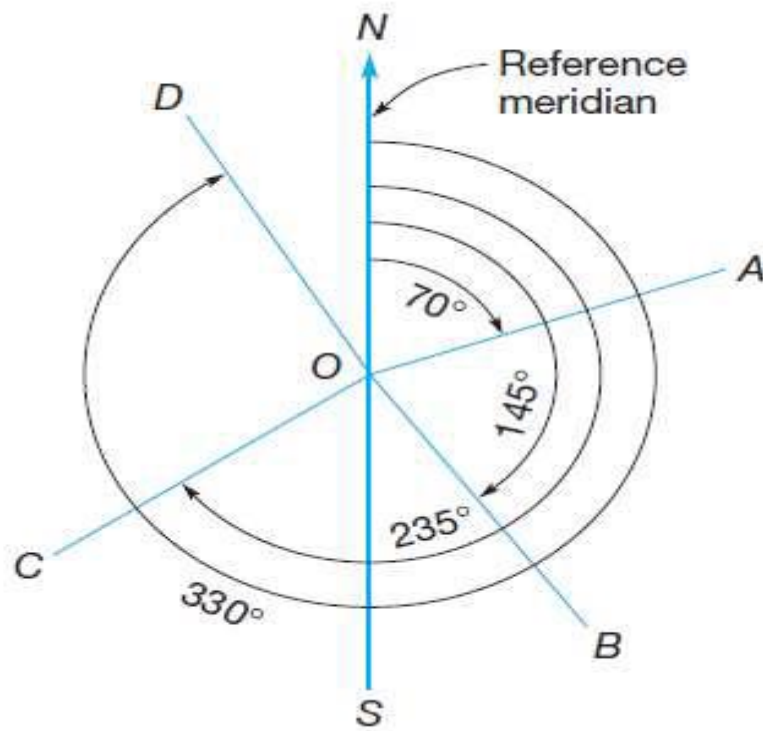
Systems of Directions (bearing) .

bearing are designated by the following system:-

1- Whole circle Bearing (W.C.B) or Azimuth :-

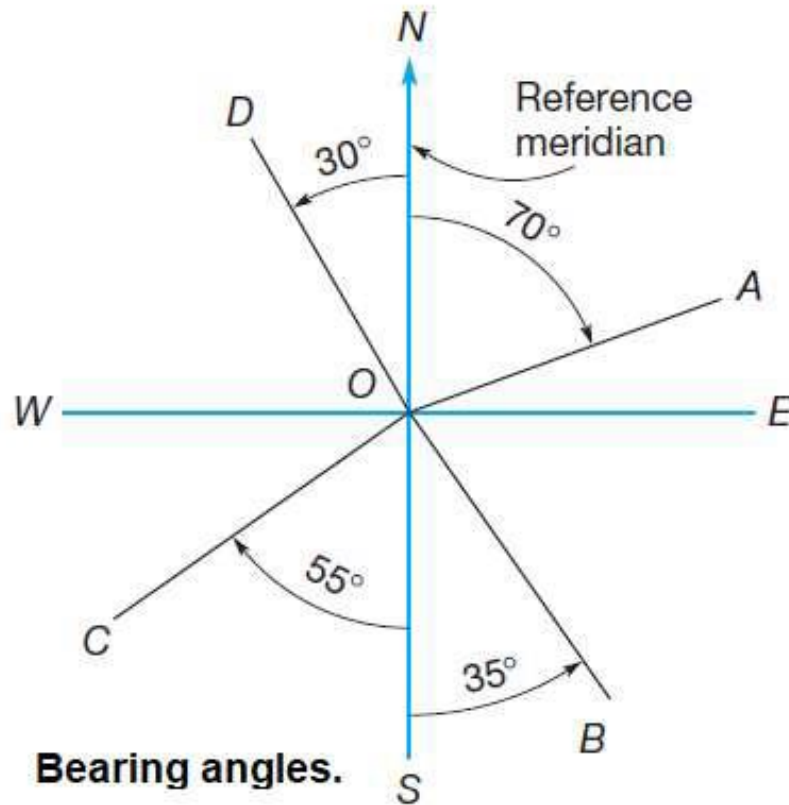
Azimuths

Azimuths are horizontal angles observed clockwise from any reference meridian. In plane surveying, azimuths are generally observed from **north** as shown in figure below, but astronomers and the military have used south as the reference direction



Bearings

Bearings are another system for designating directions of lines. *The bearing of a line is defined as the acute horizontal angle between a reference meridian and the line.* The angle is observed from either the north or south toward the east or west, to give a reading smaller than 90° . The letter N or S preceding the angle, and E or W following it shows the proper quadrant. Thus, a properly expressed bearing includes quadrant letters and an angular value.



Directions computation

Side	Azimuth computation equation	Azimuth
AB	$\angle WAE - (360 - \text{Az. AW}) + \angle EAB$	$126^{\circ}55'17''$
BC	$\angle ABC - (360 - \text{Az. BA})$	$178^{\circ}18'58''$
CD	$\angle BCD - (360 - \text{Az. CB})$	$15^{\circ}31'54''$
DE	$\text{Az. DC} + \angle CDE$	$284^{\circ}35'20''$
EA	$\text{Az. ED} + \angle DEA$	$206^{\circ}09'42''$
Checking AB	$\text{Az. AE} + \angle EAB$	$126^{\circ}55'17''$

المركبة الأفقية للضلع

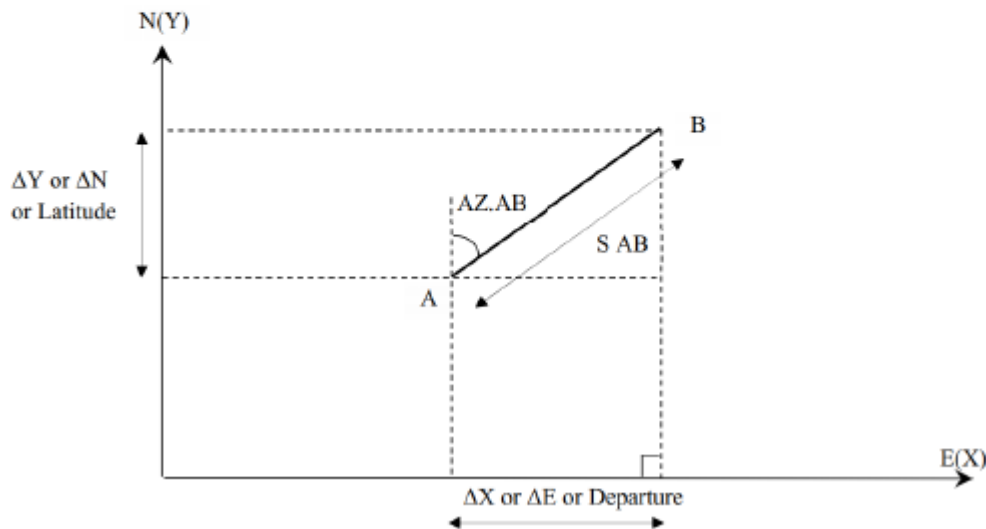
Dep. or ΔE = Length of side * sin (Az. of side)

$$\Delta E = S * \sin Az.$$

المركبة الرأسية للضلع

Lat. or ΔN = Length of side * cos (Az. of side)

$$\Delta N = S * \cos Az.$$



Side	Length(m)	Direction	Dep.(Δ Easting)	Lat. (ΔNorthing)
AB	647.286	126°55'17"	517.480	-388.837
BC	203.030	178°18'58"	5.966	-202.942
CD	720.697	15°31'54"	192.982	694.379
DE	610.210	284°35'20"	-590.536	153.701
EA	285.063	206°09'42"	-125.686	-255.895
Σ	2466.286		ΣΔE= 0.206	ΣΔN=0.442

خطأ القفل المضلع Linear misclosure

يعرف خط غلق المضلع بأنه طول الخط الذي يغلق المضلع من الناحية النظرية والذي يساوي حسابيا محصلة التصحيح الكلي.

$$\begin{aligned}
 \text{Linear misclosure} &= \sqrt{\Sigma \Delta E^2 + \Sigma \Delta N^2} \\
 &= \sqrt{0.206^2 + 0.442^2} \\
 &= 0.487 \text{ m.}
 \end{aligned}$$

$$\text{Correction for Departure} = - \frac{\text{Total Departure misclosure}}{\text{Total length Traverser}} \times \text{Length of side}$$

$$\text{Correction for Latitude} = - \frac{\text{Total Latitude misclosure}}{\text{Total length Traverser}} \times \text{Length of side}$$

$$\text{Correction for Departure} = - \left(\frac{0.206}{2466.286} \right) \times 647.286$$

$$\text{Correction for Departure} = - 0.054 \text{ m.}$$

$$\text{Correction for Latitude} = - \left(\frac{0.442}{2466.286} \right) \times 647.286$$

$$\text{Correction for Latitude} = - 0.116 \text{ m.}$$

Side	Length (m)	Computed (m)		Correction (m)		Corrected (m)	
		Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
AB	647.286	517.480	-388.837	-0.054	-0.116	517.426	-388.953
BC	203.030	5.966	-202.942	-0.017	-0.036	5.949	-202.978
CD	720.697	192.982	694.379	-0.060	-0.129	192.922	694.250
DE	610.210	-590.536	153.701	-0.051	-0.109	-590.587	153.692
EA	285.063	-125.686	-255.895	-0.024	-0.051	-125.710	-255.910
Σ	2466.286	0.206	0.442	-0.206	-0.442	0.000	0.000

Coordinates Computation

١١-٢٠-٢٠
لحساب احداثي التثريق

$$X_B = X_A + \text{Dep.}_{AB} (\Delta X_{AB})$$

Or

$$E_B = E_A + \text{Dep.}_{AB} (\Delta E_{AB})$$

$$E_B = E_A + S_{AB} * \sin (Az_{AB})$$

اما لحساب احداثي التشميل

$$Y_B = Y_A + \text{Lap.}_{AB} (\Delta Y_{AB})$$

Or

$$N_B = N_A + \text{Lap.}_{AB} (\Delta N_{AB})$$

$$N_B = N_A + S_{AB} * \cos (Az_{AB})$$

وبالتالي يمكن الحصول على جميع الاحداثيات باتتباع الاسلوب ذاته، ففي المضلعات المغلقة polygon يتم حساب احداثيات المحطة الاولى مرة اخرى بالاعتماد على المحطة الاخيرة في المضلع لغرض التحقق من حسابات المضلع.

Point	X (m)	Y (m)
A	10000	5000
B	10517.426	4611.047
C	10523.375	4408.069
D	10716.297	5102.319
E	10125.710	5255.011
A'	10000	5000.1

2-Exterior angles

EX./ Adjusted the following traverse's angles

Station	Angles to the right	correction	Adjusted angles
A	254° 30' 00"	+15'	254° 45' 00"
B	248° 15' 00"	+15'	248° 30' 00"
C	276° 30' 00"	+15'	276° 45' 00"
D	299° 45' 00"	+15'	300° 00' 00"
SUM measured	1079° 00' 00"	+ 1°	1080° 00' 00"

The sum of angle to the right should be $= (n+2)180$

The sum $= (4+2)180 = 1080^\circ$

Error = $\sum \text{theoretical} - \sum \text{measured}$

Error $= 1080^\circ 00' 00'' - 1079^\circ 00' 00'' = +1^\circ$

Total correction $= +1^\circ$

Correction angle $= \frac{+1^\circ}{4} = \frac{60'}{4} = +15'$

3-Deflection angles

The sum of right deflection angle - The sum of left deflection angle = 360°

$$\sum R - \sum L = 360^\circ$$

EX./ The following deflection angles were measured in a traverse that beginning and closes at station A what are the adjusted deflection angles .

