محاضرة رقم (6)

مشتقات الدوال المثلثية او الدائرية:-

للحفظ مهم جداً

$$y = \sin x \Rightarrow y' = \cos x$$

$$y = \cos x \Rightarrow y' = -\sin x$$

$$y = \tan x \Rightarrow y' = \sec^2 x$$

$$y = \cot x \Rightarrow y' = -\csc^2 x$$

$$y = \sec x \Rightarrow y' = \sec x * \tan x$$

$$y = \csc x \Rightarrow y' = -\csc x * \cot x$$

ملاحظة (1):- عندما نشتق أي دالة مثلثية نضرب في مشتقة الزاوية.

ملاحظة(2):- قواعد حاصل ضرب دالتين او حاصل قسمة دالتين او القوس المرفوع الى اس تنطبق على هذه الدوال.

Examples :-

Find y' for the next function.

1) $y = \sin 5x \Rightarrow y' = \cos 5x * 5$

$y' = 5 * \cos 5x$

2) $y = \cos 4x \Rightarrow y' = -\sin 4x * 4$

$y' = -4 * \sin 4x$

3) $y = \tan x^2 \Rightarrow y' = \sec^2 x^2 * 2x$

$y' = 2x * \sec^2 x^2$

4) $y = \sin^5(x^3 + 2x - 1) \Rightarrow y' = 5 * \sin^4(x^3 + 2x - 1) * \cos(x^3 + 2x - 1) * (3x^2 + 2)$

$y' = (15x^2 + 10) * \sin^4(x^3 + 2x - 1) * \cos(x^3 + 2x - 1)$

5) $y = \sin(7x^2 + 4x + 1)$

$y' = \cos(7x^2 + 4x + 1) * (14x + 4)$

$y' = (14x + 4) * \cos(7x^2 + 4x + 1)$

6) $y = \tan^2(x^2 + 3x) \Rightarrow y' = 2 * \tan(x^2 + 3x) * \sec^2(x^2 + 3x) * (2x + 3)$

$y' = (4x + 6) * \tan(x^2 + 3x) * \sec^2(x^2 + 3x)$

7) $y = \sin\sqrt[3]{x} \Rightarrow y' = \cos\sqrt[3]{x} * \dfrac{1}{3 * \sqrt[3]{x^2}}$

8) $y = \sin 3x * \cos 3x$

$y' = \sin 3x * [-\sin 3x * 3] + \cos 3x * [\cos 3x * 3]$

$y' = -3\sin^2 3x + 3\cos^2 3x$

9) $y = \sin x - \dfrac{1}{3}\sin^3 x$

$y' = \cos x - \dfrac{1}{\cancel{3}} * \cancel{3} * \sin^2 x * \cos x * (1)$

$y' = \cos x - \sin^2 x * \cos x \Rightarrow y' = \cos x\,[1 - \sin^2 x]$

where: $[\sin^2 x + \cos^2 x = 1] \Rightarrow [\cos^2 x = 1 - \sin^2 x]$ (law)

$y' = \cos x * \cos^2 x \Rightarrow y' = \cos^3 x$

10) $y = \tan x + \dfrac{1}{3}\tan^3 x$

$y' = \sec^2 x + \dfrac{1}{\cancel{3}} * \cancel{3} * \tan^2 x * \sec^2 x * (1)$

$y' = \sec^2 x + \tan^2 x * \sec^2 x$

$y' = \sec^2 x\,[1 + \tan^2 x] \Rightarrow y' = \sec^2 x * \sec^2 x$

$y' = \sec^4 x$

مشتقات الدوال اللوغاريتمية:-

تعريفها:-

$$y = \ln u \Rightarrow y' = \frac{1}{u} * u'$$

$$y = \log_b u \Rightarrow y' = \frac{1}{\ln bu} * u'$$

امثلة جد $\frac{dy}{dx} = y'$ اذا كان:-

$$1)\ y = \ln(x^2 + 3) \Rightarrow y' = \frac{1}{(x^2 + 3)} * 2x \Rightarrow y' = \frac{2x}{(x^2 + 3)}$$

$$2)\ y = \ln(\tan x) \Rightarrow y' = \frac{1}{(\tan x)} * \sec^2 x \Rightarrow y' = \frac{\sec^2 x}{(\tan x)}$$

$$3)\ y = \log_3 \sin x \Rightarrow y' = \frac{1}{\ln 3 \sin x} * \cos x \Rightarrow y' = \frac{1}{\ln 3} \cot x$$

$$4)\ y = \ln(\sin x * \cos x) \Rightarrow y = \ln \sin x + \ln \cos x$$

$$y' = \frac{1}{\sin x} * \cos x + \frac{1}{\cos x} * -\sin x \Rightarrow y' = \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$$

$$y' = \cot x - \tan x$$

$$5)\ y = \ln \sqrt[3]{4x^2 + 5x} \Rightarrow y = \ln(4x^2 + 5x)^{\frac{1}{3}} \Rightarrow y = \frac{1}{3} \ln(4x^2 + 5x)$$

$$y' = \frac{1}{3} * \frac{1}{(4x^2 + 5x)} * 8x + 5 \Rightarrow y' = \frac{8x + 5}{3(4x^2 + 5x)}$$

مشتقات الدوال الاسية:-

تعريفها :- هي نفسها ضرب مشتقة الاس.

$$y = e^{u} \Rightarrow y = e^{u} * u'$$

Examples:-

$$1)\ y = e^{x} \Rightarrow y' = e^{x} * 1 \Rightarrow y' = e^{x}$$

$$2)\ y = e^{5x} \Rightarrow y' = e^{5x} * 5 \Rightarrow y' = 5 * e^{5x}$$

$$3)\ y = e^{\sqrt{x}} \Rightarrow y = e^{x^{\frac{1}{2}}} \Rightarrow y' = e^{x^{\frac{1}{2}}} * \frac{1}{2} x^{\frac{-1}{2}} \Rightarrow$$

$$y' = e^{x^{\frac{1}{2}}} * \frac{1}{2x^{\frac{1}{2}}} \Rightarrow y' = e^{\sqrt{x}} * \frac{1}{2\sqrt{x}}$$

$$4)\ y = e^{x \ln x} \Rightarrow y' = e^{x \ln x} * (x * \frac{1}{x} * 1 + \ln x * 1) \Rightarrow y' = (\frac{1}{x} + \ln x) e^{x \ln x}$$

$$5)\ \sec(e^{4x}) \Rightarrow y' = \sec(e^{4x}) \tan(e^{4x}) * 4 \Rightarrow y' = 4 * \sec(e^{4x}) \tan(e^{4x})$$

ملاحظات مهمة جدا على قواعد (ln)

$$1)\ y = \ln \frac{a}{b} \Rightarrow y = \ln a - \ln b$$

$$2)\ y = \ln(a.b) \Rightarrow y = \ln a + \ln b$$

$$3)\ y = \ln x^{n} \Rightarrow y = n \ln x$$