الجامعة التقنية الجنوبية
المعهد التقني العمارة
قسم أنظمة الحاسوب

المحاضرات النظرية لمادة الرياضيات للمرحلة الأولى

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الرياضيات

The Matrices المصفوفات

When we solved the set of equations

$$\left.\begin{aligned} x+y&=5 \\ x-2y&=-4 \end{aligned}\right\}$$ by the short method we made use of $\begin{bmatrix} 1 & 1 & 5 \\ 1 & -2 & -4 \end{bmatrix}$. Later in the chapter we will wante to solve (حل) for this example:

$$\begin{aligned} x+2y-z&=-5 \\ 2x-y+z&=6 \\ x-y-3z&=-3 \end{aligned}$$ by using $\begin{bmatrix} 1 & 2 & -1 & -5 \\ 2 & -1 & 1 & 6 \\ 1 & -1 & -3 & -3 \end{bmatrix}$

When we speak of matrix, it is ordinarily namde by a single capital letter.

for example may say:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$

Element's position in the matrix writ with small letter (a). The first subscript number always names the (row) (صف), and the second subscript number names the (column) (عمود).

For example a_{23} row (صف) $= 2$, column (عمود) $= 3$.

We may also say that a matrix $A = [a_{ij}]$ (i) the number of row, (j) the number of columns

رتبة المصفوفة

Also calld $m \times n$ matrix A, (m) the number of rows, (n) the number of columns.

for example: $A = 2 \times 3$, $m = 2$ عدد الصفوف, $n = 3$ عدد الاعمدة

عمليات على المصفوفات:

A- Addition of Matrices: عملية الجمع للمصفوفات

When to add two matrices to be equal (تساوي) of order (الرتبة). For example

Ex (1) $A = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ -5 & 3 \end{bmatrix}$

$$A + B = \begin{bmatrix} 4 & 4 \\ -4 & -2 \end{bmatrix}$$

Ex (2) find $(A+B)$:

$A = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 4 & 1 \end{bmatrix}$; $B = \begin{bmatrix} -2 & 3 & 5 \\ 8 & -7 & 2 \end{bmatrix}$

الحل

Solution: $A + B = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 4 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 3 & 5 \\ 8 & -7 & 2 \end{bmatrix}$

او (C)

$$= \begin{bmatrix} -1 & 5 & 8 \\ 5 & -3 & 3 \end{bmatrix}$$

ملاحظة

Note: $A + B = [a_{ij}]_{m,n} + [b_{ij}]_{m,n}$

$= [a_{ij} + b_{ij}]_{m,n}$, $A + B = B + A$

(3)

Ex (3). Find $2A+B$ if: $A=\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

Solution: $2A=\begin{bmatrix} 4 & 2 \\ 6 & 10 \end{bmatrix}$

$$2A+B=\begin{bmatrix} 4 & 2 \\ 6 & 10 \end{bmatrix}+\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

$$=\begin{bmatrix} 7 & 3 \\ 8 & 14 \end{bmatrix}$$

B- Multiplication of Matrices ضرب المصفوفات

The multiply two matrices, one $3*2$ and the other $2*2$, by multiplying the elements in the rows of the first matrix by elements in columns of the second matrix, and by then adding the results.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} * \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \quad \text{النتيجة}$$

$$\begin{bmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \\ a_{31}b_{11}+a_{32}b_{21} & a_{31}b_{12}+a_{32}b_{22} \end{bmatrix}$$

Ex1: Find $A*B$: $A=\begin{bmatrix} 3 & 1 \\ -2 & 0 \\ 4 & 1 \end{bmatrix}$; $B=\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 3 & 1 & 5 \end{bmatrix}$

Solution:

$$\begin{bmatrix} 3 & 1 \\ -2 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 3+0 & 6+3 & 3+1 & 12+5 \\ -2+0 & -4+0 & -2+0 & -8+0 \\ 4+0 & 8+3 & 4+1 & 16+5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 9 & 4 & 17 \\ -2 & -4 & -2 & -8 \\ 4 & 11 & 5 & 21 \end{bmatrix}$$

EX 2: Find $A * B$: $A = \begin{bmatrix} 1 & 5 \\ -2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & -6 \\ -2 & 1 & -1 \end{bmatrix}$

So: $\begin{bmatrix} 1 & 5 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -6 \\ -2 & 1 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 1-10 & 0+5 & -6-5 \\ -2+0 & 0+0 & 12+0 \end{bmatrix} = \begin{bmatrix} -9 & 5 & -11 \\ -2 & 0 & 12 \end{bmatrix}$$

الشرط الأساسي لضرب مصفوفتين هو: عدد الأعمدة في المصفوفة الأولى = عدد الأسطر في المصفوفة الثانية

ملاحظة

Note $A * B \neq B * A$

EX3: prove $AB \neq BA$ if $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix}$

So: $AB = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \end{bmatrix} * \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3+0+2 & 1+0+0 \\ -3+6+1 & -1+3+0 \end{bmatrix}$

$$AB = \begin{bmatrix} 5 & 1 \\ 4 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3-1 & 0+9 & 6+1 \\ 2-1 & 0+3 & 4+1 \\ 1+0 & 0+0 & 2+0 \end{bmatrix} = \begin{bmatrix} 2 & 9 & 7 \\ 1 & 3 & 5 \\ 1 & 0 & 2 \end{bmatrix}$$

$\therefore AB \neq BA$

مهم

Special Types Matrices

أنواع الأساسية للمصفوفات

1- Zero Matrix: A matrix with all of its elements equal to 0, perhaps a 2x3, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

المصفوفة الصفرية

2- Square Matrix: It has the same number of rows and columns

المصفوفة المربعة

Ex: $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

3- Diagonal Matrix: If all the elements of a square matrix are zero except those falling on the main diagonal.

المصفوفة القطرية

Ex: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

4- Identity Matrix: If all of nonzero elements of a diagonal matrix are equal to (1)

المصفوفة الوحدية

Ex: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The transpose of matrix (A^T)
(منقول أو صيغة المصفوفة)

Matrix A is defined by $A^T = [a_{ij}]^T = [a_{ji}]$

Ex: $A = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix}$

Notes: $(A^T)^T = A$; $(A+B)^T = A^T + B^T$; $(AB)^T = B^T A^T$

(منقول المنقول)

Ex: $A^T = \begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix}$, $(A^T)^T = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix} = A$

The Inverse Matrix (A^{-1})
(معكوس أو مقلوبة المصفوفة)

The Inverse A call it A^{-1}

The formula (الصيغة) used to find A^{-1}, where A is 2×2 matrix is:

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

; $(ad-bc) \neq 0$, Note: $(A^{-1})^{-1} = A$, $(A^T)^{-1} = (A^{-1})^T$, $(AB)^{-1} = B^{-1}A^{-1}$

(معكوس مقلوبة المصفوفة)

Ex: Find A^{-1} if $A = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Solution: If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

We have $a=2, b=0, c=1, d=1$ and $ad-bc = 2-0 = 2$

(7)

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & -0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

Exercises:- تمارين

1- Add $\begin{bmatrix} 10 & 15 \\ 20 & 25 \end{bmatrix} + \begin{bmatrix} -24 & -16 \\ -4 & 24 \end{bmatrix}$

2- Add $\left\{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}\right\} + \begin{bmatrix} 0 & 1 \\ 8 & 2 \end{bmatrix}$

3- Find $3A+2B$ if $A = \begin{bmatrix} 5 & 6 \\ 7 & -9 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix}$

4- Find $3C+[A+2B]$, if

$A = \begin{bmatrix} 7 & 2 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$, $C = \begin{bmatrix} 6 & 7 \\ 4 & 5 \end{bmatrix}$

5- Find $A \times B$ if $A = \begin{bmatrix} -5 & 6 & 7 \\ 4 & 3 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 8 & -3 \\ 2 & 4 \\ 1 & -9 \end{bmatrix}$

6- Find A^T if $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -5 \\ 7 & -4 & 6 \end{bmatrix}$

7- Find A^{-1} if $A = \begin{bmatrix} 3 & 15 \\ 1 & 5 \end{bmatrix}$

8- If $A = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$, prove: ① $(A^{-1})^{-1} = A$, ② $(A^T)^{-1} = (A^{-1})^T$

9- Prove $(AB)^{-1} = B^{-1}A^{-1}$, when $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 1 \\ 2 & 3 \end{bmatrix}$

and prove $(A+B)^T = A^T + B^T$ and $(AB)^T = B^T A^T$

The Determinants $|A|$ المحددات

1- Determint of matrix 2x2, $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is difined to be $(ad-bc)$

EX: $\begin{vmatrix} 2 & 1 \\ 5 & -3 \end{vmatrix} = 2(-3) - 1(5) = -6-5 = -11$

2- Determint of matrix 3x3:
We used Imagine a checkboard : محدد الاشارات

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

EX①: Find $|A|$ if $A = \begin{vmatrix} 1 & 2 & -2 \\ 3 & 0 & -1 \\ 2 & 1 & 4 \end{vmatrix}$

Solution: chose a given row or column, let select the first row. نختار

$$\begin{vmatrix} 1 & 2 & -2 \\ 3 & 0 & -1 \\ 2 & 1 & 4 \end{vmatrix} = (1)\begin{vmatrix} 0 & -1 \\ 1 & 4 \end{vmatrix} - 2\begin{vmatrix} 3 & -1 \\ 2 & 4 \end{vmatrix} - 2\begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix}$$

$$= 1(0+1) - 2(12+2) - 2(3-0)$$
$$= 1 - 24 - 4 - 6$$
$$= -33$$

3- Determint of 4x4 and over:

a. Add one row (or column) to another to make Matrix become 3x3 when first column zreo

b- Multiply a row or column by a constant (عدد ثابت) and add to another row or column

Ex:

$$\begin{vmatrix} 1 & 2 & -1 & 3 \\ 2 & -3 & 5 & 1 \\ -2 & 4 & 1 & -4 \\ 3 & 4 & -2 & 8 \end{vmatrix}$$

Solution:

$$A = \begin{vmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 7 & -5 \\ 0 & 8 & -1 & 2 \\ 0 & -2 & 1 & -1 \end{vmatrix} \begin{array}{l} \text{صف} \\ -2R_1 + R_2 \\ 2R_1 + R_3 \\ -3R_1 + R_4 \end{array}$$

$$|A| = 1 \begin{vmatrix} -7 & 7 & -5 \\ 8 & -1 & 2 \\ -2 & 1 & -1 \end{vmatrix} = -7 \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} - 7 \begin{vmatrix} 8 & 2 \\ -2 & -1 \end{vmatrix} - 5 \begin{vmatrix} 8 & -1 \\ -2 & 1 \end{vmatrix}$$

$$= -7(1-2) - 7(-8+4) - 5(8-2)$$

$$= -7(-1) - 7(-4) - 5(6)$$

$$= 7 + 28 - 30 = 5$$

Note: There is second Method by multiplying and addtion the elements of main diagonals (الاقطار الرئيسية) Minus the addtion pultipying of second diagonals (الاقطار الثانوية).