Station	Deflection angles	correction	Adjusted angles
A	113° 39' 00" L	- 30"	113° 38' 30" L
B	98° 15' 30" L	- 30"	98° 15' 00" L
C	88° 19' 30" L	- 30"	88° 19' 00" L
D	117° 43' 00" L	- 30"	117° 42' 30" L
E	57° 54' 30" R	+ 30"	57° 55' 00" R
SUM			$\sum L = 417^\circ 55' 00''$ $\sum R = 57^\circ 55' 00''$ $= 360^\circ 00' 00''$

Angles to the left = $417^\circ 55' 00''$ L

Angles to the right = $57^\circ 55' 00''$ R

$$= 57^\circ 55' 00'' - 417^\circ 55' 00'' = - 360^\circ 02' 30'' \text{ (تعمل الإشارة السالبة)}$$

$$\text{Error} = 360^\circ 02' 30'' - 360^\circ = + 02' 30''$$

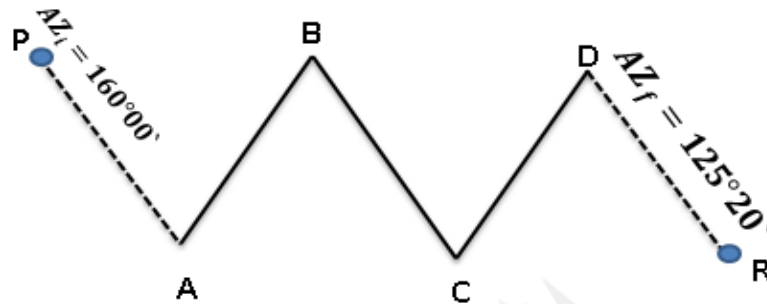
$$\text{Correction angles} = \frac{2' 30''}{5} = 30''$$

H.W: In the closed traverse ABCD. compute the Azimuth for each side $AZ_{AB} = 31^{\circ}33'$

point	Deflection angle
A	$+143^{\circ}16' R$
B	$-51^{\circ}16' L$
C	$+150^{\circ}34' R$
D	$+116^{\circ}52' R$

2- connected traverse (Link)

EX:



Sol:

point	Angle to the Right	correction	Cor.angle
A	$82^{\circ}34'$	$-00^{\circ}04'$	$82^{\circ}30'$
B	$237^{\circ}14'$	$-00^{\circ}04'$	$237^{\circ}10'$
C	$119^{\circ}44'$	$-00^{\circ}04'$	$119^{\circ}40'$
D	$246^{\circ}04'$	$-00^{\circ}04'$	$246^{\circ}00'$
$\Sigma = 685^{\circ}36'$			$685^{\circ}20'$

$$\Sigma theory = AZ_F = AZ_i + \Sigma Angles(B) - (n * 180^\circ)$$

$$AZ_F = 160^\circ 00' + \Sigma 685^\circ 36' - (4 * 180^\circ)$$

$$AZ_F = 125^\circ 36'$$

نطرح من الازيموث المعطى لاستخراج مقدار التصحيح

$$Error = AZ_F - AZ_F = \gg 125^\circ 36' - 125^\circ 20' = 00^\circ 16'$$

$$T.C = -00^\circ 16'$$

$$Correction = \frac{T.C}{n} = \frac{-00^\circ 16'}{4} = -00^\circ 04'$$

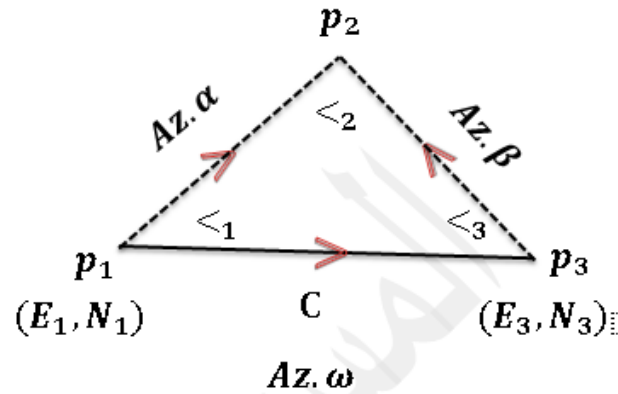
$$AZ_F = 160^\circ 00' + 685^\circ 36' - 720^\circ 00' = 125^\circ 20'$$

Lecture (2) and(3)

Intersection or Omitted measurements

التقاطعات أو القياسات المجهولة

Intersection: two lengths side are unknown (طول الضلعين المجهولان) التقاطع الاول



Trigonometric method:

1. compute, $Az.\omega$, C

$$R.B = \tan^{-1} \frac{\Delta E}{\Delta N}, R.B \rightarrow Az.\omega$$

$$C = \sqrt{(\Delta E)^2 + (\Delta N)^2}$$

2- Comput angles: < 1 , < 2 , < 3

$$< 1 = Az.\omega - Az.\alpha$$

$$< 2 = B.Az.\alpha - Az.\beta$$

$$< 3 = Az.\omega - Az.\beta \text{ or } < 3 = 180^\circ - (< 1 + < 2)$$

2- Comput A and B by using(sins Law)

$$A = \frac{c * \sin < 3}{\sin < 2}, \quad B = \frac{c * \sin < 1}{\sin < 2}$$

2- traverse Equation method

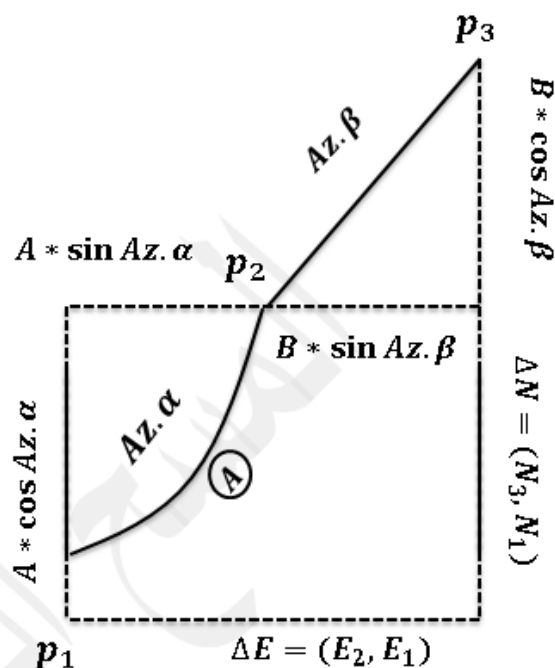
$$\sum Dep = \Delta E = (E_2 - E_1)$$

$$\sum lat = \Delta N = (N_2 - N_1)$$

or

$$\Delta E = A * \sin Az. \alpha + B * \sin Az. \beta$$

$$\Delta E = A * \cos Az. \alpha + B * \cos Az. \beta$$



(p_1, p_3)

$$A = \frac{\Delta E * \cos \beta - \Delta N * \sin \beta}{\sin AZ.(\alpha - \beta)}, \quad B = \frac{\Delta E * \cos \alpha - \Delta N * \sin \alpha}{\sin AZ.(\alpha - \beta)}$$

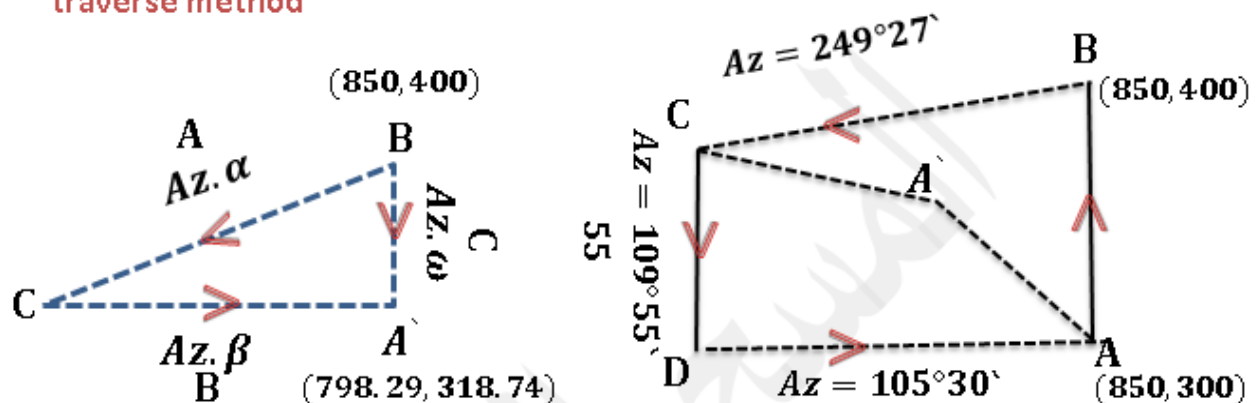
$$EP_2 = EP_1 \pm (A * \sin AZ. \alpha)$$

$$NP_2 = NP_1 + (A * \cos AZ. \alpha)$$

$$EP_3 = EP_2 + (B * \sin AZ. \beta)$$

$$NP_3 = NP_2 + (B * \cos AZ. \beta)$$

EX: Compute the coordinates of the traverse A, B, C, D if A=(850,300), B=(850,400) by using trigonometric method and check solution by using traverse method



$$\Delta E = 850(55 \sin 249^\circ 27') = 798.29$$

$$\Delta N = 300(55 \cos 249^\circ 27') = 318.74$$

$$R.B = \tan^{-1} \left| \frac{\Delta E}{\Delta N} \right| = \tan^{-1} \frac{798.29 - 850}{318.74 - 400} = S32^\circ 28' 15'' W$$

$$AZ.W = 180^\circ + 32^\circ 28' 15'' = 212^\circ 48' 15''$$

$$C = \sqrt{(\Delta E)^2 + (\Delta N)^2} = \Rightarrow \sqrt{(798.29 - 850)^2 + (318.74 - 400)^2} = 96.32 \text{ m}$$

2- Comput $\angle 1, \angle 2, \angle 3$

$$\angle 1 = Az, \alpha - Az, w = 249^\circ 27' - 212^\circ 48' 15'' = 36^\circ 58' 45''$$

$$\angle 2 = Az, \beta - B.Az, \alpha = 105^\circ 30' - (249^\circ 27' - 180^\circ) = 36^\circ 03' 00''$$

$$\angle 3 = 180^\circ - (\angle 1 + \angle 2) = 180^\circ - (36^\circ 58' 45'' + 36^\circ 03' 00'') = 106^\circ 58' 15''$$

total angle: $\angle 1, \angle 2, \angle 3$

$$36^\circ 58' 45'' + 36^\circ 03' 00'' + 106^\circ 58' 15'' = 180^\circ$$

$$\frac{A}{\sin \angle 3} = \frac{c}{\sin \angle 2} \Rightarrow A = \frac{c * \sin \angle 3}{\sin \angle 2}$$

$$A = \frac{c * \sin \angle 3}{\sin \angle 2} = \frac{96.32 * \sin 106^\circ 58' 15''}{\sin 36^\circ 03' 00''} = 156.55 \text{ m}$$

$$\frac{B}{\sin \angle 1} = \frac{c}{\sin \angle 2} \Rightarrow B = \frac{c * \sin \angle 1}{\sin \angle 2}$$

$$B = \frac{c * \sin \angle 1}{\sin \angle 2} = \frac{96.32 * \sin 36^\circ 58' 45''}{\sin 36^\circ 03' 00''} = 98.45 \text{ m}$$

- . . .

Coords. Of p_2

$$EP_2 = EP_1 \pm (A * \sin AZ. \alpha) \\ = 850 \pm (156.55 * \sin 249^\circ 27') = 798.29$$

$$NP_2 = NP_1 \pm (A * \cos AZ. \alpha) \\ = 120 \pm (156.55 * \sin 249^\circ 27') = 318.74$$

$$\therefore P_2, \quad E = 798.29, N = 318.74$$

Check:

$$EP_3 = EP_2 + (B * \sin AZ. \beta) \\ = 798.29 + (98.45 * \sin 105^\circ 30') = 798.28$$

$$NP_3 = NP_2 + (B * \cos AZ. \beta) \\ = 318.74 + (98.45 * \cos 105^\circ 30') = 318.74$$

2- check by using traverse method

$$\Delta E = 798.29 - 850 = -51.71$$

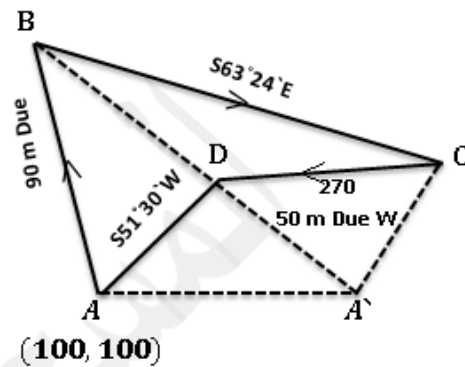
$$\Delta N = 318.74 - 400 = -81.26$$

$$A = \frac{\Delta E * \cos \beta - \Delta N * \sin \beta}{\sin AZ.(\alpha - \beta)}, \quad B = \frac{\Delta E * \cos \alpha - \Delta N * \sin \alpha}{\sin AZ.(\alpha - \beta)}$$

$$A = \frac{\Delta E * \cos \beta - \Delta N * \sin \beta}{\sin AZ.(\alpha - \beta)} \\ = \frac{(-51.71) * \cos 105^\circ 30' - (-81.26) * \sin 105^\circ 30'}{\sin AZ.(249^\circ 27' - 105^\circ 30')} = 156.54 \text{ m}$$

$$B = \frac{\Delta E * \cos \alpha - \Delta N * \sin \alpha}{\sin AZ.(\alpha - \beta)} \\ = \frac{(-51.71) * \cos 249^\circ 27' - (-81.26) * \sin 249^\circ 27'}{\sin AZ.(249^\circ 27' - 105^\circ 30')} = 98.45 \text{ m}$$

EX: compute the missing Lengths and coords. of travers shown below



$$E_B = 100, E_A = 150$$

$$N_B = 190, N_A = 100$$

$$R.B = \tan^{-1} \left| \frac{\Delta E}{\Delta N} \right| = \tan^{-1} \left| \frac{150 - 100}{100 - 190} \right| = 29^\circ 03'$$

$$AZ.W = 180^\circ - 29^\circ 03' = 150^\circ 57'$$

$$C = \sqrt{(\Delta E)^2 + (\Delta N)^2} \Rightarrow \sqrt{(150 - 100)^2 + (100 - 190)^2} = 102.90 \text{ m}$$

2- Comput < 1, < 2, < 3

$$< 1 = Az.w - Az. \alpha = 150^\circ 57' - (180^\circ - 63^\circ 24') = 34^\circ 21'$$

$$< 2 = B.Az. \alpha - Az. \beta = (360^\circ - 63^\circ 24') - (180^\circ - 51^\circ 30') = 65^\circ 06'$$

$$< 3 = 360^\circ - (B.Az.w - Az. \beta) = 360^\circ - (180^\circ + 150^\circ 57' - 51^\circ 30') = 80^\circ 33'$$

$$A = \frac{c * \sin < 3}{\sin < 2} = \frac{102.90 * \sin 80^\circ 33'}{\sin 65^\circ 06'} = 111.97 \text{ m}$$

$$B = \frac{c * \sin < 1}{\sin < 2} = \frac{102.90 * \sin 34^\circ 21'}{\sin 65^\circ 06'} = 64.05 \text{ m}$$

Coords. Of p_2

$$\begin{aligned} EP_2 &= EP_1 \pm (A * \sin AZ. \alpha) \\ &= 100 \pm (111.97 * \sin 116^\circ 36') = 200.12 \text{ m} \end{aligned}$$

$$\begin{aligned} NP_2 &= NP_1 \pm (A * \cos AZ. \alpha) \\ &= 190 \pm (111.97 * \cos 116^\circ 36') = 139.86 \text{ m} \end{aligned}$$

Check:

$$\begin{aligned} EP_3 &= EP_2 + (B * \sin AZ. \beta) \\ &= 187,63 + (64.05 * \sin 231^\circ 30') = 150 \text{ m} \end{aligned}$$

$$\begin{aligned} NP_3 &= NP_2 + (B * \cos AZ. \beta) \\ &= 120,30 + (64.05 * \cos 231^\circ 30') = 100 \text{ m} \end{aligned}$$

3- Analytic Geometry method

$$\tan. \alpha = \frac{E_2 - E_1}{N_2 - N_1}$$

$$\tan. \beta = \frac{E_3 - E_2}{N_3 - N_2}$$

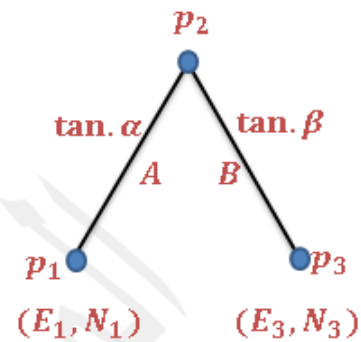
$$N_2 = \frac{E_3 - E_1 + N_1 * \tan. \alpha - N_3 * \tan. \beta}{\tan. \alpha - \tan. \beta}$$

$$\begin{aligned} E_2 &= E_1 + (N_2 - N_1) * \tan. \alpha \\ &= E_3 - (N_3 - N_2) * \tan. \beta \end{aligned}$$

Comput A and B

$$A = \frac{\Delta E}{\sin. AZ. \alpha} \Rightarrow \frac{E_2 - E_1}{\sin. AZ. \alpha}$$

$$B = \frac{\Delta N}{\cos. AZ. \beta} \Rightarrow \frac{N_3 - N_2}{\cos. AZ. \beta}$$



Solution:

1- Comput $p_2(E_2, N_2)$

2- Comput A and B

3- Comput p_2 and check on p_3

من النقوانين الاتية

$$EP_2 = EP_1 \pm (A * \sin AZ. \alpha)$$

$$NP_2 = NP_1 + (A * \cos AZ. \alpha)$$

$$EP_3 = EP_2 + (B * \sin AZ. \beta)$$

$$NP_3 = NP_2 + (B * \cos AZ. \beta)$$

4- Rotation of Coordinates Method

$\theta = \text{Rotation angle}$

$$\phi = (AZ. \beta - AZ. \alpha)$$

Comput: $\Delta E', \Delta N'$

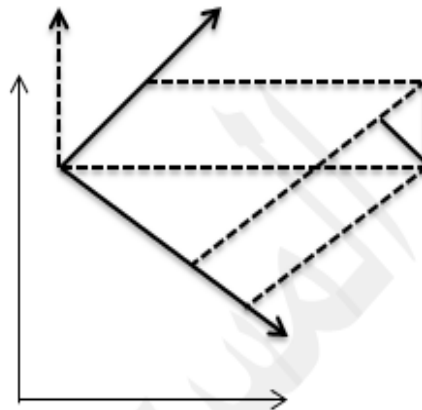
$$\Delta E' = \Delta E * \cos \theta \alpha - \Delta N * \sin \theta \beta$$

$$\Delta N' = \Delta E * \sin \theta \alpha - \Delta N * \cos \theta \beta$$

$$B = \frac{\Delta E * \cos \alpha - \Delta N * \sin \alpha}{\sin(\beta - \alpha)}$$

$$A = \Delta E * \sin \alpha - \Delta N * \cos \beta - B \cos(\beta - \alpha)$$

ملاحظة : يتم حساب (B) قبل حساب (A)



$$EP_2 = EP_1 \pm (A * \sin AZ. \alpha)$$

$$NP_2 = NP_1 + (A * \cos AZ. \alpha)$$

$$EP_3 = EP_2 + (B * \sin AZ. \beta)$$

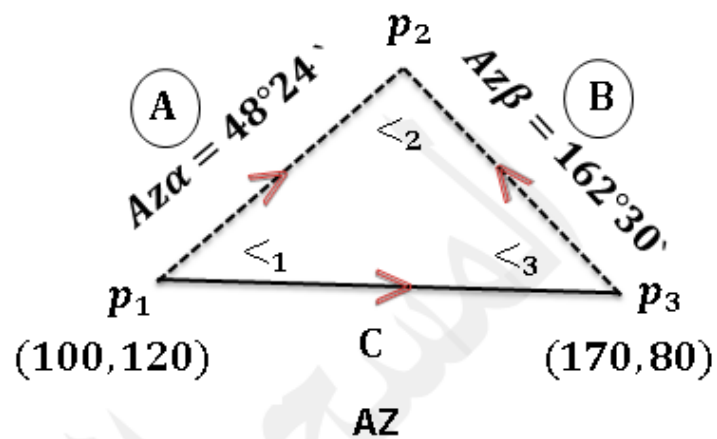
$$NP_3 = NP_2 + (B * \cos AZ. \beta)$$

Solution:

1- Comput A and B

2- Comput p_2 and check on p_3

H.W: Find the lengths (A,B) and the coords. of Point (P2) From the figure below.

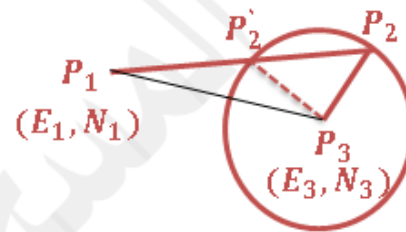


Lecture (4) and(5)

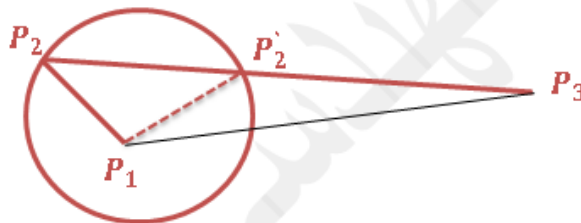
Intersection (II)

Two lengths of side and Azimuth on other are unknown

الحالة الاولى: يكون اتجاه الضلع الاول وطول الضلع الثاني معلومات



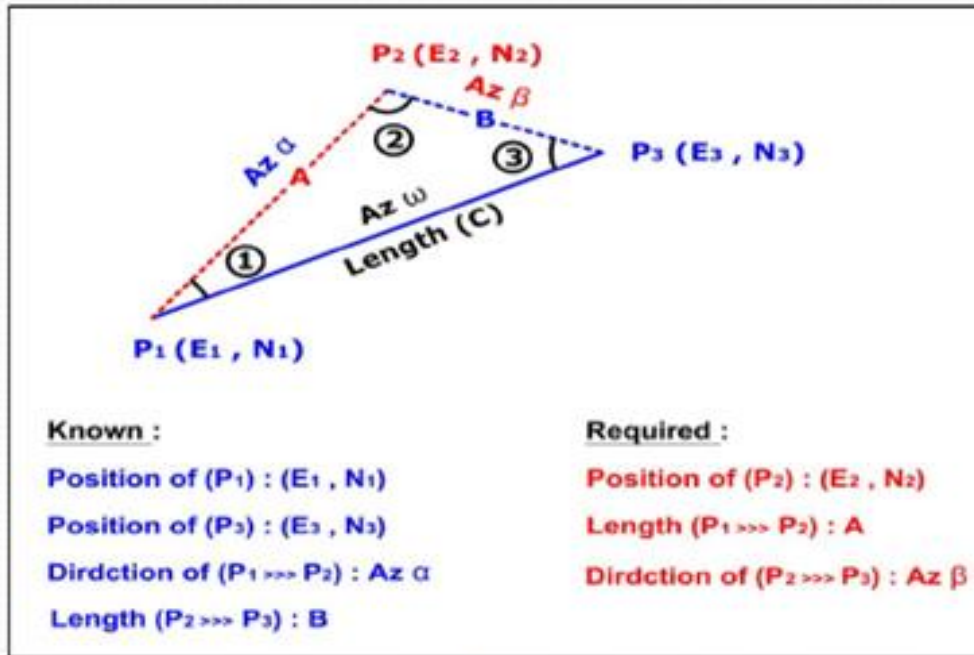
الحالة الثانية: اذا كان طول الضلع الاول واتجاه الضلع الثاني معلومات



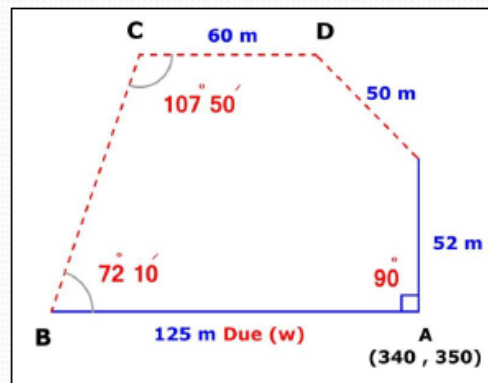
- التقاطع الثاني (II) Intersection :

يمثل هذا التقاطع هندسياً بتقاطع خط مستقيم ودائرة أحدهما معلوم الطول والثاني معلوم الاتجاه ، وبالتالي فإنه بمعلومية ميل أحدهما وطول الآخر يمكن تحديد نقطة تقاطعهما المجهولة (P_2) .