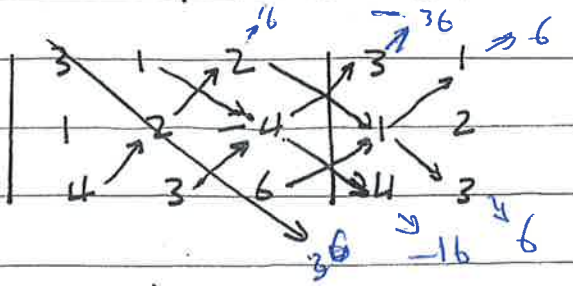
Note: Used for 3x3 Matrix only.

القاعدة: (حاصل جمع ضرب الاقطار الرئيسية) - (حاصل جمع ضرب الاقطار الثانوية)
وتستخدم في المصفوفة الثلاثية فقط

10

Ex: Find $|A|$ if $A = \begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & -4 \\ 4 & 3 & 6 \end{vmatrix}$

Solution: $|A| =$



$= (36 - 16 + 6) - (16 - 36 + 6)$

$= (26) - (-14) = 40$

Exercises: تمارين

Finde $|A|$ for these flowing used two Method?

1- $\begin{vmatrix} -1 & 2 & 4 \\ 7 & 2 & -3 \\ 1 & 4 & -3 \end{vmatrix}$ Answer = +134

2- $\begin{vmatrix} -2 & 7 \\ -3 & 5 \end{vmatrix}$ Answer = 11

3- $\begin{vmatrix} 5 & 4 & 0 & 7 \\ 4 & 5 & -2 & 2 \\ 6 & 0 & -3 & 6 \\ 2 & -3 & 5 & 4 \end{vmatrix}$ Answer = -1383 000 -3345000 ?

Cramer's Rule System طريقة كرامر لحل المعادلات الخطية

This rule use to find unknowns (يجاد المجاهيل) by using determinats (المحددات) by the constants (الثوابت).

$$a_1x_1 + b_1x_2 + c_1x_3 = d_1$$
$$a_2x_2 + b_2x_2 + c_2x_3 = d_2$$
$$a_3x_1 + b_3x_2 + c_3x_3 = d_3$$

$$x_1 = \frac{|A_1|}{|A|}, \quad x_2 = \frac{|A_2|}{|A|}, \quad x_3 = \frac{|A_3|}{|A|}$$

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$|A_1| = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}; \quad |A_2| = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}; \quad |A_3| = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

ملاحظة Note : $x_1 = x$, $x_2 = y$, $x_3 = z$

12

Ex Find x_1, x_2, x_3 for these system:

$$2x_1 + 4x_2 + x_3 = -11$$
$$-x_1 + 3x_2 - 2x_3 = -16$$
$$2x_1 - 3x_2 + 5x_3 = 21$$

Solution: Find $|A|, |A_1|, |A_2|, |A_3|$

$$|A| = \begin{vmatrix} 2 & 4 & 1 \\ -1 & 3 & -2 \\ 2 & -3 & 5 \end{vmatrix} = 2(15-6) - 4(-5+4) + 1(3-6)$$
$$= 18 + 4 - 3 = 19$$

$$|A_1| = \begin{vmatrix} -11 & 4 & 1 \\ -16 & 3 & -2 \\ 21 & -3 & 5 \end{vmatrix} = -11(15-6) - 4(-80+42) + 1(48-63)$$
$$= -99 + 152 - 15 = 38$$

$$|A_2| = \begin{vmatrix} 2 & -11 & 1 \\ -1 & -16 & -2 \\ 2 & 21 & 5 \end{vmatrix} = 2(-80+42) + 11(-5+4) + (-21+32)$$
$$= -76 - 11 + 11 = -76$$

$$|A_3| = \begin{vmatrix} 2 & 4 & -11 \\ -1 & 3 & -16 \\ 2 & -3 & 21 \end{vmatrix} = 2(63-48) - 4(-21+32) - 11(3-6)$$
$$= 30 - 44 + 33 = 19$$

$$x_1 = \frac{|A_1|}{|A|} = \frac{38}{19} = 2, \quad x_2 = \frac{|A_2|}{|A|} = \frac{-76}{19} = -4, \quad x_3 = \frac{|A_3|}{|A|} = \frac{19}{19} = 1$$

Exercises Solve these systemes with Cramer's rule:

1) $x+2y-z=5$
$2x-y+z=6$
$x-y-3z=-3$

So: $x=3.10526$, $y=1.6842$
$z=1.4736$

2) $2x+y+z=-1$
$-z+x-3y=-2$
$-y-z+2=x$

So: $x=-3$, $y=-3$
$z=8$

3) $4x+5y=10$
$3y+7x=15$

So: $x=1.95.65$
$y=0.43429$

4) $3x+y+z-2w=-3$
$x+y+z+w=4$
$-x-y+z+2w=6$
$3y-z-w+x=0$

So: $x=0.50444$
$y=0.49166$
$z=0.2$
$w=2.7639$

5) $6x+4y+z=22$
$y+2z-x=-2$
$z-x+2y=13$

So: $x=-1.26086$
$y=8.91304$
$z=-6.09695$

6) $2x+y-z=-2$
$2y+3z=5$
$2x-z=0$

So: $x=0.181818$
$y=-1.181818$
$z=2.454545$

الرياضيات

The Differntion التفاضل (الاشتقاق)

Using the definition for derivative the following theorems could be proved. These theorems quickly give the correct derivatives without going through the long limit process.

Differentiation Theorms (الاشتقاق) قواعد التفاضل

1. $\frac{d}{dx}(c) = 0$, c any constant. , $\frac{d}{dx}(x) = 1$

2. $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$, $x \neq 0$ anel n any real number.

3-a. $\frac{d}{dx}[g(x) + h(x)] = \frac{d}{dx} g(x) + \frac{d}{dx} h(x)$ or $\dot{g}(x) + \dot{h}(x)$ مجموع دالتين او اكثر

3-b. $\frac{d}{dx}[g(x) - h(x)] = \dot{g}(x) - \dot{h}(x)$ طرح دالتين او اكثر

4-a. $\frac{d}{dx}[g(x) \cdot h(x) = g(x) \cdot \frac{d}{dx} h(x) + h(x) \cdot \frac{d}{dx} g(x)$, or مشتق دالتين

$g(x) \cdot \dot{h}(x) + h(x) \cdot \dot{g}(x)$

4-b. $\frac{d}{dx}[c \cdot h(x) = c \cdot \frac{d}{dx} h(x)$, or $c \cdot \dot{h}(x)$.

5. $\frac{d}{dx}\left[\frac{g(x)}{h(x)}\right] = \frac{h(x) \cdot \frac{d}{dx} g(x) - g(x) \frac{d}{dx} h(x)}{[h(x)]^2}$, or

$\frac{h(x) \cdot \dot{g}(x) - g(x) \cdot \dot{h}(x)}{[h(x)]^2}$ قسمة دالتين

6- $\frac{d}{dx}(\ln x) = \frac{1}{x}$, $\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}$

7. $\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}e^u = e^u \cdot \frac{du}{dx}$

Examples for these theorems :

Ex1- Find $\frac{d}{dx}x^5$; $\frac{d}{dx}\sqrt{x}$, $\frac{d}{dx}\left(\frac{1}{x^2}\right)$

Solution :- In each problem use $\frac{d}{dx}x^n = nx^{n-1}$

* $\frac{d}{dx}x^5 = 5x^4$

* $\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$.

* $\frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{d}{dx}x^{-2} = -2x^{-3} = -\frac{2}{x^3}$.

Ex2- Find $\bar{y}$ for $y = x^3 - x^2 + x - 4$ and $\bar{\bar{y}}, \bar{\bar{\bar{y}}}, \bar{\bar{\bar{\bar{y}}}}$

Solution :- To find the derivative of the sum or difference of two or more functions

$\bar{y} = 3x^2 - 2x + 1 - 0 = 3x^2 - 2x + 1$

$\bar{\bar{y}} = 6x - 2$

$\bar{\bar{\bar{y}}} = 6$

$\bar{\bar{\bar{\bar{y}}}} = 0$

Ex 3 - Find $\frac{d}{dx}(x^3+7)(x^2-9x+4)$ and $\bar{y}$, $\bar{\bar{y}}$

Solution: In words the derivative of the product of two functions is equal to the first function times the derivative of the second function plus the second function times the derivative of the first function.