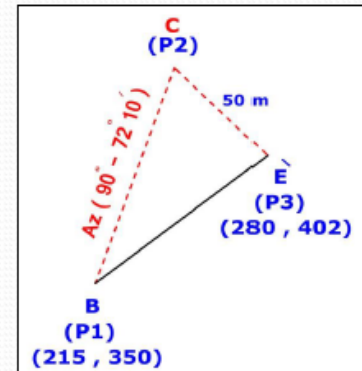
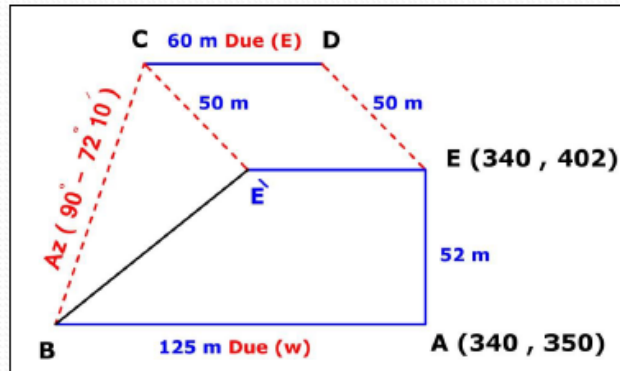
ولتبسيط هذا النوع من التقاطعات ، في حال عدم تجاوز الاضلاع الحاوية على المجاهيل (كما في الاشكال الرباعية والخماسية والسداسية .. وغيرها) يتم رسم موازيات بحسب الحالة المتوفرة بهدف تكوين حالة التقاطع الثاني ، بحيث يكون الضلعين الحاويين على تلك المجاهيل متجاوران وتشكل نقطتي بدايتهما خطاً معلوماً بالاسم (P_1 , P_3) ويتقاطعان من طرفهما الآخر في النقطة المجهولة (P_2) . وكما في الشكل التالي :



Ex : Compute the Coords of a Traverse (A,B,C,D,E) :



Sol :



(Intersections II)

$$Az_{BC} = 90^\circ - 72^\circ 10' = 17^\circ 50'$$

$$Az_{CD} = (17^\circ 50' + 180^\circ) - 107^\circ 50' = 90^\circ$$

Coords. Of (B) : (215, 350)

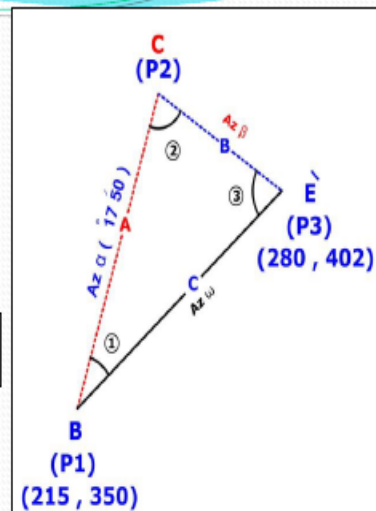
Coords. Of (E) : (340, 402)

Coords. Of (E') : (280, 402)

$$Az_{\omega} = \tan^{-1} \left(\frac{E_3 - E_1}{N_3 - N_1} \right) \rightarrow \tan^{-1} \left(\frac{280 - 215}{402 - 350} \right) = 51^\circ 20'$$

$$C = \sqrt{(280 - 215)^2 + (402 - 350)^2} = 83.24 \text{ m}$$

$$< 1 = Az_{\omega} - Az_{\alpha} \rightarrow 51^\circ 20' - 17^\circ 50' = 33^\circ 30'$$



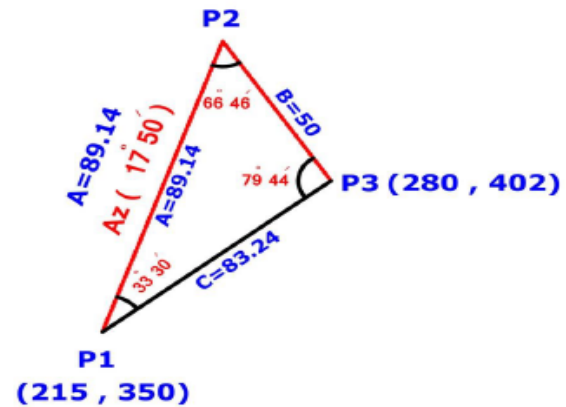
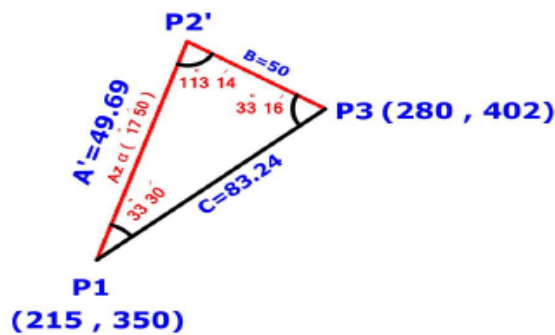
By Sin Law :

$$\frac{B}{\sin < 1} = \frac{C}{\sin < 2} \rightarrow \frac{50}{\sin 33^\circ 30'} = \frac{83.24}{\sin < 2} \rightarrow \sin < 2 = 0.92 \rightarrow < 2 = 66^\circ 46'$$

$$< 3 = 180^\circ - (< 1 + < 2) \rightarrow 180^\circ - (33^\circ 30' + 66^\circ 46') = 79^\circ 44'$$

$$\frac{50}{\sin 33^\circ 30'} = \frac{A}{\sin 79^\circ 44'} = 89.14 \text{ m}$$

$$Az_{\beta} = \text{Back } Az_{\alpha} - < 2 \rightarrow (17^\circ 50' + 180^\circ) - 66^\circ 46' = 131^\circ 04'$$



$$\angle 2' = 180^\circ - \angle 2 \rightarrow 180^\circ - 66^\circ 46' = 113^\circ 14'$$

$$\angle 3' = 180^\circ - (\angle 1 + \angle 2') \rightarrow 180^\circ - (33^\circ 30' + 113^\circ 14') = 33^\circ 16'$$

$$\frac{50}{\sin 33^\circ 30'} = \frac{A'}{\sin 33^\circ 16'} = 49.69 \text{ m}$$

$$Az \beta' = \text{Back Az } \alpha - \angle 2' \rightarrow (17^\circ 50' + 180^\circ) - 131^\circ 04' = 84^\circ 36'$$

Coords. Of P_2 :

$$E_{P_2} = E_{P_1} + A \cdot \sin Az \alpha \rightarrow 215 + 89.41 \sin 17^\circ 50' = 242.30 \text{ m}$$

$$N_{P_2} = N_{P_1} + A \cdot \cos Az \alpha \rightarrow 350 + 89.41 \cos 17^\circ 50' = 434.86 \text{ m}$$

Check P_3 From P_2 :

$$E_{P_3} = E_{P_2} + B \cdot \sin Az \beta \rightarrow 242.30 + 50 \sin 131^\circ 04' = 280 \text{ m}$$

$$N_{P_3} = N_{P_2} + B \cdot \cos Az \beta \rightarrow 434.86 + 50 \cos 131^\circ 04' = 402 \text{ m}$$

..... Checked

Coords. Of P_2' :

$$E_{P_2'} = E_{P_1} + A' \cdot \sin Az \alpha \rightarrow 215 + 49.69 \sin 17^\circ 50' = 230.22 \text{ m}$$

$$N_{P_2'} = N_{P_1} + A' \cdot \cos Az \alpha \rightarrow 350 + 49.69 \cos 17^\circ 50' = 397.30 \text{ m}$$

Check P_3 From P_2' :

$$E_{P_3} = E_{P_2'} + B \cdot \sin Az \beta' \rightarrow 230.22 + 50 \sin 84^\circ 36' = 280 \text{ m}$$

$$N_{P_3} = N_{P_2'} + B \cdot \cos Az \beta' \rightarrow 397.30 + 50 \cos 84^\circ 36' = 402 \text{ m}$$

..... Checked

Coords. Of (C) (P2) : (242.30 , 434.86)

Coords. Of (C') (P2') : (230.22 , 397.30)

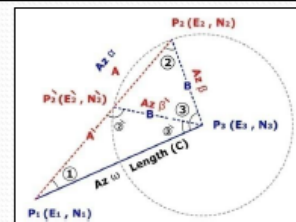
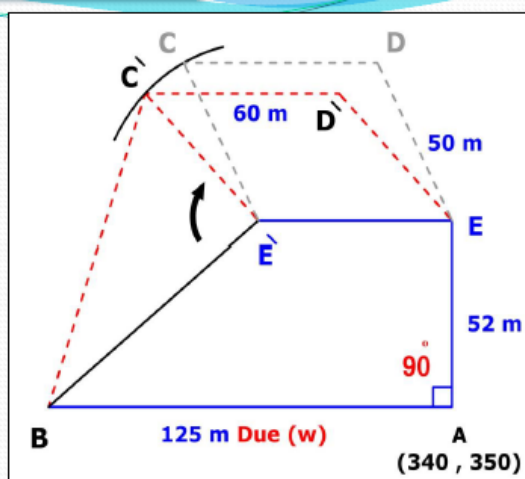
Coords. Of (D) From (C) : (302.30 , 434.86)

Coords. Of (D') From (C') : (290.22 , 397.30)

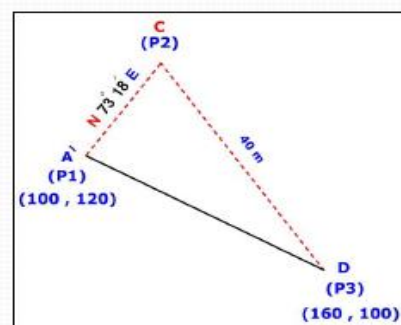
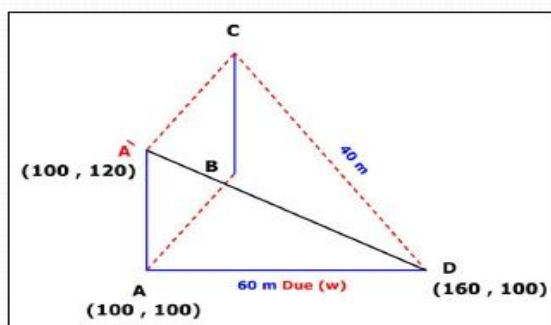
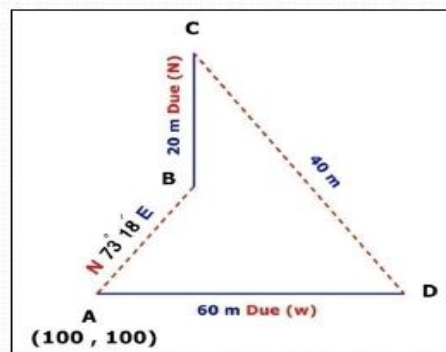
Length of DE :

$$D - E = \sqrt{(340 - 302.3)^2 + (402 - 434.86)^2} = 50.01 \text{ m}$$

$$D' - E = \sqrt{(340 - 290.22)^2 + (402 - 397.3)^2} = 50.00 \text{ m}$$



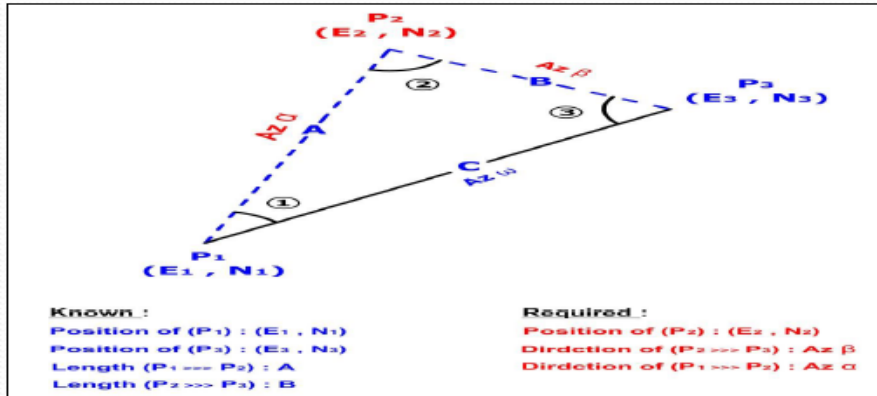
Home work : Find the Coords. of the following Traverse Points by a trigonometric method :