كلمات قاعدة الضرب:

$$= (x^3+7)\cdot\frac{d}{dx}(x^2-9x+4) + (x^2-9x+4)\cdot\frac{d}{dx}(x^3+7)$$

$$= (x^3+7)(2x-9) + (x^2-9x+4)(3x^2)$$

$$= 2x^4 - 9x^3 + 14x - 63 + 3x^4 - 27x^3 + 12x^2$$

$$= 5x^4 - 36x^3 + 12x^2 + 14x - 63$$

$\bar{\bar{y}} = 20x^3 - 108x^2 + 24x + 14 \longrightarrow \bar{y} = 6x^2 - 216x + 24$

Ex 4 - If $y = 3x^{\frac{1}{2}}$ Find $\frac{dy}{dx}$?

Solution: $\frac{dy}{dx} = 3\cdot\frac{d}{dx}x^{\frac{1}{2}} = 3\cdot\frac{1}{2}x^{-\frac{1}{2}} = \frac{3}{2\sqrt{x}}$

Ex 5 - $y = 3\sqrt{x}\cdot(4x-17)$ Find $\bar{y}$

Solution $\bar{y} = 3\sqrt{x}\cdot 4 + (4x-17)\cdot 3\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$ مشتقة الضرب

$$= 12\sqrt{x} + \frac{3(4x-17)}{2\sqrt{x}}$$

(17)

Ex6- Find $\left[\frac{5x+7}{x^2}\right]$

Solution: ① $\frac{d}{dx}(5x+7)\,x^{-2} = (5x)+7)(-2x^{-3}) + x^{-2}(5)$

$= -10x^{-2} - 14x^{-3} + 5x^{-2} = -5x^{-2} - 14x^{-3}$

① تصعيد المقام الى البسط
② وقسمة والبيتم

$= \frac{-5}{x^2} - \frac{14}{x^3} = \frac{-5x-14}{x^3}$

$\times \quad 1$

So:- ② : $\frac{d}{dx}\left[\frac{5x+7}{x^2}\right] = \frac{x^2(5)-(5x+7)(2x)}{x^4}$

تطبيق قاعدة قسمة والبيتم

$= \frac{5x^2-10x^2-14x}{x^4} = \frac{-5x^2-14x}{x^4} = \frac{-5x-14}{x^3}$

$\div x$

Ex7: Find $\bar{y}$ if $y = \frac{3x^3}{x-7}$

Solution: $\bar{y} = \frac{(x-7)\,9x^2 - 3x^3(1)}{(x-7)^2}$

$= \frac{9x^3-63x^2-3x^3}{(x-7)^2} = \frac{6x^3-63x^2}{(x-7)^2}$

$= \frac{3x^2(2x-21)}{(x-7)^2}$

Ex8: If $y=(3x^5-7x^2-4)^{10}$ find $\bar{y}$?

Solution $\bar{y} = 10(3x^5-7x^2-4)^9 \cdot (3x^5-7x^2-4)^{-}$

$= 10(3x^5-7x^2-4)^9 \cdot (15x^4-14x)$

اولاً اشتقاق قوة القوس
ثم اشتقاق ما داخل القوس

Ex 9 : Find $\frac{d}{dx}[xe^x]$

Solution : $\frac{d}{dx}[xe^x] = x\frac{d}{dx}(e^x) + e^x\frac{d}{dx}(x)$

تطبيق قاعدة ضرب دالتين

$= xe^x + e^x(1) = xe^x + e^x = e^x(x+1)$

Ex 10: Find $\frac{dy}{dx}$ for $\frac{2x}{\ln x}$

Solution :

$$\frac{d}{dx}\frac{2x}{\ln x} = \frac{\ln x \cdot \frac{d}{dx}(2x) - 2x \cdot \frac{d}{dx}(\ln x)}{(\ln x)^2}$$

تطبيق قاعدة قسمة دالتين

$$= \frac{\ln x(2) - 2x\cdot\frac{1}{x}}{(\ln x)^2} = \frac{2\ln x - 2}{(\ln x)^2} = \frac{2(\ln x - 1)}{(\ln x)^2}$$

Ex 11 : Find $\bar{y}$ for $y = (3+6x-2x^2)(\sqrt{x}-4)$

ضرب دالتين

Solution : $\bar{y} = (3+6x-2x^2)\cdot\frac{1}{2}x^{-\frac{1}{2}} + (\sqrt{x}-4)(6-4x)$

$$= \frac{(3+6x-2x^2)}{2\sqrt{x}} + (\sqrt{x}-4)(6-4x)$$

Ex 12: $y = \frac{3x^2+16}{(8-x^3)}$

Solution : $\bar{y} = \frac{(8-x^3)(6x) - (3x^2+16)(-3x^2)}{(8-x3)^2} = \frac{48x - 6x^4 + 9x^4 + 48x^2}{(8-x^3)^2}$

قسمة دالتين

$$= \frac{3x^4 + 48x^2 + 48x}{(8-x3)^2}$$

Exercise

Find $\frac{dy}{dx}$ for these Followings:

1- $y = (x^2 + \sqrt{x} - \frac{1}{x^2})$

2- $\frac{d}{dx}[(x^5 - x)(x^2 + 7)]$

3- $y = 5x^{10} + 5x^{-9} + \sqrt[3]{2x}$

4- $\frac{d}{dx}\left[\frac{x^2 - 8}{3x^3 - 7}\right]$

5- $y = \frac{1}{6}(3 + 3x - x^2)$

6- $y = \frac{3 + 2x - x^2}{6x}$

7- $y = (\ln x)\sqrt[3]{x^4}$

8- $y = (16x^2 - x)(e^x)$

9- $y = \frac{x^2 - x - 2}{-2x}$

10- $y = \frac{\ln x}{3 + x}$

11- $y = \frac{(e^x)(\ln x)}{3 - x^3}$

12- $y = \sqrt{3x^2 + 5x}$

13- $y = x^2 e^{3x}$

14- $y = \ln(2x^2 + x)$

15- $y = \frac{1}{3x^2 - 7}$

16- $y = \frac{3x^2 + 16}{8 - x^3}$

17- prove $y = 0$ if $y = 3x^4 - 2x^3 + 3x^2 - x + 1$

(20)

الرياضيات

تفاضل الدوال المثلثية Trigonometric Function

أشتقاق الدوال المثلثية:- If u derivative for x i.e.

الدالة function	الدالة المشتقة The Derivative
1- $y = \sin u$	$\bar{y} = \cos u \cdot \bar{u}$
2- $y = \cos u$	$\bar{y} = -\sin u \cdot \bar{u}$
3- $y = \tan u$	$\bar{y} = \sec^2 u \cdot \bar{u}$
4- $y = \cot u$	$\bar{y} = -\csc^2 u \cdot \bar{u}$
5- $y = \sec u$	$\bar{y} = \sec u \tan u \cdot \bar{u}$
6- $y = \csc u$	$\bar{y} = -\csc u \cot u \cdot \bar{u}$

Exa: find $\bar{y}$ for these following:

1- $y = \sin(2-3x)$ ($u = 2-3x$, $\bar{u} = -3$)

So: $\bar{y} = \cos(2-3x) \cdot (2-3x) = -3\cos(2-3x)$

2- $y = \sin(x^2+5)$

So: $\bar{y} = \cos(x^2+5) \cdot 2x = 2x\cos(x^2+5)$

3- $y = \cos(x^2+x)^3$ ($\bar{u} = 2x+1$)

So: $\bar{y} = -\sin(x^2+x)^3 \cdot 3(x^2+x)^2 \cdot (2x+1)$

$= -3(x^2+x)^2 \sin(x^2+x)^3 (2x+1)$

في المثال السابق: تم اشتقاق cos ثم اشتقاق القوى للقوس ثم اشتقاق ما بداخل القوس.

4- $y = \tan^3 \frac{2}{5}x$

So: $y = (\tan \frac{2x}{5})^3$

$\bar{y} = 3(\tan \frac{2x}{5})^2 \sec^2(\frac{2x}{5}) \cdot (\frac{2}{5})$

T u u

$= \frac{6}{5} \tan^2 \cdot \frac{2x}{5} \sec^2 \frac{2x}{5}$

5- $y = \sec(\sin x^2)$

So: $\bar{y} = \sec(\sin x^2) \tan(\sin x^2) \cos x^2 \cdot 2x$ ج

6- $y = \tan 2x \cdot \sec 3x$ ضرب متغيرين

So: $\bar{y} = \tan 2x \cdot \sec 3x \tan 3x \cdot 3 + \sec 3x \cdot \sec^2 2x \cdot 2$

$= 3\tan 2x \cdot \sec 3x \tan 3x + 2 \sec 3x \cdot \sec^2 2x$

7- $y = \sqrt{\csc x}$

So: $y = (\csc x)^{\frac{1}{2}}$

$\bar{y} = \frac{1}{2}(\csc x)^{-\frac{1}{2}} \cdot -\csc x \cot x$

$= -\frac{1}{2} \frac{1}{\sqrt{\csc x}} \cdot \csc x \cot x$

8- $y = \sin 3x + \cos 2x$ جمع متغيرين

So: $\bar{y} = \cos 3x \cdot 3 - \sin 2x \cdot 2 = 3\cos 3x - 2\sin 2x$

9- $y = \tan x^2$

So: $\bar{y} = \sec^2 x^2 \cdot 2x = 2x \sec^2 x^2$

10- $y = \tan^2 x$

So: $y = (\tan x)^2$

$\bar{y} = 2 \tan x \cdot \sec^2 x$

11- $y = \cot(1 - 2x^2)$

So: $\bar{y} = -\csc^2(1-2x^2) \cdot -4x$

$= 4x \csc^2(1-2x^2)$

12- $y = \sec^3 \sqrt{x}$

So: $y = \sec^3 x^{\frac{1}{2}}$

$\bar{y} = 3 \sec^2 x^{\frac{1}{2}} \cdot \sec x^{\frac{1}{2}} \tan x^{\frac{1}{2}} \cdot \frac{1}{2} x^{-\frac{1}{2}}$

$= \frac{3}{2\sqrt{x}} \sec^3 \sqrt{x} \tan \sqrt{x}$

13- $y = x^2 \sin x$

So: $\bar{y} = x^2 \cos x + 2x \sin x$ حاصل ضرب مشتقين

14- $y = \frac{\cos x}{x}$

So: $\bar{y} = \frac{x(-\sin x) - \cos x}{x^2} = \frac{-x \sin x - \cos x}{x^2}$ قسمة مشتقين

Logarithmis Function الدالة اللوغاريتمية

اشتقاق الدوال اللوغاريتمية: If u function derivative for x

1- $\ln(x \cdot y) = \ln x + \ln y$ لحالة الضرب

2- $\ln \frac{x}{y} = \ln x - \ln y$ لحالة القسمة

3- $\ln x^r = r \ln x$

4- $\ln 1 = 0$

5- $\log_a x = \frac{\ln x}{\ln a}$

الدالة Function — The Derivative

1- $y = \ln u$ $\quad \bar{y} = \frac{1}{u} \cdot \bar{u}$

2- $y = \log_a u$ $\quad \bar{y} = \frac{1}{\ln a \, u} \cdot \bar{u}$

Exa: Find $\bar{y}$ for these Followring:

1- $y = \ln(3x^2)$ $\quad \bar{u} = 6x$

So: $\bar{y} = \frac{1}{3x^2} \cdot 6x = \frac{6x}{3x^2} = \frac{2}{x}$

2- $y = \log_5(\tan x^2)$

So: $\bar{y} = \frac{1}{\ln 5 \tan x^2} \cdot \sec^2 x^2 = \frac{\sec^2 x^2}{\ln 5 \tan x^2}$

(24)

3- $y = \ln^2(x+3)$

So: $y = 2\ln(x+3) \cdot \frac{1}{x+3} \cdot (1) = \frac{2\ln(x+3)}{(x+3)}$ ج3

4- $y = \ln \sin 3x$

So: $\bar{y} = \frac{1}{\sin 3x} \cdot \cos 3x = \cot 3x$; $\cot = \frac{\cos x}{\sin x}$ ن/ب

Exercise

Find $\bar{y}$ for these following:

1- $y = \ln(4x-5$

2- $y = \ln\sqrt{3-x^2}$

3- $y = (\ln x^2)/x^2$

4- $y = x \cdot \ln x - x$

5- $y = \ln(\sec x + \tan x)$

6- $y = 3\sin x$

7- $y = 4\tan 5x$

8- $y = \sin^2(3x-2)$

9- $y = \frac{1}{2}\tan x \sin 2x$

10- $y = \cos(1-x)^2$

- $y = x^2 \sin x + 2x\cos x - 2\sin x$

- $y = \frac{1}{4}\csc 4x$

- $y = \sin x - x\cos x + x^2 + 4x + 3$

- $y = \cos(1-x^2)$

- $y = \sqrt{\sin x}$

(*) 1

Partal Derivative التفاضل الضمني (الجزئي)

Another use of derivatives is in the study of functions where the value of the dependent variable may depend on the values of two or more independent variables.

يستخدم التفاضل الضمني (الجزئي) عندما يراد ايجاد التفاضل لمتغيرين أو اكثر.

If $z = f(x,y)$, then we might wish to study the rate at which z changes as x changes if y is held constant.

أولاً ايجاد تفاضل المتغير x ونعتبر المتغير y هو الثابت.

ثانياً ايجاد " " y " " x هو الثابت.

$z = f(x,y)$ as it:

نقرأ بارتشل

(1) $\frac{\partial z}{\partial x}$ or $f_x(x,y) \Rightarrow$ Take the derivative of f and treat y as a constant.

نؤخذ تفاضل x ونعتبر y ثابت.

(2) $\frac{\partial z}{\partial y}$ or $f_y(x,y) \Rightarrow$ Take the derivative of f and treat x as constant.

عكس الحالة الأولى نؤخذ تفاضل y ونعتبر x ثابت.

Examples:

* Find partial derivatives for certain functions ?

1- $z = 3x^2y + 5x^2 + 4y^2$

So: $\frac{\partial z}{\partial x} = 3y \frac{d}{dx}(x^2) + \frac{d}{dx}(5x^2) + \frac{d}{dx}(4y^2)$

Here $3y$ and $4y^2$ are treated as constants.

$$\frac{\partial z}{\partial x} = 3y(2x) + 10x$$

ثم أجراء المشتقة للمتغير x

$$= 6xy + 10x$$

$$\frac{\partial z}{\partial y} = 3x^2 \frac{d}{dy}(y) + \frac{d}{dy}(5x^2) + \frac{d}{dy}(4y^2)$$

Here $3x^2$ and $5x^2$ are treated as constants.

$$\frac{\partial z}{\partial y} = 3x^2(1) + 0 + 8y$$

ثم أجراء المشتقة للمتغير y

$$= 3x^2 + 8y$$

(3)

2- Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if:

$$z = f(x,y) = 2x^2y + y^2 + x^2$$

So: To find $\frac{\partial z}{\partial x}$ treat y as a constant.

$$f_x(x,y) = \frac{\partial z}{\partial x} = 4xy + 2x$$

Notice that $\frac{\partial}{\partial x} y^2 = 0$

To find $\frac{\partial z}{\partial y}$ treat x as a constant.

$$f_y(x,y) = \frac{\partial z}{\partial y} = 2x^2 + 2y$$

3 Find the second differentiation of this function

$$z = 2xy + x^3 - y^3$$

So: $f_x(x,y) = 2y + 3x^2 \Rightarrow f_{xx}(x,y) = 6x$

$f_y(x,y) = 2x - 3y^2 \Rightarrow f_{yy}(x,y) = -6y$

$\Rightarrow f_{xy}(x,y) = 2$

Exercises:

1- Given $f(x,y) = x^2 + xy + y^2$

2- Find $f_{xy}(x,y)$ for these functions?

- $f(x,y) = x^3 - 3x^3y + 2y$
- $f(x,y) = 3x^2 + x^2y^2 - 2y^2$
- $f(x,y) = \frac{y - 2x}{xy}$
- $f(x,y) = (x^3 + y^2)^4$
- $z = 3x^2y - 2y^2$
- $z = 4x^3 + \frac{1}{2}y^2 - xy$
- $z = 6xy - x^3 - y^3$
- $z = 2x - 4y - x^2 - y^2 - 2$

The Integral التكامل

1- The indefinite Integral التكامل الغير محدود

Properties of Indefinite integral :

قواعد التكامل الغير محدود (للدوال الخطية) :

1- $\int dx = x + c$

2- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $n \neq -1$

3- $\int a\, f(x) dx = a \int f(x) dx = ax + c$

4- $\int [f(x) \mp g(x) dx] = \int f(x) dx \mp \int g(x) dx$

5- $\int [f(x)]^n \cdot f'(x) dx = \frac{[F(x)]^{n+1}}{n+1} + c$; $n \neq -1$

6- $\int \frac{dx}{x} = \ln|x| + c$

ملاحظة: c عدد ثابت اختياري والسبب يرجع لكون مشتقة العدد الثابت يساوي الصفر ويسمى بثابت التكامل.

Note: The Integral opisit of differintion

التكامل عكس التفاضل أو الاشتقاق

ملاحظة

Examples:

1- $\int x\,dx = \frac{1}{2}x^2 + c$

2- $\int x^{10}\,dx = \frac{1}{11}x^{11} + c = \frac{x^{11}}{11} + c$

3- $\int (\sqrt{x})^3\,dx = \int x^{\frac{3}{2}}\,dx = \frac{1}{\frac{5}{2}}x^{\frac{5}{2}} + c = \frac{2}{5}x^{\frac{5}{2}} + c$

4- $\int dx = x + c$

5- $\int (2x^2 - 3 - 5\sqrt{x})\,dx = \int 2x^2\,dx - \int 3\,dx - \int 5x^{\frac{1}{2}}\,dx$

$= 2\int x^2\,dx - 3\int dx - 5\int x^{\frac{1}{2}}\,dx$

$= 2\cdot\frac{x^3}{3} - 3x - \frac{5x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{2}{3}x^3 + 3x - \frac{10}{3}x^{\frac{3}{2}} + c$

6- $\int \left(\frac{x^4 + 2x^2 + 1}{x^2}\right)dx = \int \left(\frac{x^4}{x^2} + \frac{2x^2}{x^2} + \frac{1}{x^2}\right)dx$

$= \int (x^2 + 2 + x^{-2})\,dx$

$= \frac{x^3}{3} + 2x + \frac{x^{-1}}{-1} + c$

$= \frac{1}{3}x^3 + 2x - x^{-1} + c = \frac{1}{3}x^3 + 2x - \frac{1}{x} + c$

(27)

7- $\int \left(\frac{1}{\sqrt{x}} - \frac{1}{x^2}\right) dx = \int x^{-\frac{1}{2}} dx - \int x^{-2} dx$

$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{x^{-1}}{-1} + c = 2\sqrt{x} + \frac{1}{x} + c$

8- $\int \sqrt{x^2+1}\, dx = \int (x^2+1)^{\frac{1}{2}} dx = \frac{(x^2+1)^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}(x^2+1)^{\frac{3}{2}} + c$