Lecture (6) to (8)

- التقاطع الثالث (III) Intersection :

يمثل هذا التقاطع هندسياً بتقاطع خطين مستقيمين معلومي الطول ومجهولي الاتجاه ، وبالتالي فإنه بمعلومية طوليها يمكن تحديد نقطة تقاطعهما المجهولة (P₂) .
ولتبسيط هذا النوع من التقاطعات ، في حال عدم تجاوز الاضلاع الحادية على المجاهيل (كما في الاشكال الرباعية والخماسية والسداسية .. وغيرها) يتم رسم موازيات بحسب الحالة المتوفرة بهدف تكوين حالة التقاطع الثالث ، بحيث يكون الضلعين الحاديين على تلك المجاهيل متجاوران وتشكل نقطتي بدايتهما خطاً معلوماً بالاسم (P₁ , P₃) ويتقاطعان من طرفهما الاخر في النقطة المجهولة (P₂) . وكما في الشكل التالي :



هنالك طريقتين لحساب المجاهيل في هذا النوع من التقاطعات :

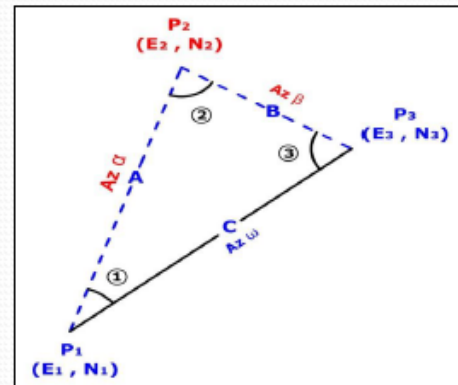
أولاهما (وهي الطريقة المهمة) : طريقة المثلثات (Triangometric Method) :

يستخدم في هذه الطريقة قانون جيب التمام (Cos Law) . وكما يلي :

١- بحسب إتجاه الضلع (P₁ → P₃) (Az ω) وطوله (C) بالعلاقتين التاليتين :

$$Az \omega = \tan^{-1} \left(\frac{E_3 - E_1}{N_3 - N_1} \right)$$

$$Length (C) = \left(\frac{E_3 - E_1}{\sin Az \omega} \right) = \left(\frac{N_3 - N_1}{\cos Az \omega} \right)$$



٢- تحسب الزوايا الداخلية للمثلث (< 1 , < 2 , < 3) باستخدام قانون جيب التمام .

$$B^2 = A^2 + C^2 - 2A.C.Cos < 1$$

$$Cos < 1 = \frac{A^2 + C^2 - B^2}{2A.C}$$

$$Cos < 2 = \frac{A^2 + B^2 - C^2}{2A.B}$$

$$Cos < 3 = \frac{B^2 + C^2 - A^2}{2B.C}$$

٣- بحسب الاتجاهان المجهولان ($Az \beta$, $Az \alpha$) من الزوايا الداخلية التي تم حسابها في الخطوة السابقة ،

$$Az \alpha = Az \omega \pm < 1 \quad Az \beta = Back Az \alpha \pm < 2 \quad . (Az \omega) \text{ والاتجاه}$$

٤- بحسب الاتجاهان المحتملان ($Az \beta'$, $Az \alpha'$) وهما نفس الزوايا السابقة ، لكن بالاتجاه المعاكس لخط

القاعدة ، وكما في الشكل التالي .

$$Az \alpha' = Az \omega \pm < 1$$

$$Az \beta' = Back Az \alpha' \pm < 2 \dots (or) \dots Back Az \omega \pm < 3$$

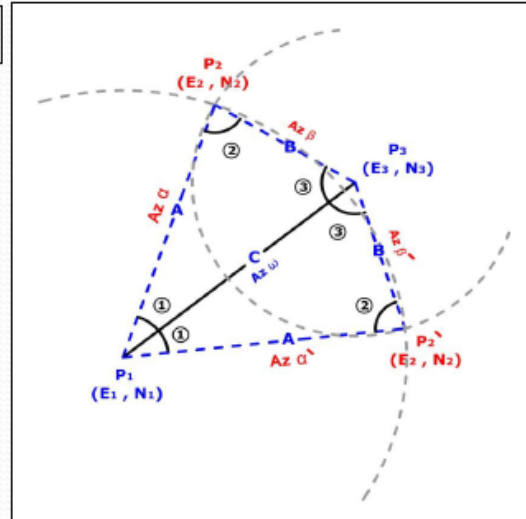
٥- بحسب الموقع (P_2) من خلال (P_1 , A , $Az \alpha$) .

ويتم التحقق من (P_3) بواسطة (P_2 , B , $Az \beta$) .

٦- بحسب الموقع المحتمل (P_2') من خلال (P_1 , A , $Az \alpha'$) .

ويتم التحقق من (P_3) بواسطة (P_2' , B , $Az \beta'$) .

٧- يتم إختيار أحد الاحتمالين إما بالاعتماد على الرسم أو الواقع .



Ex : Find the coords. of the traverse (J , K , L , M) by using triangometric method .

Sol : Coords. Of (M) :

$$E_M = E_J + L_{(J \rightarrow M)} \sin Az_{(J \rightarrow M)}$$

$$E_M = 250 + 56 \sin (50^\circ 24' + 180^\circ) = 206.85 \text{ m}$$

$$N_M = N_J + L_{(J \rightarrow M)} \cos Az_{(J \rightarrow M)}$$

$$N_M = 200 + 56 \cos (50^\circ 24' + 180^\circ) = 164.30 \text{ m}$$

Coords. Of (J') : (220 , 200)

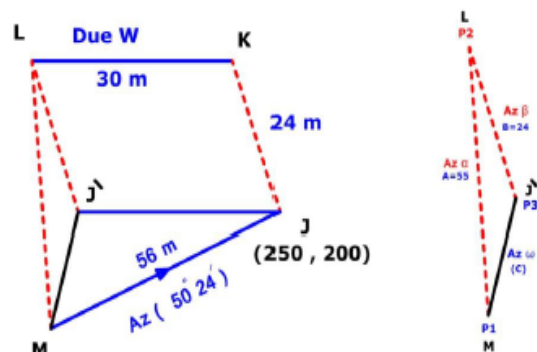
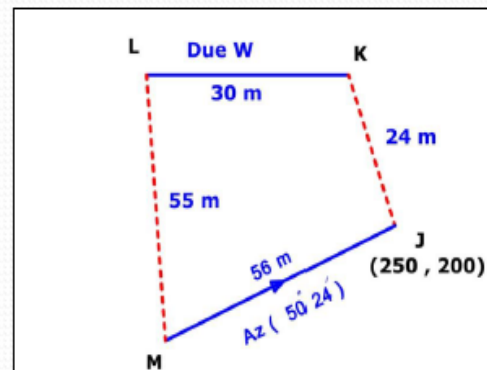
$$\text{length}_{(C)} = \sqrt{(220 - 206.85)^2 + (200 - 164.30)^2} = 38.05 \text{ m}$$

$$Az_\omega = \tan^{-1} \frac{(220 - 206.85)}{(200 - 164.30)} = 20^\circ 13'$$

$$\cos < 1 = \frac{A^2 + C^2 - B^2}{2A.C} \rightarrow \frac{55^2 + 38.05^2 - 24^2}{2 \times 55 \times 38.05} = 21^\circ 24'$$

$$\cos < 2 = \frac{A^2 + B^2 - C^2}{2A.B} \rightarrow \frac{55^2 + 24^2 - 38.05^2}{2 \times 55 \times 24} = 35^\circ 21'$$

$$\cos < 3 = \frac{B^2 + C^2 - A^2}{2B.C} \rightarrow \frac{24^2 + 38.05^2 - 55^2}{2 \times 24 \times 38.05} = 123^\circ 15'$$



$$Az_{\alpha} = Az_{\omega} - < 1$$

$$Az_{\alpha} = 20^{\circ} 13' - 21^{\circ} 24' \rightarrow -1^{\circ} 11' + 360^{\circ} = 358^{\circ} 49'$$

$$Az_{\beta} = \text{Back } Az_{\alpha} - < 2$$

$$Az_{\beta} = (358^{\circ} 49' - 180^{\circ}) - 35^{\circ} 21' = 143^{\circ} 28'$$

Coords. Of (P2) From (P1) :

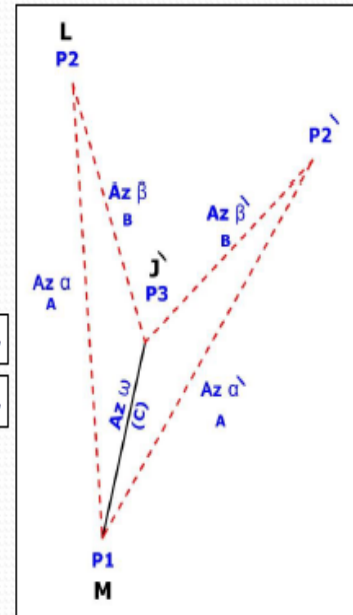
$$E_{P2} = E_{P1} + A \cdot \sin Az_{\alpha} \rightarrow 206.85 + 55 \sin 358^{\circ} 49' = 205.71 \text{ m}$$

$$N_{P2} = N_{P1} + A \cdot \cos Az_{\alpha} \rightarrow 164.30 + 55 \cos 358^{\circ} 49' = 219.29 \text{ m}$$

Check Of (P3) From (P2) :

$$E_{P3} = E_{P2} + B \cdot \sin Az_{\beta} \rightarrow 205.71 + 24 \sin 143^{\circ} 28' = 220 \text{ m}$$

$$N_{P3} = N_{P2} + B \cdot \cos Az_{\beta} \rightarrow 219.29 + 24 \cos 143^{\circ} 28' = 200 \text{ m}$$



(((Checked)))

(((الاحتمال الثاني)))

$$Az_{\alpha}' = Az_{\omega} + < 1$$

$$Az_{\alpha}' = 20^{\circ} 13' + 21^{\circ} 24' = 41^{\circ} 37'$$

$$Az_{\beta}' = \text{Back } Az_{\omega} - < 3$$

$$Az_{\beta}' = (20^{\circ} 13' - 180^{\circ}) - 123^{\circ} 15' = 76^{\circ} 58'$$

$$(\text{or}) Az_{\beta}' = \text{Back } Az_{\alpha}' + < 2 - 180^{\circ}$$

$$Az_{\beta}' = (41^{\circ} 37' + 180^{\circ}) + 35^{\circ} 21' - 180^{\circ} = 76^{\circ} 58'$$

Coords. Of (P2 ') From (P1) :

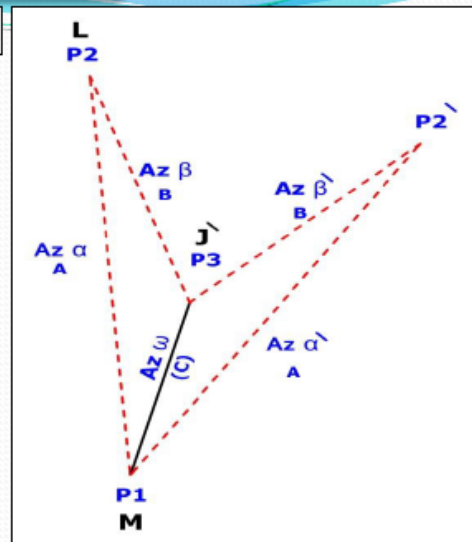
$$E_{P2}' = E_{P1} + A \cdot \sin Az_{\alpha}' \rightarrow 206.85 + 55 \sin 41^{\circ} 37' = 243.38 \text{ m}$$