9- $\int \frac{1}{2x} dx = \frac{1}{2}\int \frac{1}{x} dx = \frac{1}{2} \ln|x| + c$

10- $\int \frac{4x+7}{2x^2+7x+3} dx = \ln|2x^2+7x+3| + c$

لوجود مشتقة المقام في البسط

11- $\int \frac{x^2 dx}{x^3-1} dx = \frac{1}{3}\int \frac{3x^2 dx}{x^3-1} = \frac{1}{3}\ln|x^3-1| + c$

[illegible]

12- $\int (x^2+1)^2 dx = \int (x^4+2x^2+1) dx$ مفكوك مربع

$= \frac{x^5}{5} + \frac{2x^3}{3} + x + c$

13- $\int 3x^{-4} dx = \cancel{3}\frac{x^{-3}}{-\cancel{3}} + c = \frac{-1}{x^3} + c$

14- $\int \frac{5}{\sqrt{1-x^3}} dx = \int 5 (1-x^3)^{-\frac{1}{2}} dx$

$= 5\int -3x^2 (1-x^3)^{-\frac{1}{2}} dx$

$= 5\left(\frac{(1-x^3)^{\frac{1}{2}}}{\frac{1}{2}}\right. = 10\sqrt{1-x^3} + c$

القاعدة
تكامل حسب (5)
ص 20

(28)

Integration of Trigonometric function

تكامل الدوال المثلثية: صيغ

1- $\int \sin x\,dx = -\cos x + c$

2- $\int \cos x\,dx = \sin x + c$

3- $\int \sec^2 x\,dx = \tan x + c$

4- $\int \sec x \tan x\,dx = \sec x + c$

5- $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$

6- $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$

7- $\int \cos ax\,dx = \frac{1}{a}\sin ax + c$

8- $\int \tan x\,dx = \ln|\sec x| + c$

9- $\int \cot x = -\ln|\operatorname{cosec} x| + c$

صيغ "علاقات مثلثية"

1- $\tan x = \frac{\sin x}{\cos x} \longleftrightarrow \sin x = \cos x \tan x \longleftrightarrow \cos x = \frac{\sin x}{\tan x}$

2- $\cot x = \frac{\cos x}{\sin x} \longleftrightarrow \cos x = \sin x \cot x \longleftrightarrow \sin x = \frac{\cos x}{\cot x}$

3- $\sin^2 x = \frac{1}{2}(1 - \cos 2x) \longleftrightarrow \sin^2 x = \frac{1-\cos 2x}{2}$

4- $\cos^2 x = \frac{1}{2}(1 + \cos 2x) \longleftrightarrow \cos^2 x = \frac{1+\cos 2x}{2}$

5- $\tan^2 x = \sec^2 x - 1 \longleftrightarrow \sec^2 x = \tan^2 x + 1$

6- $\cot^2 x = \operatorname{cosec}^2 x - 1 \longleftrightarrow \operatorname{cosec}^2 x = \cot^2 x + 1$

7- $\sin^2 x + \cos^2 x = 1 \longleftrightarrow \sin^2 x = 1 - \cos^2 x \longleftrightarrow \cos^2 x = 1 - \sin^2 x$

(29)

Examples

1- $\int \cos 4x \, dx = \frac{1}{4} \sin 4x + c$ (ق 6) ص 25

2- $\int \sec^2\left(\frac{3x}{2}\right) dx = \frac{2}{3} \tan\left(\frac{3x}{2}\right) + c$ (ق 6) ص 25

3- $\int \sqrt{\cos x} \sin x \, dx = \int (\cos x)^{\frac{1}{2}} \sin x \, dx$

$= -\int (\cos x)^{\frac{1}{2}} \sin x \, dx$ (1-)*

لأجل الحصول على مشتقة $\cos x$ وهي $-\sin x$

$= \frac{-(\cos x)^{\frac{3}{2}}}{\frac{3}{2}} + c = -\frac{2}{3}(\cos x)^{\frac{3}{2}} + c$ وتكامل حسب قاعدة (5) ص 20

4- $\int \frac{\sin x + \cos x}{\cos x} dx = -\int \frac{\sin x}{\cos x} dx + \int \frac{\cos x}{\cos x} dx$

$= -\int \frac{\sin x}{\cos x} dx + dx$

$= -\ln|\cos x| - x + c$ ثم ضرب الدالة في (1-)

للحصول على مشتقة $\cos x$ في البسط وهي $(-\sin x)$ وأجري التكامل حسب قاعدة (6) ص 25

5- $\int \tan^2 3x \, dx = \int (\sec^2 3x - 1) dx$

تم تحويل الدالة إلى الشكل المعلوم حسب العلاقات المثلثية العامة علاقة (5) ص 26 لأنه لا يوجد تكامل للدالة $\tan^2 x$

$= \frac{1}{3} \tan 3x - x + c$ اجري التكامل حسب ق (5) ص 26

6- $\int \cos^2 x \, dx = \frac{1}{2} \int (1 + \cos 2x) dx$ من العلاقة (4) ص 26 لأنه لا توجد تكامل $\cos^2 x$

$= \frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right] + c = \frac{1}{2}x + \frac{1}{4}\sin 2x + c$ ق (6) ص 26

Exercise

1- $\int x^{\frac{1}{3}} dx$

2- $\int (2x^2 - 5x + 3) dx$

3- $\int (1-x)\sqrt{x}\, dx$

4- $\int (3x+4)^2 dx$

5- $2\int x^2 dx$

6- $\int \frac{x^2 dx}{\sqrt[4]{(x^2+2)}}$

7- $\int \frac{x^3 + 5x^2 - 4}{x^2} dx$

8- $\int \frac{dx}{x+2} dx$

9- $\int \frac{dx}{2x-3} dx$

10- $\int \frac{x^2 dx}{1-2x^3} dx$

11- $\int \frac{x+2}{} dx$

12- $\int \cos 3x\, dx$

13- $\int \sin^2 x \cos x\, dx$

14- $\int \tan x\, dx$

15- $\int (\sin x + \sec^2 x) dx$

16- $\int (\cot^2 x + \tan^2 x) dx$

17- $\int (\sin^2 x - \cos^2 x + \tan x) dx$

18- $\int \frac{x}{x^2+2} dx$

19- $\int \tan^3 x dx$

20- $\int \frac{2x+7}{x^2+7x+15} dx$

21- $\int \left(\frac{1}{x^4} + x^{\frac{2}{5}} - 7x^4\right) dx$

22- $\int (x^2+1)\sqrt{x^3+3x+1}\, dx$

23- $\int \frac{2x^3+1}{x^4+2x+1} dx$

24- $\int \frac{x+\cos 2x}{x^2+\sin 2x} dx$

25- $\int x \tan(2x^2+1) dx$

26- $\int x^{\frac{2}{5}} dx + \int \sqrt{5x^3}^2 dx$

2- The definite integral

التكامل المحدد (المحدود)

It is possible to use this same type of procedure for a function f defined on [a, b] without ever mentioning concept of area.

the definite integral to the f(x)dx from a to b for x is:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

To find the definite integral:

1- Find the indinite integral for function. ايجاد التكامل اللامحدد

2- Replaced the variable x may be in front of $\int_a^b$. تعويض

ملحوظة 1- $\int_a^a f(x)dx = 0$ 2- $\int_a^b f(x)dx = -\int_b^a f(x)dx$

Examples

Find I for these following:-

1- $\int_1^2 x^3\,dx = \left[\frac{x^4}{4}\right]_1^2 = \frac{2^4}{4} - \frac{1}{4} = \frac{16}{4} - \frac{1}{4} = \frac{15}{4} = 3\frac{3}{4}$

2- $\int_1^3 (x^2 + 2x - 1)\,dx$

$$I = \left[\frac{x^3}{3} + \frac{2x^2}{2} - x\right]_1^3 = \left(\frac{3^3}{3} + 3^2 - 3\right) - \left(\frac{1}{3} + 1 - 1\right)$$

$$= \left(\frac{27}{3} + 9 - 3\right) - \left(\frac{1}{3}\right) = 9 + 9 - 3 - \frac{1}{3} = 15 - \frac{1}{3} = \frac{45-1}{3} = \frac{44}{3} = 14\frac{2}{3}$$

(21)

3- $\int_2^3 2x\,dx$

$I = \left[\frac{2x^2}{2} \right]_2^3 = 3^2 - 2^2 = 9 - 4 = 5$

4- $\int_0^{2\pi} \sin x\,dx$

$I = \left[-\cos x \right]_0^{2\pi} = -\cos 2\pi + \cos 0 = -1 + 1 = 0$

5- $\int_0^1 (3-x)\,dx$

$I = \left[\frac{1}{2}(3-x)^2 \right]_0^1 = \left[\frac{1}{2}(3-1)^2 \right] - \left[\frac{1}{2}(3-0)^2 \right]$

$= \frac{4}{2} - \frac{9}{2} = -\frac{5}{2} = -2\frac{1}{2}$

6- $\int_{-1}^{4} 3(x+2)^2\,dx$

$I = \left[\frac{3}{3}(x+2)^3 \right]_{-1}^{4} = (4+2)^3 - (-1+2)^3 = 6^3 - 1^3 = 216 - 1$

$= 215$

7- $\int_1^3 (x^2+2)\,dx = \int_1^3 x^2\,dx + \int_1^3 2\,dx$

$I = \left[\frac{x^3}{3} + 2x \right]_1^3 = \left[\left(\frac{3^3}{3} - \frac{1}{3} \right) \right] + \left[(2(3) - 2(1)) \right]$

$= \left(\frac{27}{3} - \frac{1}{3} \right) + (6-2) = \left(9 - \frac{1}{3} \right) + 4 = \frac{38}{3} = 12\frac{2}{3}$

Exercise

Find I for these following:

1- $\int_0^{\frac{\pi}{2}} \sin 2x\,dx$

2- $\int_{\frac{\pi}{2}}^{\pi} \cos x\,dx$

3- $\int_1^4 \left(\frac{1}{\sqrt{x}} - \frac{1}{x^2}\right)dx$

4- $\int_0^2 \frac{x^2 dx}{\sqrt{x^3+1}}$

5- $\int_0^3 (x^2 + \sqrt{3x+1})\,dx$

6- $\int_1^5 (7-x)\,dx$

7- $\int_0^9 \frac{1}{2}\sqrt{x}\,dx$

8- $\int_1^3 (2x - x^2)\,dx$

9- $\int_{-1}^2 (6x^2+1)\,dx$

10- $\int_1^9 (3\sqrt{x} - 2)\,dx$

11- $\int_4^9 12\sqrt{x}\,dx$

(34)

Applications of The Definite Integral
تطبيقات التكامل المحدود

1- The Area Between Two Curves: المساحة بين المنحنيين

We use this properties to find the area between two curves.

$$A = \int_a^b [|f(x)| - |g(x)|]\,dx$$

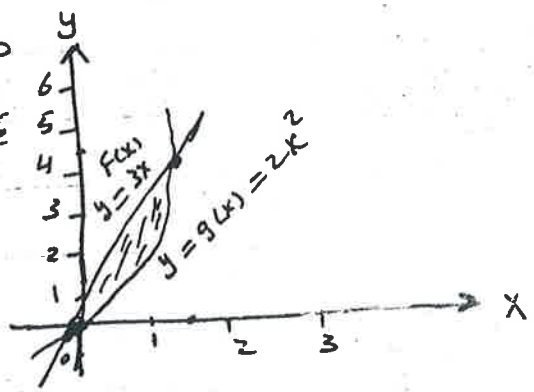
Exp:1 Find the area between the curves, $y = 2x^2$ and $y = 3x$

So: first, we find the points of intersection of $y = 2x^2$ and $y = 3x$ by solving the two equations simultaneously

نقاط التقاطع المنحنيين لايجاد نقاط التقاطع

$$3x = 2x^2 \Rightarrow 2x^2 - 3x = 0$$

$$\Rightarrow x(2x-3) = 0 \Rightarrow x = 0 \text{ or } x = \frac{3}{2}$$



$$A = \int_0^{\frac{3}{2}} [|f(x)| - |g(x)|]\,dx$$

$$= \int_0^{\frac{3}{2}} |(3x - 2x^2)|\,dx$$

$$= \left[\frac{3}{2}x^2 - \frac{2}{3}x^3\right]_0^{\frac{3}{2}} = \left[\frac{3}{2}\left(\frac{3}{2}\right)^2 - \frac{2}{3}\left(\frac{3}{2}\right)^3\right] - \left[\frac{3}{2}(0)^2 - \frac{2}{3}(0)^3\right]$$

$$= \frac{27}{8} - \frac{2}{3}\left(\frac{27}{8}\right) = \frac{1}{3}\left(\frac{27}{8}\right) = \frac{9}{8} \Rightarrow$$

$$= 1\frac{1}{8} \text{ square units}$$

Ex#2: Find the area of the region bounded by the curves $y = x^2$ and $y = 8x$

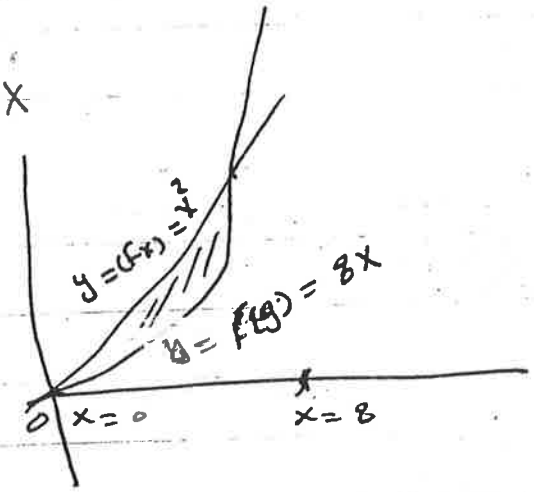
So: First find the points of intersection:

$$x^2 = 8x \Rightarrow x^2 - 8x = 0 \Rightarrow x(x-8) = 0$$

$$x = 0 \quad \text{or} \quad x - 8 = 0 \Rightarrow x = 8$$

$$A = \int_a^b [|f(x)| - |g(x)|]\, dx$$

$$A = \int_0^8 |(x^2 - 8x)|\, dx$$



$$A = \left[\frac{x^3}{3} - \frac{8}{2} x^2 \right]_0^8$$

$$= \left[\frac{8^3}{3} - 4(8)^2 \right] - [0]$$

$$= \left[\frac{512}{3} - 4(64) \right] = \frac{512}{3} - 256 = 170.666 - 256$$

$$= |-85.333| = 85.333 \text{ Unit}^2$$

Exercise:

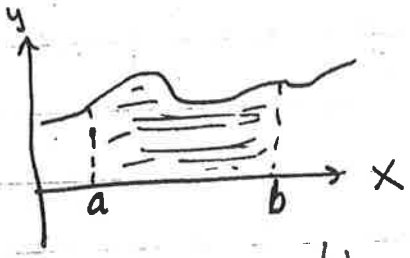
1- Find the area between two curves $y = x^2$, $y = 2x$?

2- Find the area between two curves $y = x^2$, $y = 3x$ when points $x = 1$, $x = 2$?

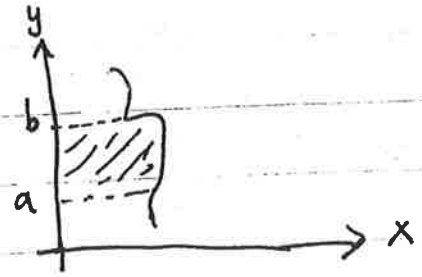
2- Find the area under the curve : ايجاد المساحة تحت المنحنى :

We use this properties to find the area under the curve:

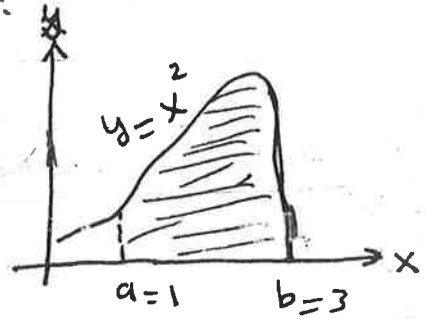
a- $A = \int_a^b |F(x)|\,dx$, when $y = F(x)$ for axis X



b- $A = \int_a^b |g(y)|\,dx$, when $y = g(y)$ for axis y



Exa:1 : Find the area between $y = x^2$ and X axis when lines $a = 1$, $b = 3$?



So: $A = \int_a^b |F(x)|\,dx$

$$A = \int_1^3 |x^2|\,dx = \left[\frac{x^3}{3}\right]_1^3$$

$$= \frac{3^3}{3} - \frac{1}{3} = 9 - \frac{1}{3} = \frac{27-1}{3} = \frac{26}{3} = 8\frac{2}{3} \text{ unit}^2$$

37

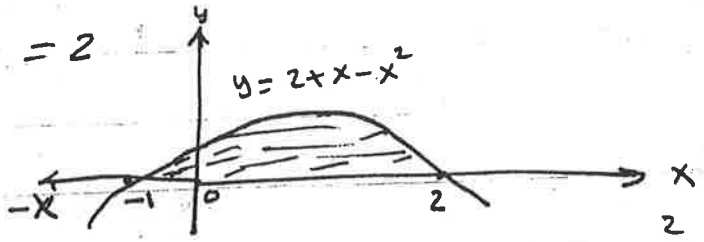
Exa:2 Find the earea bounded by the $y = 2 + x - x^2$ and the x axis.

So: First solve the system to find a, b.

$$x^2 - x - 2 = 0 \Rightarrow (x+1)(x-2) = 0$$

$$\Rightarrow x + 1 = 0 \Rightarrow x = -1 \quad , \quad x - 2 = 0 \Rightarrow x = 2$$

$$a = -1 \quad , \quad b = 2$$



$$A = \int_a^b |F(x)| dx$$

$$A = \int_{-1}^{2} (2 + x - x^2) dx = \left[2x + \frac{x^2}{2} - \frac{x^3}{3} \right]_{-1}^{2}$$

$$A = \left(2(2) + \frac{2^2}{2} - \frac{2^3}{3}\right) - \left(-2 + \frac{1}{2} + \frac{1}{3}\right) = 4\frac{1}{2} \text{ unit}^2$$

Exercise:

1- Find the earea between curve $y = x^3 - 6x^2 + 8x$ and x axis?

2- Find the eared between curve $x^2 - 7x + 6$ and y aixs?

38

Find the volums of the cylinders bodies

ايجاد حجوم الاجسام الدورانية

1-A: If the move around x axis is the volum:

الحجم $V = \int_a^b \pi y^2 dx = \pi \int_a^b y^2 dx$, π الثابت

B: If the move around y axis is the volum:

$$V = \int_a^b \pi x^2 dy = \pi \int_a^b x^2 dy$$

When the lines $x = a$, $x = b$

Exa: 1 Find V if $y = \sqrt{x}$ on X, when $x = 0$, $x = 3$?