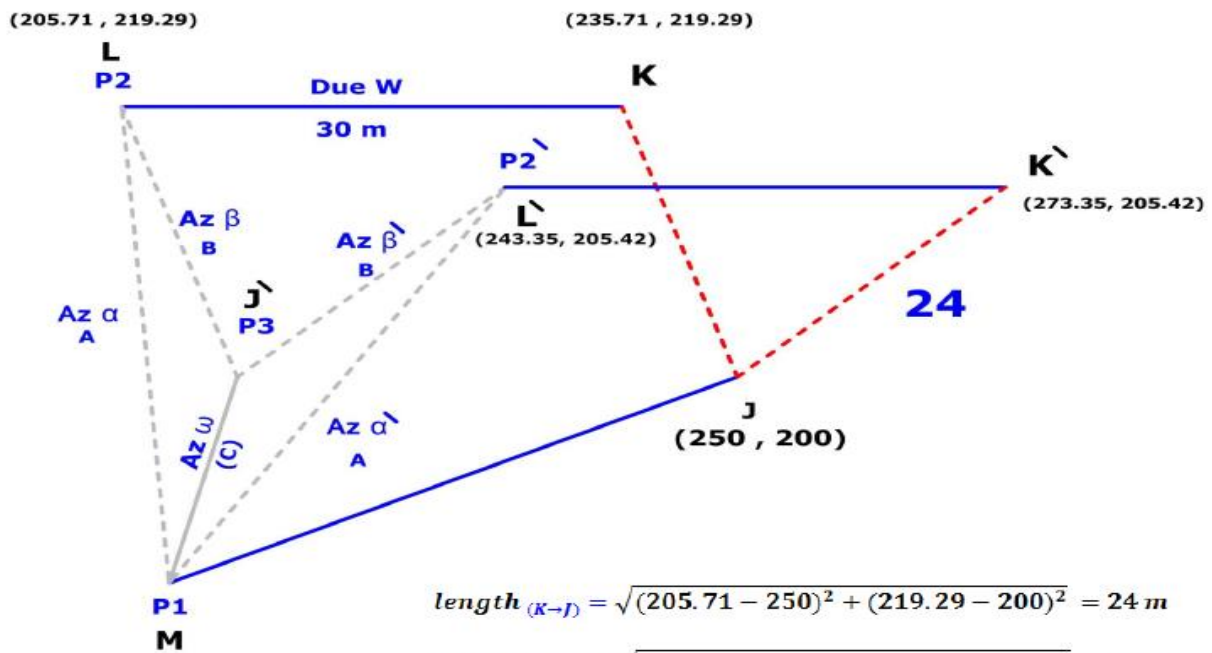
$$N_{P2}' = N_{P1} + A \cdot \cos Az_{\alpha}' \rightarrow 164.30 + 55 \cos 41^{\circ} 37' = 205.42 \text{ m}$$

Check Of (P3) From (P2 ') :

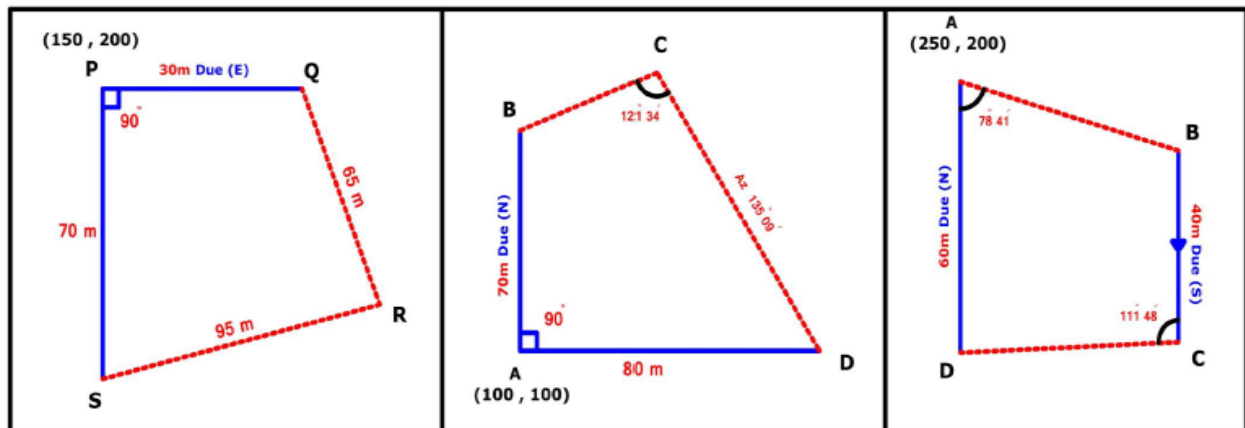
$$E_{P3} = E_{P2}' + B \cdot \sin Az_{\beta}' \rightarrow 243.38 + 24 \sin (76^{\circ} 58' + 180^{\circ}) = 220 \text{ m}$$

$$N_{P3} = N_{P2}' + B \cdot \cos Az_{\beta}' \rightarrow 205.42 + 24 \cos (76^{\circ} 58' + 180^{\circ}) = 200 \text{ m}$$





Examples : Find the solving triangle of the traverses .



Lecture (9) to (15)

(Resections)

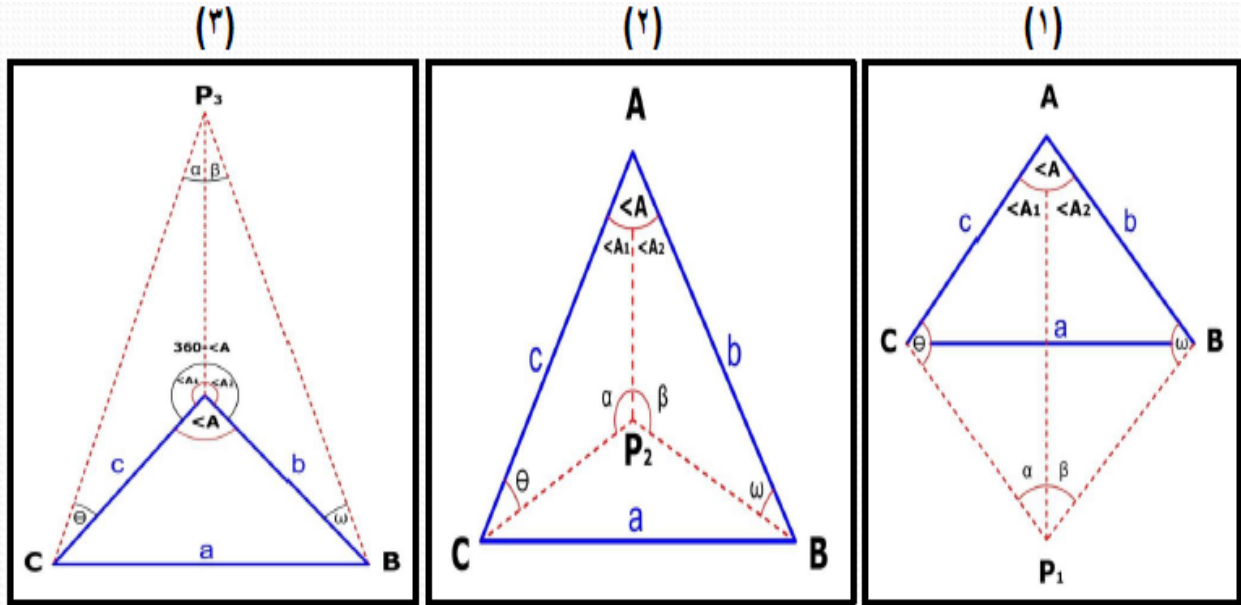
يقصد بـ (Resection) التقاطع العكسي عملية حساب موقع نقطة معينة يمكن نصب الجهاز عليها (إحداثياتها) وإيجاد إحداثياتها من خلال ثلاث نقاط معلومة الاحداثيات ، والزوايتين (α, β) الناتجتين من رصد النقاط الثلاث.

ويستخدم التقاطع الخلفي عادة لربط احداثيات منطقة حديثة الانشاء بمدينة قائمة ، أو للبدء بمشروع معين من خلال نقاط معلومة الاحداثيات محيطة بالموقع .

هناك ثلاثة حالات للتقاطع العكسي بالاعتماد على النقطة المطلوب حساب احداثياتها ، وهي كما موضحة في

الاشكال التالية :

- ١- النقطة المطلوبة (P_1) خارج المثلث باتجاه القاعدة.
- ٢- النقطة المطلوبة (P_2) داخل حدود المثلث .
- ٣- النقطة المطلوبة (P_3) خارج حدود المثلث باتجاه الرأس



Resection Case (1)

١- طريقة كاستنر : KAESTNER METHOD

المعطيات : (احداثيات A,B,C) والزواويتان (α, β) .

المطلوب : الزاويتان (ω, θ) والموقع (P) .

((خطوات الحل في الحالة (١) طريقة كاستنر / النقطة المطلوبة اسفل قاعدة المثلث)))

١- من احداثيات اركان المثلث نحسب الاطوال (a , b , c) والاتجاهات لاضلاع المثلث ثم الزوايا الداخلية للمثلث .

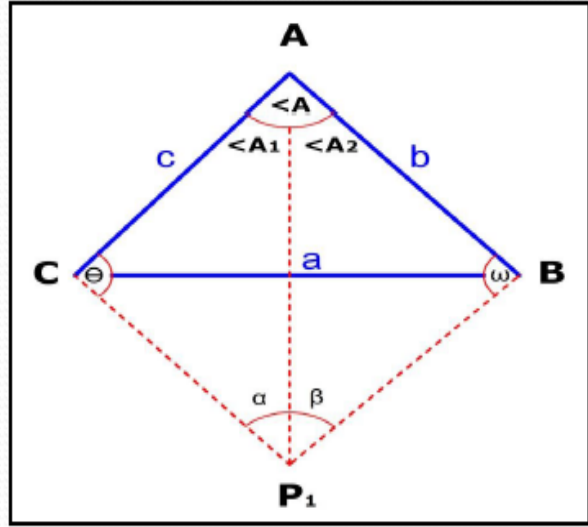
٢- لحساب الزاويتين (ω, θ) يجب حساب (ϕ, S) . ويتم ذلك من خلال :

$$\angle S = \theta + \omega = 360 - (a + \beta + \angle A) \dots (1)$$

$$\tan \phi = \frac{\sin \omega}{\sin \alpha} = \frac{c \cdot \sin \beta}{b \cdot \sin \alpha} \dots\dots\dots(2)$$

$$\tan \theta = \frac{\sin \angle S}{\tan \phi + \cos \angle S} \dots\dots\dots(3)$$

$$\tan \omega = \frac{\sin \angle S}{\cot \phi + \cos \angle S} \dots\dots\dots(4)$$



٣- بحسب طول الضلع (AP_1) بإستخدام قانون الجيوب .

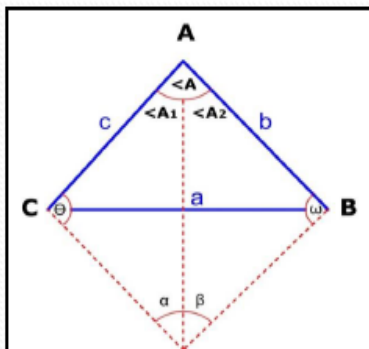
$$\text{للمثلث الايسر} \quad \frac{AP_1}{\sin \theta} = \frac{c}{\sin \alpha} \quad \text{OR} \quad \frac{AP_1}{\sin \omega} = \frac{b}{\sin \beta}$$

٤- بحسب إتجاه الضلع AP_1 من الإتجاه (AZ_{AC} والزاوية A_1) أو (AZ_{AB} والزاوية A_2) .

$$< A_1 = 180 - (\theta + \alpha) \quad , \quad < A_2 = 180 - (\omega + \beta)$$

$$AZ_{AP_1} = AZ_{AC} \pm < A_1 \quad , \quad AZ_{AP_1} = AZ_{AB} \pm < A_2$$

٥- تحسب احداثيات (P_1) من الإتجاه AZ_{AP_1} والطول AP_1 واحداثيات A .



$$EP_1 = EA + (AP_1) \sin AZ_{AP_1}$$

$$NP_1 = NA + (AP_1) \cos AZ_{AP_1}$$

Ex : For the fig. below , find the coords. of point (P_1) .

Sol. : 1- Calculating (Length , Azimuth of a triangle Sides) :

$$a = \sqrt{(722.178 - 240.983)^2 + (454.913 - 502.233)^2} = 483.516 \text{ m}$$

$$b = \sqrt{(722.178 - 500)^2 + (454.913 - 750)^2} = 369.377 \text{ m}$$

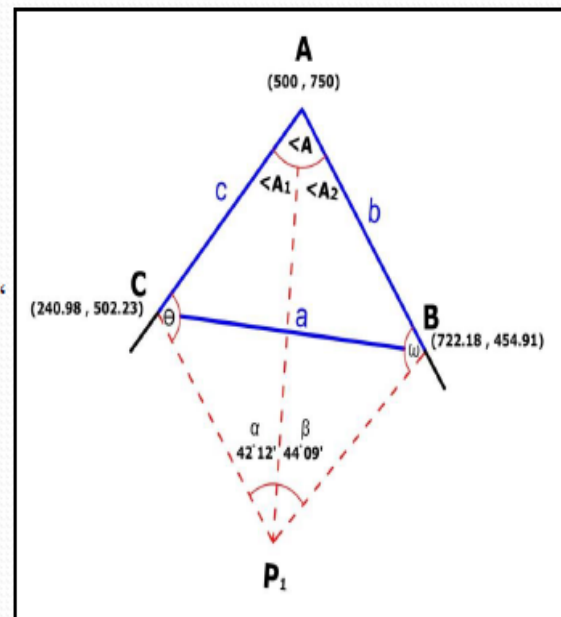
$$c = \sqrt{(500 - 240.983)^2 + (750 - 502.233)^2} = 358.439 \text{ m}$$

$$Az_{CA} = \tan^{-1} \left[\frac{(500 - 240.983)}{(750 - 502.233)} \right] = 46^\circ 16' 18''$$

$$Az_{AB} = \tan^{-1} \left[\frac{(722.178 - 500)}{(454.913 - 750)} \right] = 180^\circ - 36^\circ 58' 37'' = 143^\circ 01' 22''$$

Compute $< A$ from Az_{CA} & Az_{AB}

$$< A = (46^\circ 16' 18'' + 180^\circ) - 143^\circ 01' 22'' = 83^\circ 14' 56''$$



2- The angles (θ & ω):

$$< S = \theta + \omega = 360 - (\alpha + \beta + < A) \dots (1)$$

$$< S = 360^\circ - (42^\circ 12' 17'' + 44^\circ 09' 09'' + 83^\circ 14' 56'') = 190^\circ 23' 38''$$

$$\tan \phi = \frac{\sin \omega}{\sin \alpha} = \frac{c \cdot \sin \beta}{b \cdot \sin \alpha} \dots \dots \dots (2)$$

$$\tan \phi = \frac{358.439 \cdot \sin 44^\circ 09' 09''}{369.377 \cdot \sin 42^\circ 12' 17''} \rightarrow \tan \phi = 1.006 \rightarrow \phi = 45^\circ 10' 37''$$

$$\tan \theta = \frac{\sin < S}{\tan \phi + \cos < S} \dots \dots \dots (3)$$

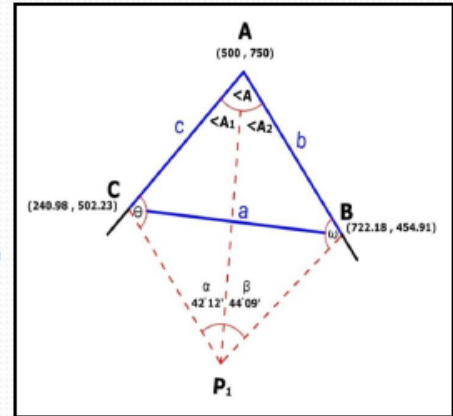
$$\tan \theta = \frac{\sin 190^\circ 23' 38''}{1.006 + \cos 190^\circ 23' 38''} \rightarrow \theta = -82^\circ 55' 10''$$

$$\theta = 180^\circ - 82^\circ 55' 10'' \rightarrow \theta = 97^\circ 04' 50''$$

$$\tan \omega = \frac{\sin < S}{\cot \phi + \cos < S} \dots \dots \dots (4)$$

$$\tan \omega = \frac{\sin 190^\circ 23' 38''}{\frac{1}{1.006} + \cos 190^\circ 23' 38''} \rightarrow \omega = -86^\circ 41' 52''$$

$$\omega = 180^\circ - 86^\circ 41' 52'' \rightarrow \omega = 93^\circ 18' 08''$$



3- The length (AP₁) by Sin Law :

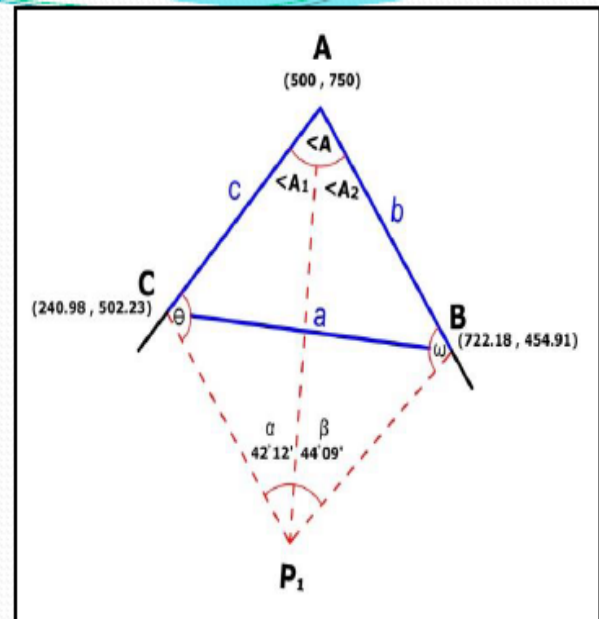
$$\text{(للمثلث الايسر)} \quad \frac{AP_1}{\sin \theta} = \frac{c}{\sin \alpha}$$

$$AP_1 = \frac{358.439 \cdot \sin 97^\circ 04' 50''}{\sin 42^\circ 12' 17''} \rightarrow AP_1 = 529.496 \text{ m}$$

$$\text{(Or) (للمثلث الايمن)} \quad \frac{AP_1}{\sin \omega} = \frac{b}{\sin \beta}$$

$$AP_1 = \frac{369.377 \cdot \sin 93^\circ 18' 08''}{\sin 44^\circ 09' 09''} \rightarrow AP_1 = 529.399 \text{ m}$$

$$\text{Avg. of } AP_1 = 529.448 \text{ m}$$



4- The direction (AP₁) :

$$< A_1 = 180 - (\theta + \alpha) \quad , \quad < A_2 = 180 - (\omega + \beta)$$

$$AZ_{AP_1} = AZ_{AC} \pm < A_1 \quad , \quad AZ_{AP_1} = AZ_{AB} \pm < A_2$$

$$< A_1 = 180^\circ - (42^\circ 12' 17'' + 97^\circ 04' 50'') \rightarrow < A_1 = 40^\circ 42' 53''$$

$$AZ_{AP_1} = (46^\circ 16' 18'' + 180^\circ) - 40^\circ 42' 53'' \rightarrow AZ_{AP_1} = 185^\circ 33' 25''$$

5- Coordinate of point (P₁) :

$$EP_1 = EA + (AP_1) \sin AZ_{AP_1} \quad , \quad NP_1 = NA + (AP_1) \cos AZ_{AP_1}$$

$$EP_1 = 500 + 529.448 \sin 185^\circ 33' 25'' \rightarrow EP_1 = 448.731 \text{ m}$$

$$NP_1 = 750 + 529.448 \cos 185^\circ 33' 25'' \rightarrow NP_1 = 223.04 \text{ m}$$

Resection Case (2)

طريقة تينستر (TIENSTRA'S METHOD)

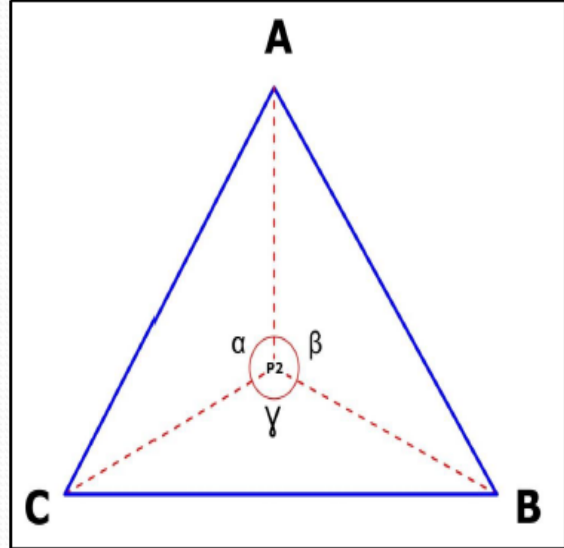
المعطيات: (احداثيات A,B,C) والزوايا (γ, β, α) .

المطلوب: الموقع (P_2) .. وخطوات الاجابة كما يلي:

$$a = \tan^{-1} \left[\frac{E_C - E_A}{N_C - N_A} \right] - \tan^{-1} \left[\frac{E_B - E_A}{N_B - N_A} \right]$$

$$b = \tan^{-1} \left[\frac{E_A - E_B}{N_A - N_B} \right] - \tan^{-1} \left[\frac{E_C - E_B}{N_C - N_B} \right]$$

$$c = \tan^{-1} \left[\frac{E_B - E_C}{N_B - N_C} \right] - \tan^{-1} \left[\frac{E_A - E_C}{N_A - N_C} \right]$$



$$K_1 = \frac{1}{\cot a - \cot \gamma}, \quad K_2 = \frac{1}{\cot b - \cot \alpha}, \quad K_3 = \frac{1}{\cot c - \cot \beta}$$

$$E_{P3} = \frac{K_1 * E_A + K_2 * E_B + K_3 * E_C}{K_1 + K_2 + K_3}, \quad N_{P3} = \frac{K_1 * N_A + K_2 * N_B + K_3 * N_C}{K_1 + K_2 + K_3}$$

Ex : For the fig. below , find the coords. of point (P₂) .

Sol. :

$$a = \tan^{-1} \left[\frac{E_C - E_A}{N_C - N_A} \right] - \tan^{-1} \left[\frac{E_B - E_A}{N_B - N_A} \right] \rightarrow a = \tan^{-1} \left[\frac{240.983 - 500}{502.233 - 750} \right] - \tan^{-1} \left[\frac{722.178 - 500}{454.913 - 750} \right] = 83.249$$

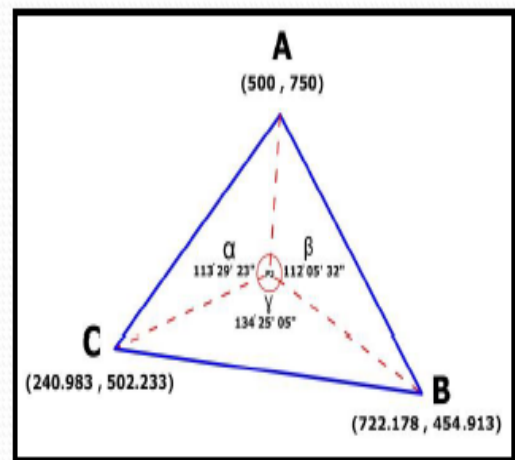
$$b = \tan^{-1} \left[\frac{E_A - E_B}{N_A - N_B} \right] - \tan^{-1} \left[\frac{E_C - E_B}{N_C - N_B} \right] \rightarrow b = \tan^{-1} \left[\frac{500 - 722.178}{750 - 454.913} \right] - \tan^{-1} \left[\frac{240.983 - 722.178}{502.233 - 454.913} \right] = 47.407$$

$$c = \tan^{-1} \left[\frac{E_B - E_C}{N_B - N_C} \right] - \tan^{-1} \left[\frac{E_A - E_C}{N_A - N_C} \right] \rightarrow c = \tan^{-1} \left[\frac{722.178 - 240.983}{454.913 - 502.233} \right] - \tan^{-1} \left[\frac{500 - 240.983}{750 - 502.233} \right] = -130.655$$

$$K_1 = \frac{1}{\cot a - \cot \gamma} \rightarrow K_1 = \frac{1}{\cot 83.249 - \cot 143^\circ 25' 05''} = 0.912$$

$$K_2 = \frac{1}{\cot b - \cot \alpha} \rightarrow K_2 = \frac{1}{\cot 47.407 - \cot 113^\circ 29' 23''} = 0.739$$

$$K_3 = \frac{1}{\cot c - \cot \beta} \rightarrow K_3 = \frac{1}{\cot 130.655 - \cot 112^\circ 05' 32''} = 0.791$$



$$E_{P3} = \frac{K_1 * E_A + K_2 * E_B + K_3 * E_C}{K_1 + K_2 + K_3} \rightarrow E_{P3} = \frac{0.912 * 500 + 0.739 * 722.178 + 0.791 * 240.983}{0.912 + 0.739 + 0.791} = 483.336 \text{ m}$$

$$N_{P3} = \frac{K_1 * N_A + K_2 * N_B + K_3 * N_C}{K_1 + K_2 + K_3} \rightarrow N_{P3} = \frac{0.912 * 750 + 0.739 * 454.913 + 0.791 * 502.233}{0.912 + 0.739 + 0.791} = 580.445 \text{ m}$$

Resection Case (3)

المعطيات : احداثيات (A,B,C) والزائيتان (β, α) .