So: $V = \pi \int_a^b y^2 dx$

$$V = \pi \int_0^3 (\sqrt{x})^2 dx = \pi \int_0^3 x\, dx$$

$$V = \pi \left[\frac{x^2}{2} \right]_0^3 = \pi \left[\frac{3^2}{2} - 0 \right] = \frac{9\pi}{2} \text{ unit}^3$$

الزاوية	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π
	0°	30°	45°	60°	90°	180°	270°	360°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	—	0	—	0

(39)

Exa: 2 Find V if $y = x^3$ on y axis when $y = 3$?

So: we saw $a = 0, b = 3$

$$V = \pi \int_a^b x^3 \, dy$$

$\because y = x^3$, $\therefore x = \sqrt[3]{y} \Rightarrow x = y^{\frac{1}{3}}$

$$V = \pi \int_0^3 (y^{\frac{1}{3}})^2 \, dy$$

$$= \pi \int_0^3 y^{\frac{2}{3}} \, dy = \pi \left[\frac{3}{5} y^{\frac{5}{3}} \right]_0^3$$

$$= \pi \left[\frac{3}{5} (3)^{\frac{5}{3}} - 0 \right]$$

$$= \pi \left[\frac{3}{5} (\sqrt[3]{3})^5 \right] = \pi \left[\frac{3}{5} (\sqrt[3]{3^5}) \right]$$

$$= \pi \left(\frac{3}{5} (3) (\sqrt[3]{9}) \right)$$

$$= \frac{9\pi}{5} \sqrt[3]{9} \text{ unit}^3$$

2- Find the volum between two curves:
ايجاد الحجم بين المنحنيين.

A: If around x axis use:

$$V = \pi \int_a^b \left[(F(x))^2 - (g(x))^2 \right] dx$$

$y_1 = F(x)$, $y_2 = g(x)$

B: If around y axis use:

$$V = \pi \int_a^b \left[(F(x))^2 - (g(x))^2 \right] dy$$

Exa: 1 Find V between $y = 2x$, $y = x^2$ on x axis?

So: $x^2 = 2x \Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0$ (بمساواة المعادلتين)

$\Rightarrow x = 0$, $x = 2$

$a = 0$, $b = 2$

$$V = \pi \int_a^b \left[(F(x))^2 - (g(x))^2 \right] dx$$

$$V = \pi \int_0^2 \left[(2x)^2 - (x^2)^2 \right] dx$$

41

$$= \pi \int_0^2 (4x^2 - x^4)\, dx$$

$$= \pi \left[\frac{4x^3}{3} - \frac{x^5}{5} \right]_0^2 = \pi \left[\frac{4(2^3)}{3} - \frac{2^5}{5} \right] - 0$$

$$= \frac{32}{3} - \frac{32}{5} = \frac{64\pi}{15} \text{ unit}^3$$

Exercises:

1- Find V between $y = 4x$, $y = 2x^2$ on x axis?

2- Find V if $y^2 = 8x$ on x, when $x = 3$?

3- Find V between $y = \sin x$, $y = \cos x$, when points $[0, \frac{\pi}{4}]$ on X ?

4- Find V when $y = x$, $y = x^2$ on x ?

*

الرياضيات

The Nearly Integral التكامل التقريبي (العددي)

Which is the second mathed to find the definite integral.

There are two mathedes to find it:

first (Trapezoidal); and second (prismoidal)

شبه المنحرف — شبه المنشور (سيمسون)

Integration (التكامل) take $\int_a^b f(x)\,dx$; and the area (المساحة)

divided $[a, b]$ to equal (متساوية) parts (اقسام) long (الطول) from n

$$\therefore h = \frac{b-a}{n}.$$

طريقة شبه المنحرف

1- Trapezoidal mathed:

$$\int_a^b f(x)\,dx \cong \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + 2y_3 + \cdots + y_n\right]$$

2- prismoidal (Semson) mathed: سيمسون (شبه المنشور)

$$\int_a^b f(x)\,dx \cong \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + y_n\right]$$

43

Ex 1: Find the value of nearly Integral by the Trapezoidal and prismoidal methods if $\frac{dy}{dx} = \int_0^2 x^4 dx$? take $n = 4$.

So: $h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$

$\because \frac{dy}{dx} = x^4 dx \Rightarrow y = x^4$

n	X	$y = x^4$
0	$x_0 = 0$	$y_0 = 0$
1	$x_1 = 0.5$	$y_1 = 0.0625$
2	$x_2 = 1$	$y_2 = 1$
3	$x_3 = 1.5$	$y_3 = 5.0625$
4	$x_4 = 2$	$y_4 = 16$

1- by Trapezoidal:

$$\int_0^2 f(x)dx \simeq \frac{h}{2}\left[(y_0)^4 + 2(y_1)^4 + 2(y_2)^4 + 2(y_3)^4 + (y_4)^4 \right]$$

$$\int_0^2 \simeq \frac{0.5}{2}\left[0 + 2(0.0625) + 2(1) + 2(5.0625) + 16\right]$$

$$\simeq \frac{0.5}{2}\left[0 + 0.125 + 2 + 10.125 + 16\right]$$

$$\simeq \frac{0.5}{2}\left[28.25\right) \simeq 7.0625$$

(Simpson) قاعدة سمبسون

2- by prismoidal:

$$\int_a^b f(x)\,dx \cong \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$

$$\int_0^2 x^4\,dx \cong \frac{0.5}{3}\left[0 + 4(0.0625) + 2(1) + 4(5.0625) + 16\right]$$

$$\cong \frac{0.5}{3}\left[0 + 0.25 + 2 + 20.25 + 16\right]$$

$$\cong \frac{0.5}{3}(38.5) \cong 6.416$$

Exa 2 : If $\int_0^t \frac{1}{(x^2+1)}\,dx = \frac{t}{4}$ Find t by the trapzoidal and prismoidal method? take $n = 4$?

So: take $n = 4 \longrightarrow h = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4} = 0.25$

n	X	$y = X^2$
0	0	$\frac{1}{0+1} = 1$ ⟷ y_0
1	$\frac{1}{4}$	$\frac{1}{\frac{1}{16}+1} = 0.941$ ⟷ y_1
2	$\frac{1}{2}$	$\frac{1}{\frac{1}{4}+1} = 0.800$ ⟷ y_2
3	$\frac{3}{4}$	$\frac{1}{\frac{9}{16}+1} = 0.640$ ⟷ y_3
4	1	$\frac{1}{141} = 0.500$ ⟷ y_4

4.5

We See : $t = 4\int_0^1 \frac{1}{x^2+1}$ (من ضرب الوسطين X الطرفين)

1- by Trapezoidal : يترك الحل للطالب !!!

النتيجة هي (3.131)

2- by prismoidal :

$$t = \cong \frac{4}{12}\left[1 + 4(0.941) + 2(0.800) + 4(0.640) + 0.500\right]$$

$$t = \cong 3.141$$

توضيح : تم الحصول على $\frac{4}{12}$ من خلال $4(\frac{1}{3} * \frac{1}{4})$ ومن العلاقة $4(\frac{h}{3})$

علماً بأن قيمة h هي $\frac{1}{4}$ أو (0.25)

يمثل العدد القياسي (k)

Exercise :

Find the Inegral by Trapezoidal and prismoidal matheds :-

1- $\int_0^1 (2x+1)\,dx$; take $n = 3$

2- $\int_0^1 \frac{dx}{1+x}$; take $n = 6$

3- $\int_1^7 \frac{x^3+3}{x+1}\,dx$; take $n = 4$

الرياضيات

The Differentail Equations : المعادلات التفاضلية

1- The order of D.E : Which is high function in the equation.

هي رتبة اعلى مشتقة تظهر في المعادلة.

Exa: 1- $y'' + 3y = x^2$; second order الرتبة الثانية

2- $(3x+2y)^2 \bar{y} = 1$; frist order ، الأولى

3- $(1+2y^2)^3 = 8\bar{\bar{y}}$; third order ، الثالثة

2- The digree of D.E : Which is high exponential (اس) of function in the equation.

هي أكبر قوة (اس) مرفوع لها أعلى مشتقة في المعادلة التفاضلية وبعد إزالة الجذور والاسس الكسرية أن وجدت.

Exa: 1- $(\bar{y})^2 + (\bar{y})^3 = x^2$; second order, second digree

الرتبة الثانية ، الدرجة الثانية.

2- $\bar{y} = 5 + x$ frist order ; first digree

الرتبة الأولى ، الدرجة الأولى

The mothedes of solution the D.E:-

1- Separable First-order Equations:

المعادلات من الرتبة الأولى مفصولة المتغيرات.

This a simple kind of the differenntial Equations replac with:

$$g(x)\,dx + h(y)\,dy = 0$$

Exa: 1: Find the solution to this differential equation?

$$x^2(1-y^2)\,dx + y(1+x^2)\,dy = 0$$

نقسم

So: divid equation on $(1-y^2)(1+x^2)$

$$\frac{x^2\cancel{(1-y^2)}\,dx}{\cancel{(1-y^2)}(1+x^2)} + \frac{y\cancel{(1+x^2)}\,dy}{(1-y^2)\cancel{(1+x^2)}} = 0$$

$$\frac{x^2\,dx}{1+x^2} + \frac{y\,dy}{1-y^2} = 0$$

then: $$\int\left(1-\frac{1}{1+x^2}\right)dx - \frac{1}{2}\int\frac{-2y\,dy}{1-y^2} = 0$$

حيث تم أرجاع الحد الأول الى أصله وتم ضرب الحد الثاني $-\frac{2}{2}$ لأجل ايجاد مشتقة المقام في البسط.

للحصول على صيغة تكامل غير محدود.

$$x - \tan^{-1}x - \frac{1}{2}\ln|1-y^2| = c$$

هذا هو الحل العام للمعادلة التفاضلية.

Notes: ملاحظات

1- $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + c \Rightarrow \int \frac{dx}{\sqrt{1+x^2}} = \cos^{-1}x + c$

2- $\int \frac{dx}{1+x^2} = \tan^{-1}x + c \Rightarrow \int \frac{dx}{1-x^2} = \cot^{-1}x + c$

3- $\int \frac{dx}{\sqrt{1+x^2}} = \sinh^{-1}x + c$

4- $\int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1}x + c$

5- $\int \frac{dx}{\sqrt{1-x^2}} = \tanh^{-1}x + c$

غير مطلوبة

Exa: 2 Find the solution to this differential equation:

$$xy\,dy + (2yx^2 + 4x^2 - y - 2)\,dx = 0$$

So: from $(2yx^2 + 4x^2 - y - 2)$ تم ارجاعه الى اصله

Find $xy\,dy + (2x^2 - 1)(y+2)\,dx = 0$

divid equation on $(x(y+2)$:

$$\frac{xy\,dy}{x(y+2)} + \frac{(2x^2-1)\cancel{(y+2)}\,dx}{x\cancel{(y+2)}} = 0$$

$$\int \frac{y\,dy}{y+2} + \int \frac{2x^2-1}{x}\,dx = 0$$

$$\int \left(1 - \frac{2}{y+2}\right)dy + \int \left(2x - \frac{1}{x}\right)dx = 0$$

تم ارجاع الحد الاول والحد الثاني الى اصلهما لأجل الحصول على مقدار قابل للتكامل الغير محدود.

$$y - 2\ln|y+2| + (x^2 - \ln|x| = c$$

Exercises: Find the solution to these D.E:

1- $x(1-y)^2\,dx + y(1+x^2)\,dy = 0$

2- $x\,dy + (1+x)y\,dx = 0$

3- $(x^2+1)(y^2-1)\,dx + xy\,dy = 0$

2- Hemogeneity Equations : المعادلات المتجانسة

Which is exponential equl to them.
هي المعادلة التي تكون فيها اسس كل حدودها متساوية.

Such as: $k^4[x^4 - x^3y] = k^4 f(x,y)$

$(x^2+y^2)dy + (xy)dx = 0$ متجانسة

$(x^2+y)dy + xy\,dy = 0$ غير متجانسة

* Steps of Solution : خطوات الحل:

1- Replace $y = vx$ then $dy = v\,dx + x\,dv$ in equtien
نعوض بدل كل $y = vx$ ، عن كل $dy = v\,dx + x\,dv$ في المعادلة المتجانسة.

2- Separatbe dx , dv. فصل المتغيرات

3- Find the indefinite Intergral. إيجاد التكامل الغير محدود

4- Replace $v = \frac{y}{x}$ بدل $v = \frac{y}{x}$ نعوضها.

Exa: 1 Find the solution to this D.E.

$$(y^2 - x^2)\,dx - 2xy\,dy = 0$$

So: $y = vx \Rightarrow [dy = v\,dx + x\,dv]$ بعد اشتقاق $y = vx$

Replace to y, dy. نعوض

$$(v^2x^2 - x^2)\,dx - 2x^2(v)(v\,dx + x\,dv) = 0$$

divid on x^2 : $(v^2 - 1)\,dx - 2v(v\,dx + x\,dv) = 0$ نقسم على

$$\Rightarrow (v^2-1)\,dx - 2v^2\,dx - 2vx\,dv = 0$$

$$\Rightarrow (v^2 - 1 - 2v^2)\,dx - 2vx\,dv = 0$$

$$\Rightarrow (-v^2 - 1)\,dx - 2vx\,dv = 0$$

$* -1 \Rightarrow (v^2 + 1)\,dx + 2vx\,dv = 0$ نضرب

divid on $x(1+v^2)$: ← لنفصل المتغيرات

$$\frac{(v^2+1)\,dx}{x(v^2+1)} + \frac{2vx\,dv}{x(v^2+1)} = 0 \Rightarrow \int \frac{dx}{x} + \int \frac{2v\,dv}{1+v^2} = 0$$

$$\ln|x| + \ln|1+v^2| = \ln|c|$$

$$\Rightarrow x(1+v^2) = c$$ (حسب قاعدة قوانين اللوغاريتمية)

$\Rightarrow v = \frac{y}{x}$ بالتعويض في المعادلة الأولى

$$\Rightarrow x\left(1 + \frac{y^2}{x^2}\right) = c \Rightarrow \left(x + \frac{xy^2}{x^2}\right) = c \Rightarrow \left(x + \frac{y^2}{x}\right) = c$$

$* x \Rightarrow x^2 + y^2 = cx$ نضرب ← بعد التبسيط $\times x \Rightarrow \left(x^2 + \frac{xy^2}{x}\right) = cx$

Exercises Find the solution for D.E:

1- $(y^2+x^2)dx + xy\,dy = 0$

2- $(y^2-2xy)dx = (x^2-2xy)dy$

3- $x\,dy + (x+y)dx = 0$

4- $(2y^2-3xy+x^2)dx + x(x-2y)dy = 0$

5- $x\,dx + y\,dy = \sqrt{x^2+y^2}\,dy$

3 – Total Diff. Equations :

المعادلات التفاضلية التامة

Which is the Diff. Equation if:

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

إن الشرط الكافي لكي تكون المعادلة التفاضلية تامة أن تكون بالصيغة أعلاه.

Exa: 1 : Find the So. to this E.D?

$$\underbrace{(3x^2 + 3xy^2)}_{M}dx + \underbrace{(3x^2y + 3y^2 + 2y)}_{N}dy = 0$$

So: $M = (3x^2 + 3xy^2)$; $N = (3x^2y - 3y^2 + 2y)$

$$\frac{\partial M}{\partial y} = 6xy$$ تفاضل جزئي أو ضمني لـ y

$$\frac{\partial N}{\partial x} = 6xy$$ تفاضل جزئي أو ضمني لـ x

Then to $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ تساوي التفاضل العظمي للدالتين

Then: $F(x,y) = \int_0^x (3t^2 + 3ty^2)\,dt + \int_0^y (0 + 3t^2 + 2t)\,dt$

تم أخذ النقطة (0,0)

وبالتعويض عن كل x بـ t في الحد الأول، وكذلك عن كل y في t في الحد الثاني

بعد إجراء التكامل الغير محدود $\Rightarrow$ $t^3 + \frac{3}{2}t^2y^2 + t^3 + t^2 = C$

وبالتعويض عن كل t بـ x و y في المعادلة وبعد إجراء التكامل فنحصل الحل العام للمعادلة التفاضلية بـ

نعم التعويض بـ x و y عن t

$$x^3 + \frac{3}{2}x^2y^2 + y^3 + y^2 = c$$

ولتعيين تحقيق صحة هذا الجواب بحساب التفاضل التام للمعادلة التفاضلية نحصل على الاصلية

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

$$df = (3x^2 + 3xy^2)dx + (3x^2y - 3y^2 + 2y)dy = 0$$

Exa2 Find the So. to E.D ?

$$\underbrace{(2xy^2 - 2y)}_{M}dx + \underbrace{(3x^2y - 4x)}_{N}dy = 0$$

So:

نقارنها مع $Mdx + Ndy$ نستنتج :

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} =$$

ومن أجل الحصول على المعادلة التفاضلية التامة نضرب طرفي المعادلة في y لتصبح معادلة تامة وهنا y يسمى بالعامل المكامل .

$$(2xy^3 - 2y^2)dx + (3x^2y^2 - 4xy)dy = 0$$

حيث أن

$$\frac{\partial N}{\partial x} = 6xy^2 \quad , \quad \frac{\partial M}{\partial y} = 6xy^2$$

وبالتعويض لـ t عن y و x وأخذ النقطة (0 6 0) فنحصل على

$$f(x,y) = \int_0^x (2ty^3 - 2y^2)\,dt + \int_0^y (3(0)^2t^2 - 4(0)t)\,dt$$

$$= t^2y^3 - 2y^2 + 0 = c$$

بعد اجراء التكامل لـ t فقط

وبالتعويض x و y عن t نحصل على الحل للمعادلة : $x^2y^3 - 2y^2 = c$

4- Linear differential E. : المعادلات التفاضلية الخطية

الصيغة العامة للمعادلة الخطية من الرتبة الأولى هي:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

حيث أن $P(x)$ و $Q(x)$ دوال للمتغير x.

ولكي نحصل على حل المعادلة التفاضلية العامة نضرب طرفيها في عامل التكامل فتتحول إلى معادلة تامة.

Exa: 1 Find the So. of this E.D ?

$$\frac{dy}{dx} + \frac{1}{x}y = x^2 - 3$$

So:

نلاحظ أن المعادلة هي خطية ومعامل التكامل فيها هو

$$e^{\int \frac{1}{x}dx} = e^{\ln x} = x$$

نضرب المعادلة في x

$$xy = \int x(x^2-3)dx = \int (x^3 - 3x)dx$$

فيكون الحل هو

$$xy = \frac{1}{4}x^4 - \frac{3}{2}x^2 + c$$

Exa: 2 Find So. of E.D ?

$$y\frac{dx}{dy} + 2x = y^3$$

So:

نقسم الطرفين على y لكي نحصل على المعادلة الخطية على الصيغة العامة $\frac{dy}{dx} + P(x)y = Q(x)$

$$\frac{dx}{dy} + \frac{2}{y}x = y^2$$

وهذه المعادلة خطية متغيرها التابع x ومعامل التكامل هو

$$e^{\int (\frac{2}{y})dy} = e^{2\ln y} = y^2$$

فيكون الحل بعد ضربه بـ y^2 $\Leftarrow$ $xy^2 = \int y^4 dy = \frac{y^5}{5} + c$ وبضرب طرفي (5) نحصل على:

$$5xy^2 = y^5 + c$$