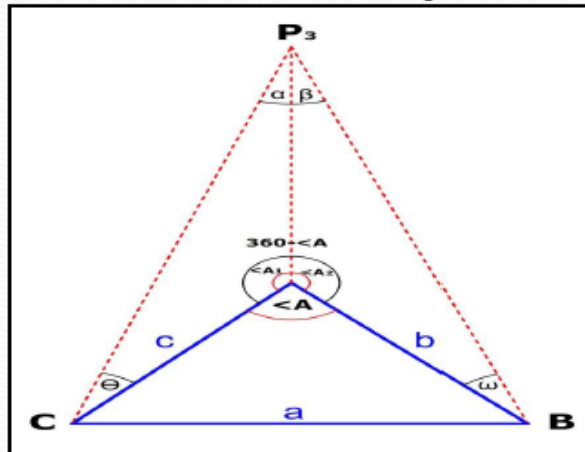
المطلوب : (ω, θ) والموقع (P) .

((خطوات الحل في الحالة (٣) / النقطة المطلوبة داخل المثلث)))

* الخطوات مطابقة لخطوات الحالة الاولى ، الاختلاف فقط في قيمة (S) وهي كما يلي :

$$\angle S = \theta + \omega = 360 - (\alpha + \beta + 360 + \angle A)$$

$$\angle S = \angle A - \alpha - \beta$$



Ex : For the fig. below , find the coords. of point (P₃) .

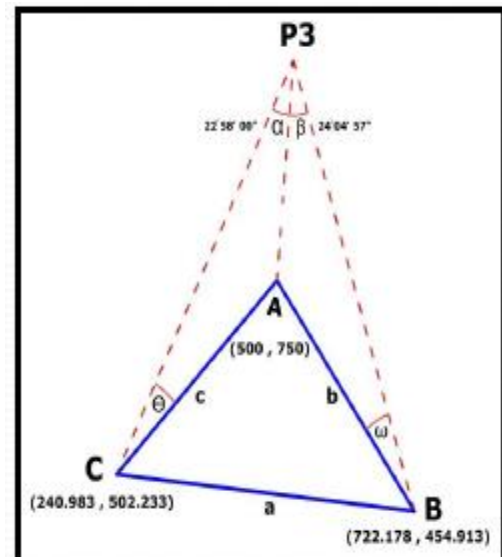
Sol : from the Coords. we find the requirement of point (1) in solution steps , as follow :

$$c = 358.44 \text{ m .}$$

$$b = 369.38 \text{ m .}$$

$$Az_{CA} = 46^{\circ} 16'$$

$$\angle A = 83^{\circ} 15'$$



(2) Compute the value of $(\phi + w)^\circ$ -

$$\angle S = \phi + w = 360 - (\alpha + \beta + 360 + \angle A) \Rightarrow \angle S = \angle A - \alpha - \beta$$

$$\angle S = 83^\circ 14' 55'' - 22^\circ 58' - 24^\circ 4' 57'' = 36^\circ 11' 58''$$

$$\tan \phi = \frac{c \cdot \sin \beta}{b \cdot \sin \alpha} = \frac{358.42 \sin 24^\circ 4' 57''}{369.377 \sin 22^\circ 58'} \Rightarrow \tan \phi = 1.0156 \Rightarrow \phi = 45^\circ 25' 9''$$

$$\tan \theta = \frac{\sin \angle S}{\tan \phi + \cos \angle S} = \frac{\sin 36^\circ 11' 58''}{1.015 + \cos 36^\circ 11' 58''} \Rightarrow \theta = 17^\circ 57' 37''$$

$$\tan w = \frac{\sin \angle S}{\phi + \cos \angle S} = \frac{\sin 36^\circ 11' 58''}{1.015 + \cos 36^\circ 11' 58''} \Rightarrow w = 18^\circ 14' 21''$$

(3) Compute the length AP_3 by Sin law -

$$\frac{AP_3}{\sin \theta} = \frac{c}{\sin \alpha} \Rightarrow AP_3 = \frac{\sin 17^\circ 57' 37'' \times 358.42}{\sin 22^\circ 58'} \Rightarrow AP_3 = 283.247 \text{ m}$$

$$\text{or } \frac{AP_3}{\sin w} = \frac{b}{\sin \beta} \Rightarrow AP_3 = \frac{\sin 18^\circ 14' 21'' \times 369.377}{\sin 24^\circ 04' 57''} \Rightarrow AP_3 = 283.32 \text{ m}$$

$$AP_3 = 283.284 \text{ m}$$

(4)

$$\angle A_1 = 180^\circ - (\alpha + \theta) \Rightarrow 180^\circ - (22^\circ 58' + 17^\circ 57' 37'') \Rightarrow \angle A_1 = 139^\circ 4' 23''$$

$$\angle AP_3 = \angle A_2 A + \angle A_1 \Rightarrow (80^\circ + 46^\circ 16' 18'') + 139^\circ 4' 23'' = 365^\circ 20' 41''$$

$$\angle AP_3 = 365^\circ 20' 41'' - 360^\circ = 5^\circ 20' 41''$$

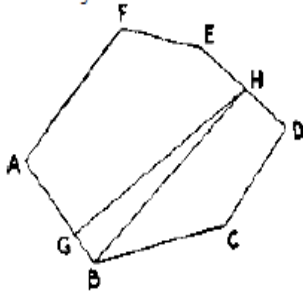
(5)

$$E_B = E_A + AP_3 \sin \angle AP_3 \Rightarrow 500 + 283.284 \sin 5^\circ 20' 41'' = 526.387 \text{ m}$$

$$NP_3 = N_A + AP_3 \cos \angle AP_3 \Rightarrow 750 + 283.284 \cos 5^\circ 20' 41'' = 1032.052 \text{ m}$$

. Subdivision of an Area into Given Parts from a Point on Boundary.

Let ABCDEFA (Fig. 202) be a plot of land, and let be required to cut off a definite area by a line drawn from the H on the boundary.



Calculate the area of the figure ABCDEFA from the coordinates and also plot the figure on a fairly large scale. By inspection or by trial and error on the plotted plan, find the station B so th the area bounded on one side by the line HB is nearer in value u» the given area than that bounded by a line from H to any other I

The length and bearing of the line BH have been computed from the coordinates of H and B and, since the bearing of BG is known, the angle HBG is known. Consequently, BG can be computed and the coordinates of G found.

406. Subdivision of an Area into Given Parts by a Line of Given Bearing. Let it be required to divide the area ABCDEFGA (Fig. 203) into two parts by a line whose bearing is given.

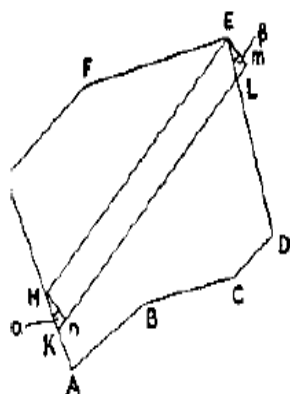


FIG. 203.

Calculate the area of the figure from the coordinates and plot it on a fairly large scale. Draw the line EH from one station E, and in the given direction, so that it cuts off an area HEFGH approximately equal to A , the area required. Calculate the bearing and distance of GE. Then, since the bearings of the lines GE, EH, and HG are known, the three angles of the triangle GEH are known and, from these and the computed distance GE, the lengths HE and GH can be calculated. Hence, the coordinates of H can be found. Using these coordinates, and those of the points E, F, and G, calculate the area of the figure HEFGH. Let A' be this area. Then, if LK is the line needed to cut off the area A , we must have:

$$A - A' = \text{Area of figure HKLEH.}$$

From E draw Em perpendicular to EH to meet KL in m, and from H draw Hn perpendicular to KL. Let Em = Hn = x and angle

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KHn = a , and $ELm = \theta$, these angles being known bearings of the different lines are known.

Then, length HE = $x \cdot \tan \theta + x \cdot \tan a$. Hence,

$$\begin{aligned} \text{Area of figure HKLEH} &= \frac{1}{2}(HE + KL) \\ &= \frac{1}{2}(2HE + x(\tan a - \tan \theta)) \\ &= x \cdot HE + \frac{x^2}{2}(\tan a - \tan \theta). \end{aligned}$$

Hence,

$$A - A' =$$

$$x(\tan a - \tan \theta).$$

This is a quadratic equation which can be solved for x . having found x , we have:

$$EL = x \cdot \sec \theta; \quad HK = x \cdot \sec a. \text{ Hence, the coordinates of K and L}$$

can be found.

سؤال: 11.10/ قطعة ارض على شكل مضلع مغلق JKLM في الشكل ادناه ، تمر خلالها قناة الرأي عرضها 6م باتجاه : $N 70^{\circ} W$ وإبتداءً من نقطة K . اريد تقسيم الجزء الواقع الى جنوب تلك القناة . الى مساحتين متساويتين من نقطة P الذي تقع في منتصف KH والمطلوب حساب : أ) مساحات اجزاء JGF و GKHF و KLMH .
 ب) طول واتجاه الخط PR وموقع نقطة R .

J(100,100)

$$E_k = 100 + 40 \cdot \sin 180^{\circ} = 100 \text{ m}$$

$$N_k = 100 + 40 \cdot \cos 180^{\circ} = 60 \text{ m}$$

$$E_L = 100 + 50 \cdot \sin 126^{\circ} 45' = 140.063 \text{ m}$$

$$N_L = 60 + 50 \cdot \cos 126^{\circ} 45' = 30.083 \text{ m}$$

$$E_M = 140.063 + 100 \cdot \sin 270^{\circ} = 40.063 \text{ m}$$

$$N_M = 30.083 + 100 \cdot \cos 270^{\circ} = 30.083 \text{ m}$$

$$E_J = 40.063 + 92 \cdot \sin 40^{\circ} 35' = 99.914 \text{ m}$$

$$N_J = 30.083 + 92 \cdot \cos 40^{\circ} 35' = 99.953 \text{ m}$$

Point	Sild	Length	AZ.	Dep.	Lat	X	Y
J						100	100
	JK	40	180°	0	-40		
K						100	60
	KL	50	$126^{\circ} 45'$	40.063	-29.916		
L						140.063	30.084
	LM	100	270°	-100	0		
M						40.063	30.084
	MJ	92	$40^{\circ} 35'$	59.851	69.870		
J						99.914	99.954
		282		-0.086	-0.046		

Corr. Dep.	Corr. Lat.	Corrected Dep.	Corrected Lat.	X	Y
				100	100
0.012	0.006	0.013	-39.994	100.013	60.006
0.015	0.008	40.078	-29.908	140.091	30.098

References

- Engineering Surveying, Zeyad AL Bakr, 1989,Baghdad ,Technical Institute
- Web sites.